Instructions

This exam contains 14 pages, including this one. Make sure you have all the pages. Write your name, directory ID, and university ID number on the top of this page, and write your directory ID at the bottom left of every page, before starting the exam.

Write your answers on the exam sheets. If you finish at least 15 minutes early, bring your exam to the front when you are finished; otherwise, wait until the end of the exam to turn it in. Please be as quiet as possible.

If you have a question, raise your hand. If you feel an exam question assumes something that is not written, write it down on your exam sheet. Barring some unforeseen error on the exam, however, you shouldn’t need to do this at all, so be careful when making assumptions.

<table>
<thead>
<tr>
<th>Question</th>
<th>Score</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>20</td>
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<tr>
<td>2</td>
<td></td>
<td>20</td>
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<td>3</td>
<td></td>
<td>20</td>
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<tr>
<td>4</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Question 1. Short Answer (20 points).

a. (3 points) Briefly describe the difference between a function call and a system call. Explain whether the two have the same calling convention and why or why not.

Answer: Function calls transfer control from one function to another, within the program’s own code or between the code and a library. System calls are used when the program needs the operating system kernel to perform some computation or action on its behalf, with privilege.

The conventions for function calls and system calls are typically different. One is defined by the language or compiler for interoperability of program modules. The other is defined by the operating system and must be followed by all programs, regardless of implementation language.

b. (3 points) Briefly describe two code generation techniques for compiling C switch statements. Explain how the two techniques compare, i.e., in what situation(s) one technique should be favored over the other.

Answer:

- Cascaded if-then-else. Check each option in succession.
- Binary search. Order the guards and then use a search algorithm.
- Jump table. Use the guard as an index into code or a set of code pointers.

Jump tables are most effective when the guards are “dense”, i.e., when they are numerically close. Jump tables are more time efficient, since they require only a single constant-time operation to compute the code location. However, they can be inefficient for space if the guards are spread out. Other answers that describe memory or time efficiency, or ease of implementation, are also acceptable.

c. (3 points) Briefly describe symbolic execution and name one application of the technique. What is the fundamental challenge that prevents exhaustive symbolic execution for most real-world programs?

Answer: Symbolic execution is a program analysis technique that treats program inputs as symbolic variables, rather than concrete values. Program expressions are computed in terms of the symbolic variables and execution is logically forked at branch points to represent all possible paths. Symbolic execution can simultaneously explore multiple paths that a program could take under different inputs, without testing every possible input. It is a popular technique with testing and bug finding tools.

The primary challenge is path/state space explosion. Real programs have an exponential number of paths, making it impossible to symbolically execute every path. Therefore, approximation, path pruning, and search strategies have become a critical component of real symbolic execution systems.
d. (5 points) The following C program contains two functions, `bar()` and `foo()`. `bar()` calls `foo()`. Draw a representation of a 32-bit x86 stack just after the assignment on line 9 is executed. Include as much stack history as you can infer from the code snippet. Clearly indicate the current “top” of the stack (`esp`), the current frame pointer (`ebp`), and which direction the stack grows. You can assume the code is compiled without any optimizations and that `main()` calls `bar()`. If you make any other assumptions, state those as well.

```c
1 int bar(int x, int y) {
2     int z;
3     z = foo(x + 10, y - 10);
4     return z;
5 }
6
7 int foo(int a, int b) {
8     int c = 100;
9     int d = a * b + c;
10    // ...
11    return d;
12 }
```

