

**CMSC 430**  
**Introduction to Compilers**  
**Fall 2018**

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**Lexing and Parsing**

# Overview

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- Compilers are roughly divided into two parts
  - Front-end — deals with surface syntax of the language
  - Back-end — analysis and code generation of the output of the front-end



- Lexing and Parsing translate source code into form more amenable for analysis and code generation
- Front-end also may include certain kinds of semantic analysis, such as symbol table construction, type checking, type inference, etc.

# Lexing vs. Parsing

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- Language grammars usually split into two levels
  - Tokens — the “words” that make up “parts of speech”
    - Ex: Identifier `[a-zA-Z_]+`
    - Ex: Number `[0-9]+`
  - Programs, types, statements, expressions, declarations, definitions, etc — the “phrases” of the language
    - Ex: `if (expr) expr;`
    - Ex: `def id(id, ..., id) expr end`
- Tokens are identified by the lexer
  - Regular expressions
- Everything else is done by the parser
  - Uses grammar in which tokens are primitives
  - Implementations can look inside tokens where needed

# Lexing vs. Parsing (cont'd)

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- Lexing and parsing often produce abstract syntax tree as a result
  - For efficiency, some compilers go further, and directly generate intermediate representations
- Why separate lexing and parsing from the rest of the compiler?
- Why separate lexing and parsing from each other?

# Parsing theory

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- Goal of parsing: Discovering a parse tree (or derivation) from a sentence, or deciding there is no such parse tree
- There's an alphabet soup of parsers
  - Cocke-Younger-Kasami (CYK) algorithm; Earley's Parser
    - Can parse *any* context-free grammar (but inefficient)
  - LL(k)
    - top-down, parses input left-to right (first L), produces a leftmost derivation (second L), k characters of lookahead
  - LR(k)
    - bottom-up, parses input left-to-right (L), produces a rightmost derivation (R), k characters of lookahead
- We will study only some of this theory
  - But we'll start more concretely

# Parsing practice

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- Yacc and lex — most common ways to write parsers
  - yacc = “yet another compiler compiler” (but it makes parsers)
  - lex = lexical analyzer (makes lexers/tokenizers)
- These are available for most languages
  - bison/flex — GNU versions for C/C++
  - ocamlyacc/ocamllex — what we’ll use in this class

# Example: Arithmetic expressions

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- High-level grammar:
  - $E \rightarrow E + E \mid n \mid (E)$
- What should the tokens be?
  - Typically they are the terminals in the grammar
    - $\{+, (, ), n\}$
    - Notice that  $n$  itself represents a set of values
    - Lexers use *regular expressions* to define tokens
  - But what will a typical input actually look like?

1	+	2	+	\n	(	3		+		4	2	)	eof
---	---	---	---	----	---	---	--	---	--	---	---	---	-----

- We probably want to allow for whitespace
  - Notice not included in high-level grammar: lexer can discard it
- Also need to know when we reach the end of the file
  - The parser needs to know when to stop

# Lexing with ocamllex (.mll)

---

```
(* Slightly simplified format *)
{ header }
rule entrypoint = parse
    regexp_1 { action_1 }
    | ...
    | regexp_n { action_n }
and ...
{ trailer }
```

- Compiled to .ml output file
  - **header** and **trailer** are inlined into output file as-is
  - **regexps** are combined to form one (big!) finite automaton that recognizes the union of the regular expressions
    - Finds *longest* possible match in the case of multiple matches
    - Generated regexp matching function is called **entrypoint**

# Lexing with ocamllex (.mll)

---

```
(* Slightly simplified format *)  
{ header }  
rule entrypoint = parse  
    regexp_1 { action_1 }  
    | ...  
    | regexp_n { action_n }  
and ...  
{ trailer }
```

- When match occurs, generated **entrypoint** function returns value in corresponding action
  - If we are lexing for **ocamlyacc**, then we'll return tokens that are defined in the **ocamlyacc** input grammar

# Example

---

```
{
  open Ex1_parser
  exception Eof
}
rule token = parse
  [' ' '\t' '\r']      { token lexbuf }  (* skip blanks *)
| ['\n' ]              { EOL }
| ['0'-'9']+ as lxm    { INT(int_of_string lxm) }
| '+'                  { PLUS }
| '('                  { LPAREN }
| ')'                  { RPAREN }
| eof                  { raise Eof }
```

```
(* token definition from Ex1_parser *)
type token =
  | INT of (int)
  | EOL
  | PLUS
  | LPAREN
  | RPAREN
```