<table>
<thead>
<tr>
<th>Registers</th>
<th>Stack</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>higher addresses go up</td>
<td>[\ldots]</td>
<td>main's local variables</td>
</tr>
<tr>
<td>[\ldots]</td>
<td></td>
<td>main's activation record</td>
</tr>
<tr>
<td>[y]</td>
<td></td>
<td>arguments to bar, pushed in reverse order</td>
</tr>
<tr>
<td>[x]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>main's return address</td>
<td>bar's activation record</td>
<td></td>
</tr>
<tr>
<td>main's saved ebp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>bar's local variables</td>
<td></td>
</tr>
<tr>
<td>[\ldots]</td>
<td>register save space</td>
<td></td>
</tr>
<tr>
<td>[b = y - 10]</td>
<td>arguments to foo, pushed in reverse order</td>
<td></td>
</tr>
<tr>
<td>[a = x + 10]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bar's return address</td>
<td>foo's activation record</td>
<td></td>
</tr>
<tr>
<td>bar's saved ebp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c = 100</td>
<td>foo's local variables</td>
<td></td>
</tr>
<tr>
<td>d = a * b + c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>stack grows down</td>
<td>esp could also point lower</td>
<td></td>
</tr>
<tr>
<td></td>
<td>if space was pre-allocated</td>
<td></td>
</tr>
</tbody>
</table>

Answer:
e. **(6 points)** Translate the following program to 3-address code. Draw the control-flow graph for the resulting program.

```plaintext
x = 10
y = 3
z = x + (y * 10) + 100
while ((z > y) && (z > x)) {
    if (a > 10) {
        z = z - 100
    } else {
        z = z - x * 2
    }
}
```

![Control-flow graph](image-url)
Question 2. Code Generation (20 points). Below (left) is a set of types representing a small machine
instruction set, followed by (right) a type representing expression ASTs for a small language.

| type instr = |
| ILoad of reg * val (* dst, src *) |
| IStore of id * reg (* dst, src *) |
| IAdd of reg * reg * reg (* dst, src1, src2 *) |
| ISub of reg * reg * reg (* dst, src1, src2 *) |
| IIfZero of reg * int (* guard, target *) |
| IImp of int (* target *) |
| IMov of reg * reg (* dst, src *) |

| type expr = |
| EInt of int (+ integers *) |
| EId of string (+ variables *) |
| EAdd of expr * expr (+ addition *) |
| ESub of expr * expr (+ subtraction *) |
| EAssn of string * expr (+ assignment *) |
| ESeq of expr * expr (+ sequences *) |
| EWhile of expr * expr (+ while loops *) |
| EDoWhile of expr * expr (+ do-while loops *) |

The instruction set has direct support for named locations (i.e., variables), which can be read from or written
to via the ILoad and IStore instructions, respectively. ILoad also supports loading a register with a constant
integer. IAdd and ISub implement register addition and subtraction, respectively. In both cases, the result
is stored in a register. The IImp (absolute jump) and IIfZero (conditional jump) instructions adjust the PC
relative to the current instruction’s PC. IMov copies the value stored in one register to another. The machine
supports an unlimited number of registers.

The expressions EInt and EId represent constant integers and variables, respectively. EAdd and ESub are the
standard binary addition and subtraction expressions. EAssn represents an assignment to a variable. ESeq is
the sequence of two expressions. The expression EWhile(e1, e2) executes the body e2 as long as the guard e1
is not zero, and the whole expression evaluates to 0. EDoWhile(e1, e2) executes e1 until e2 becomes non-zero.
Note that a do-while loop always executes the body e1 at least once (so it evaluates e1; checks if e2 is
non-zero; if not evaluates e1 again; etc). The loop itself should evaluate to 0.

Question is on the next page.
Write a function \( \text{comp_expr} : \text{expr} \rightarrow \text{reg} \times (\text{instr list}) \) that takes a single expression and returns the output register and a list of instructions that compute the expression. You may define as many helper functions as you need, and you may also use OCaml standard library functions.

```ocaml
let next_reg =
let n = ref 0 in
  fun () -> (let temp = !n in n:=!n+1; temp)

let rec comp_expr = function
| EInt n -> let r = next_reg () in
  (r, [ILoad ('Reg r, 'Int n)])
| EAdd (e1, e2) ->
  let (r1, p1) = comp_expr e1 in
  let (r2, p2) = comp_expr e2 in
  let r = next_reg () in
  (r, p1 @ p2 @ [IAdd ('Reg r, 'Reg r1, 'Reg r2)])
| ESub (e1, e2) ->
  let (r1, p1) = comp_expr e1 in
  let (r2, p2) = comp_expr e2 in
  let r = next_reg () in
  (r, p1 @ p2 @ [ISub ('Reg r, 'Reg r1, 'Reg r2)])
| EId x ->
  let r = next_reg () in
  (r, [ILoad ('Reg r, 'Id x)])
| EAssn (x, e) ->
  let (r, p) = comp_expr e in
  (r, p @ [IStore ('Id x, 'Reg r)])
| ESeq (e1, e2) ->
  let (r1, p1) = comp_expr e1 in
  let (r2, p2) = comp_expr e2 in
  (r2, p1 @ p2)
| EWhile (e1, e2) ->
  let (r1, ccode) = comp_expr e1 in
  let (r2, ecode) = comp_expr e2 in
  let r3 = next_reg () in
  let elength = List.length ecode in
  let clength = List.length ccode in
  let guard = [IIfZero ('Reg r1, elength+1)] in
  let loopback = [IJmp (−1 * (elength + clength + 2))] in
  let return_zero = [ILoad('Reg r3, 'Int 0)] in
  (r3, List.concat [ccode;guard;ecode;loopback;return_zero])
| EDoWhile (e1, e2) ->
  let (r1, ccode) = comp_expr e1 in
  let (r2, ecode) = comp_expr e2 in
  let r3 = next_reg () in
  let elength = List.length ecode in
  let clength = List.length ccode in
  let guard = [IIfZero ('Reg r2, 1)] in
  let loopback = [IJmp (-1 * (elength + clength + 2))] in
  let returnzero = [ILoad('Reg r3, 'Int 0)] in
  (r3, List.concat [ecode;ccode;guard;loopback;returnzero])
```

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CMSC430, Fall 2018, Midterm 2
Question 3. Data Flow (20 points).

In the following table, show each iteration of *reaching definitions* for the control-flow graph on the right. For each iteration, list the statement taken from the worklist in that step, the value of *out* computed for that statement, and the new worklist at the end of the iteration. You may or may not need all the iterations; you may also add more iterations if needed. Do not add the entry node to the worklist.

Use $\emptyset$ for the set of no definitions, and $\top$ for the set of all definitions.

What is $\top$?

\[
\top = 1(a), 2(b), 4(a), 6(b), 7(a), 8(a)
\]

What are the initial *out’s* for each statement?