# Generated code

---

```
# 1 "ex1_lexer.mll"  (* line directives for error msgs *)

open Ex1_parser
exception Eof

# 7 "ex1_lexer.ml"
let __ocaml_lex_tables = {...}  (* table-driven automaton *)
let rec token lexbuf = ...      (* the generated matching fn *)
```

- You don't need to understand the generated code
  - But you should understand it's not magic
- Uses [Lexing](#) module from OCaml standard lib
- Notice that [token](#) rule was compiled to [token](#) fn
  - Mysterious [lexbuf](#) from before is the argument to [token](#)
  - Type can be examined in [Lexing](#) module `ocamldoc`

# Lexer limitations

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- Automata limited to 32767 states
  - Can be a problem for languages with lots of keywords

```
rule token = parse
  "keyword_1"    { ... }
| "keyword_2"    { ... }
| ...
| "keyword_n"    { ... }
| ['A'-'Z' 'a'-'z'] ['A'-'Z' 'a'-'z' '0'-'9' '_'] * as id
                        { IDENT id}
```

- Solution?

# Parsing

---

- Now we can build a parser that works with lexemes (tokens) from [token.mll](#)
  - Recall from 330 that parsers work by consuming one character at a time off input while building up parse tree
  - Now the input stream will be tokens, rather than chars

1	+	2	+	\n	(	3		+		4	2	)	eof
---	---	---	---	----	---	---	--	---	--	---	---	---	-----

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof
--------	------	--------	------	--------	--------	------	---------	--------	-----

- Notice parser doesn't need to worry about whitespace, deciding what's an [INT](#), etc

# Suitability of Grammar

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- Problem: our grammar is ambiguous
  - $E \rightarrow E + E \mid n \mid (E)$
  - Exercise: find an input that shows ambiguity
- There are parsing technologies that can work with ambiguous grammars
  - But they'll provide multiple parses for ambiguous strings, which is probably not what we want
- Solution: remove ambiguity
  - One way to do this from 330:
  - $E \rightarrow T \mid E + T$
  - $T \rightarrow n \mid (E)$

# Parsing with ocaml yacc (.mly)

```
%{  
  header  
%}  
  declarations  
%%  
  rules  
%%  
  trailer
```

.mly input

```
type token =  
  | INT of (int)  
  | EOL  
  | PLUS  
  | LPAREN  
  | RPAREN  
  
val main :  
  (Lexing.lexbuf -> token) ->  
    Lexing.lexbuf -> int
```

.mli output

- Compiled to .ml and .mli files
  - .mli file defines `token` type and entry point `main` for parsing
    - Notice first arg to `main` is a fn from a `lexbuf` to a `token`, i.e., the function generated from a .mll file!

# Parsing with ocaml yacc (.mly)

```
%{  
  header  
}%  
  declarations  
%%  
  rules  
%%  
  trailer
```

.mly input

```
(* header *)  
type token = ...  
...  
let yytables = ...  
(* trailer *)
```

.ml output

- .ml file uses **Parsing** library to do most of the work
  - **header** and **trailer** copied direct to output
  - **declarations** lists tokens and some other stuff
  - **rules** are the productions of the grammar
    - Compiled to **yytables**; this is a table-driven parser Also include *actions* that are executed as parser executes
    - We'll see an example next

# Actions

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- In practice, we don't just want to check whether an input parses; we also want to do something with the result
  - E.g., we might build an AST to be used later in the compiler
- Thus, each production in `ocamlyacc` is associated with an *action* that produces a result we want
- Each rule has the format
  - `lhs: rhs {act}`
  - When parser uses a production `lhs → rhs` in finding the parse tree, it runs the code in `act`
  - The code in `act` can refer to results computed by actions of other non-terminals in `rhs`, or token values from terminals in `rhs`

# Example

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }          (* 1 *)
expr:
| term               { $1 }          (* 2 *)
| expr PLUS term     { $1 + $3 }     (* 3 *)
term:
| INT                { $1 }          (* 4 *)
| LPAREN expr RPAREN { $2 }          (* 5 *)
```

- Several kinds of declarations:
  - %token — define a token or tokens used by lexer
  - %start — define start symbol of the grammar
  - %type — specify type of value returned by actions

# Actions, in action

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof
--------	------	--------	------	--------	--------	------	---------	--------	-----

. 1+2+(3+42)\$
term[1].+2+(3+42)\$
expr[1].+2+(3+42)\$
expr[1]+term[2].+(3+42)\$
expr[3].+(3+42)\$
expr[3]+(term[3].+42)\$
expr[3]+(expr[3].+42)\$
expr[3]+(expr[3]+term[42].)\$
expr[3]+(expr[45].)\$
expr[3]+term[45].\$
expr[48].\$
main[48]

```

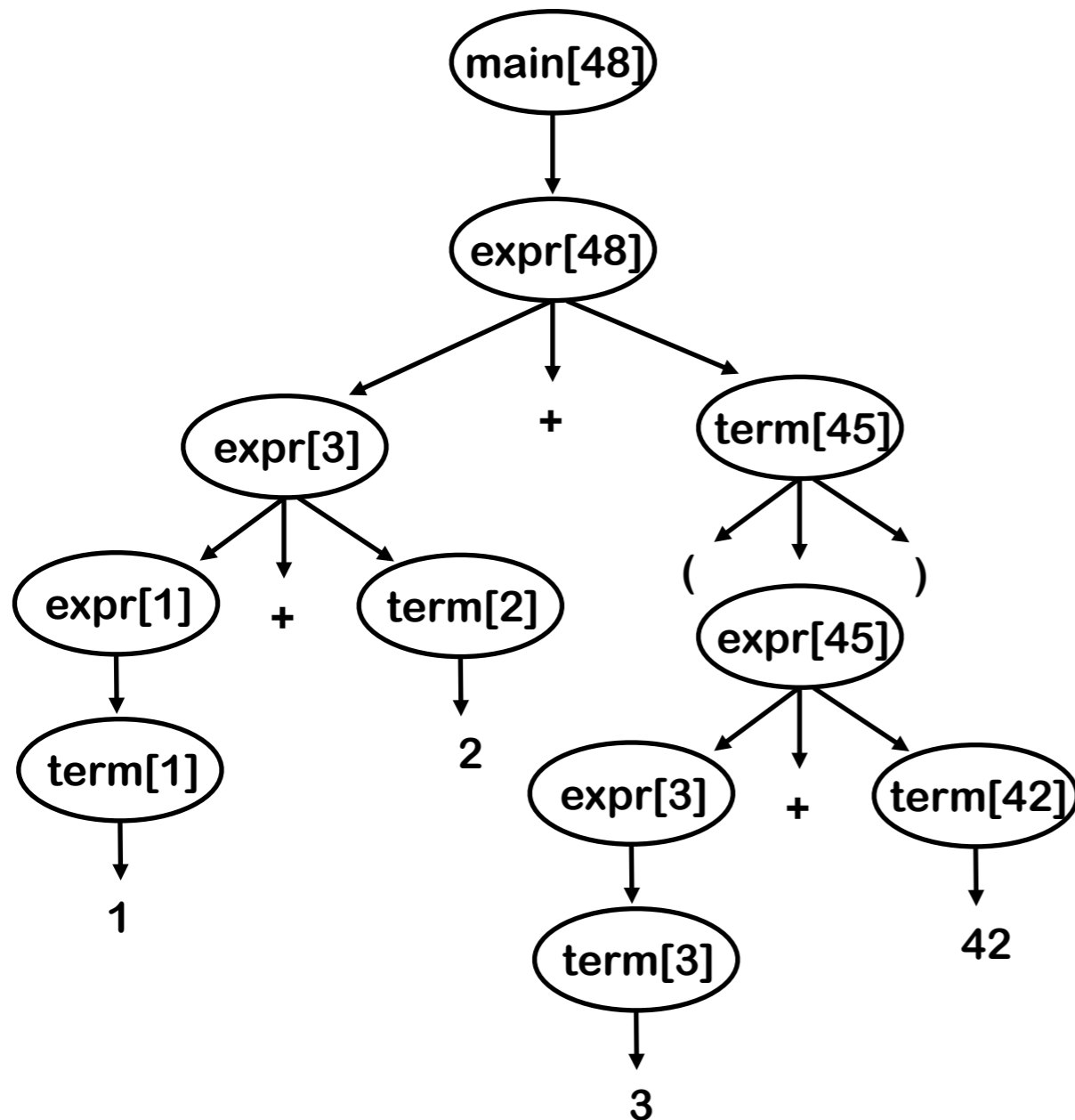
main:
| expr EOL           { $1 }
expr:
| term               { $1 }
| expr PLUS term     { $1 + $3 }
term:
| INT                 { $1 }
| LPAREN expr RPAREN { $2 }

```

- The “.” indicates where we are in the parse
  - We’ve skipped several intermediate steps here, to focus only on actions
  - (Details next)

# Actions, in action

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof
--------	------	--------	------	--------	--------	------	---------	--------	-----