<table>
<thead>
<tr>
<th>Stmt</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial out</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Iteration</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stmt taken from worklist</td>
<td>N/A</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>out of taken stmt</td>
<td>N/A</td>
<td>1(a)</td>
<td>1(a),2(b)</td>
<td>1(a),2(b)</td>
<td>2(b),4(a)</td>
</tr>
<tr>
<td>New worklist</td>
<td>1,2,3,4,5,6,7,8</td>
<td>2,3,4,5,6,7,8</td>
<td>3,4,5,6,7,8</td>
<td>4,5,6,7,8</td>
<td>5,6,7,8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Iteration</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stmt taken from worklist</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>out of taken stmt</td>
<td>2(b),4(a)</td>
<td>4(a),6(b)</td>
<td>1(a),2(b),4(a),6(b)</td>
<td>2(b),4(a),6(b)</td>
<td>2(b),4(a),6(b)</td>
</tr>
<tr>
<td>New worklist</td>
<td>6,7,8</td>
<td>3,7,8</td>
<td>4,7,8</td>
<td>5,7,8</td>
<td>6,7,8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Iteration</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stmt taken from worklist</td>
<td>6</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>out of taken stmt</td>
<td>4(a),6(b)</td>
<td>2(b),6(b),7(a)</td>
<td>1,2,4,6,7</td>
<td>2(b),4(a),6(b)</td>
<td>2(b),6(b),8(a)</td>
</tr>
<tr>
<td>New worklist</td>
<td>7,8</td>
<td>3,8</td>
<td>4,8</td>
<td>8</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>
Question 4. Optimization (20 points). Below are (left) a Simpl function that I wrote to test my project and (right) the RubeVM code that my compiler generated.

```
def foo(x):
t = mktab();
i = 0;
while (i < x) do
  a = x + 10;
  if a < 5 then
    t[i] = a + x
  else
    t[i] = a
  end;
i = i + 1
end
```

```
foo:
  mov r7, r0
  const r10, mktab
  call r10, 9, 8
  const r11, 0
  lt r12, r11, r7
  if_zero r12, 15
  const r14, 10
  add r13, r7, r14
  const r17, 5
  lt r16, r13, r17
  if_zero r16, 4
  add r18, r13, r7
  wr_tab r9, r11, r18
  mov r15, r18
  jmp 2
  wr_tab r9, r11, r13
  mov r15, r13
  const r20, 1
  add r19, r11, r20
  mov r11, r19
  jmp −17
  const r21, 0
  ret r21
```

In the space below, identify two (2) local and two (2) global optimizations that could be applied to this code. Clearly identify the type of optimization, the code location, and how the code would change. Hint: you may want to convert the RubeVM code to a more familiar three-address form and draw a control-flow graph first.
a. (5 points) Local optimization 1

**Answer:** Using the hint, we converted the RubeVM code into a CFG (see previous page). Now we can easily see statements 13-15 can leverage copy propagation – we don’t need to use \( r_{18} \) at all. The block becomes:

<table>
<thead>
<tr>
<th>Three-address</th>
<th>RubeVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>13. ( r_{15} = r_{13} + 7 )</td>
<td>13. add ( r_{15}, r_{13}, r_{7} )</td>
</tr>
<tr>
<td>14. ( r_{9}[r_{11}] = r_{15} )</td>
<td>14. wr tab ( r_{9}, r_{11}, r_{15} )</td>
</tr>
<tr>
<td>15. &lt;removed&gt;</td>
<td>15. &lt;removed&gt;</td>
</tr>
</tbody>
</table>

b. (5 points) Local optimization 2

**Answer:** Copy propagation, like above, but to remove \( r_{19} \). The block becomes:

<table>
<thead>
<tr>
<th>Three-address</th>
<th>RubeVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>19. ( r_{20} = 1 )</td>
<td>19 const ( r_{20}, 1 )</td>
</tr>
<tr>
<td>20. ( r_{11} = r_{11} + r_{20} )</td>
<td>20 add ( r_{11}, r_{11}, r_{20} )</td>
</tr>
<tr>
<td>21. &lt;removed&gt;</td>
<td>21 &lt;removed&gt;</td>
</tr>
</tbody>
</table>

c. (5 points) Global optimization 1

**Answer:** Loop invariant code motion. The assignment to variable \( a \) (\( r_{17} \)) does not depend on any modifications inside the loop, so it can be hoisted above the loop (e.g., just after statement 5) to save cycles during the loop. Similarly, \( a + x \) (\( r_{18} \)) is loop invariant and could be computed once, before the loop.

d. (5 points) Global optimization 2

**Answer:** Code specialization. There is a condition inside the loop (\( a < 5 \), \( \text{lt \( r_{16}, r_{13}, r_{17} \)} \)) that does not vary depending on the loop. We can invert the loop and conditional to make one loop for the true case and one for the false, which will run faster during the loop’s execution. Alternatively, rather than specialize the loop itself, a more complicated optimization could be performed following the code motion optimization described in part c. Since both branches of the conditional assign to the same table location \( t[i] \), the conditional could be used to compute the correct right-hand side (\( a \) or \( a + x \)) and put the result in the same register. Then, the loop can just iterate through assigning to a constant value. Logically, the same as this:

```python
def foo(x):
    t = mktab();
    i = 0;
    if a < 5 then
        temp = a + x
    else
        temp = a
    end;
    while (i < x) do
        t[i] = temp
        i = i + 1
    end
end```
**Question 5. Type Systems (20 points).** Here is the simply typed lambda calculus with integers, floats, and pairs.

\[
e ::= v \mid x \mid e \mid (e, e) \quad v ::= n \mid f \mid \lambda x : t.e \quad t ::= \text{int} \mid \text{float} \mid t \rightarrow t \mid t \times t \quad A ::= \cdot \mid x : t, A
\]

\[
\begin{array}{c|c|c|c}
\text{Int} & \text{Float} & \text{Var} & \text{Pair} \\
\hline
A \vdash n : \text{int} & A \vdash f : \text{float} & A \vdash x : A(x) & A \vdash (e_1, e_2) : t \times t' \\
\hline
\hline
\text{Lam} & \text{App} & \text{Lam} & \text{App} \\
\hline
x : t, A \vdash e : t' & A \vdash e_1 : t \rightarrow t' & A \vdash \lambda x : t.e : A \rightarrow t & A \vdash e_1 \ e_2 : t' \\
\hline
\end{array}
\]

**a. (5 points)** Draw a derivation showing that the following term is well-typed in the given type environment, where we use \(i\) instead of \(\text{int}\) and \(f\) instead of \(\text{float}\) to save space. You need not label the uses of the rules with their names.

\(A = + : i \rightarrow i \rightarrow i, \oplus : f \rightarrow f \rightarrow f\)

\(A \vdash ((\lambda x : i.\lambda y : f. (+ x 5, \oplus y 5.0)) \ 7 \ 7.0) : i \times f\)

**Answer:**

\[
A = + : i \rightarrow i \rightarrow i, \oplus : f \rightarrow f \rightarrow f \\
B = x : i, A \\
C = y : f, x : i, A
\]

\[
\begin{array}{c|c|c|c|c|c}
C \vdash + : i \rightarrow i \rightarrow i & C \vdash x : i & C \vdash + : f \rightarrow f \rightarrow f & C \vdash \oplus : f \rightarrow f \rightarrow f & C \vdash y : f & C \vdash 5.0 : f \\
\hline
C \vdash (+ x) : i \rightarrow i & C \vdash 5 : i & C \vdash (\oplus y) : f \rightarrow f & C \vdash (\oplus y 5.0) : f & C \vdash (\oplus y 5.0) : f & C \vdash (+ x 5) : i \\
\hline
C \vdash (+ x 5) : i \\
\hline
B \vdash \lambda y : f. (+ x 5, \oplus y 5.0) : f \rightarrow i \times f \\
\hline
A \vdash (\lambda x : i.\lambda y : f. (+ x 5, \oplus y 5.0)) : i \rightarrow f \rightarrow i \times f & A \vdash 7 : i & A \vdash 7.0 : f \\
\hline
A \vdash ((\lambda x : i.\lambda y : f. (+ x 5, \oplus y 5.0)) \ 7) : f \rightarrow i \times f & A \vdash ((\lambda x : i.\lambda y : f. (+ x 5, \oplus y 5.0)) \ 7 \ 7.0) : i \times f
\end{array}
\]
b. **(5 points)** If we make `int` a subtype of `float`, we get the following additional type rules (note that application has been updated):

\[
\begin{align*}
\text{S-NUM} & \quad \frac{t_1 \leq t'_1 \quad t_2 \leq t'_2}{\text{int} \leq \text{float}} \\
\text{S-PROD} & \quad \frac{t_1 \times t_2 \leq t'_1 \times t'_2}{\text{int} \leq \text{float}} \\
\text{APP} & \quad \frac{A \vdash e_1 : t_1 \rightarrow t'_1 \\ A \vdash e_2 : t_2}{A \vdash e_1, e_2 : t'_1} \\
\text{S-ARROW} & \quad \frac{\text{int} \leq \text{float} \quad t_1 \rightarrow t'_1 \leq t_2 \rightarrow t'_2}{t_1 \rightarrow t'_1 \leq t_2 \rightarrow t'_2}
\end{align*}
\]

i. **(2 points)** The rule S-ARROW, which extends the subtyping relationship to arrow types (functions), is incomplete. Fill in the rest of the rule.

**Answer:**

\[
\begin{align*}
\text{S-ARROW} & \quad \frac{t_2 \leq t_1 \quad t'_1 \leq t'_2}{t_1 \rightarrow t'_1 \leq t_2 \rightarrow t'_2}
\end{align*}
\]

ii. **(3 points)** Given your rule, can `(+: \text{int} \rightarrow \text{int} \rightarrow \text{int})` be used in computations where `(⊕: \text{float} \rightarrow \text{float} \rightarrow \text{float})` is expected? Explain why or why not. If not, give an example of a type that can be used where `⊕` is expected.

**Answer:** No. The `⊕` function requires a `float` input. The `+` function requires an `int`. The subtype rule for arrow requires that `float \leq int`, which is not true by the other type rules. `float \rightarrow float \rightarrow int` is the only other type of function that can be used in place of `float \rightarrow float \rightarrow float`. 

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c. (10 points) Consider type inference for the simply-typed lambda calculus, this time without the float, pair, and subtyping extensions.

\[ e ::= v \mid x \mid ee \]
\[ v ::= n \mid \lambda x.e \]
\[ t ::= \alpha \mid \text{int} \mid t \rightarrow t \]
\[ A ::= \cdot \mid x : t, A \]

\[ \text{Int} \quad \text{Var} \quad \text{Lam} \quad \text{App} \]
\[ A \vdash n : \text{int} \quad A \vdash x : A(x) \quad A \vdash \lambda x.e : t \rightarrow t' \quad A \vdash e_1 : t \quad A \vdash e_2 : t' \]
\[ t = t' \rightarrow \beta \quad \beta \text{ fresh} \]

i. (5 points) Draw a derivation showing type inference applied to the following term. Use subscripts to distinguish new instances of type inference variables. For example, \( \alpha_1, \alpha_2, \beta_1, \beta_2 \), etc.

\[ +: \text{int} \rightarrow \text{int} \rightarrow \text{int} \vdash (\lambda x. + 5) 37 \]

**Answer:**

\[
\begin{align*}
A &= +: i \rightarrow i \rightarrow i \\
B &= x : \alpha, A
\end{align*}
\]

\[
\begin{align*}
B \vdash +: \text{int} \rightarrow \text{int} \rightarrow \text{int} & \quad B \vdash x : \alpha \quad i \rightarrow i \rightarrow i = \alpha \rightarrow \beta_3 \\
B \vdash (+: \beta_3) & \quad B \vdash 5 : \text{int} \quad \beta_3 = \text{int} \rightarrow \beta_2 \\
\hline
B \vdash (+x) : \beta_2 & \quad A \vdash (\lambda x. + 5) : \alpha \rightarrow \beta_2 \\
A \vdash 37 : \text{int} & \quad \alpha \rightarrow \beta_2 = \text{int} \rightarrow \beta_1
\end{align*}
\]

\[
A \vdash (\lambda x. + 5) 37 : \beta_1
\]

ii. (3 points) Draw a union find data structure representing the constraints.

**Answer:**

![Union Find Diagram]

iii. (2 points) Write down a solution to the associated constraints.

**Answer:** \( \alpha = \beta_2 = \beta_1 = \text{int} \quad \beta_3 = \text{int} \rightarrow \text{int} \)