```

main:
| expr EOL           { $1 }
expr:
| term               { $1 }
| expr PLUS term     { $1 + $3 }
term:
| INT                { $1 }
| LPAREN expr RPAREN { $2 }
  
```

# Invoking lexer/parser

---

```
try
  let lexbuf = Lexing.from_channel stdin in
  while true do
    let result = Ex1_parser.main Ex1_lexer.token lexbuf in
    print_int result; print_newline(); flush stdout
  done
with Ex1_lexer.Eof ->
  exit 0
```

- Tip: can also use `Lexing.from_string` and `Lexing.from_function`

# Terminology review

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- Derivation
  - A sequence of steps using the productions to go from the start symbol to a string
- Rightmost (leftmost) derivation
  - A derivation in which the rightmost (leftmost) nonterminal is rewritten at each step
- Sentential form
  - A sequence of terminals and non-terminals derived from the start-symbol of the grammar with 0 or more reductions
  - I.e., some intermediate step on the way from the start symbol to a string in the language of the grammar
- Right- (left-)sentential form
  - A sentential form from a rightmost (leftmost) derivation
- FIRST( $\alpha$ )
  - Set of initial symbols of strings derived from  $\alpha$

# Bottom-up parsing

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- ocaml yacc builds a bottom-up parser
  - Builds derivation from input back to start symbol

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{input}$$

← bottom-up

- To reduce  $\gamma_i$  to  $\gamma_{i-1}$ 
  - Find production  $A \rightarrow \beta$  where  $\beta$  is in  $\gamma_i$ , and replace  $\beta$  with  $A$
- In terms of parse tree, working from leaves to root
  - Nodes with no parent in a partial tree form its *upper fringe*
  - Since each replacement of  $\beta$  with  $A$  shrinks upper fringe, we call it a reduction.
- Note: need not actually build parse tree
  - $|\text{parse tree nodes}| = |\text{input}| + |\text{reductions}|$

# Bottom-up parsing, illustrated

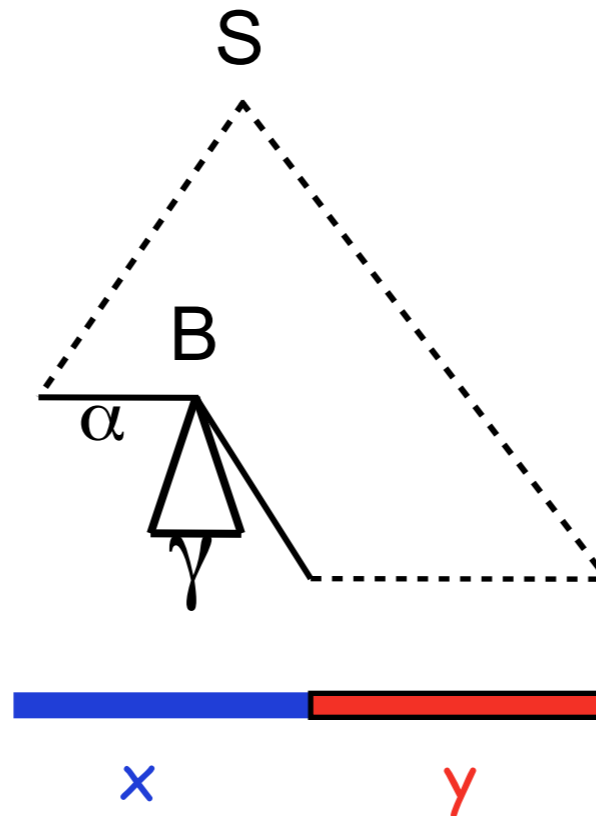
LR(1) parsing

- Scan input left-to-right
- Rightmost derivation
- 1 token lookahead

$$S \Rightarrow^* \alpha B y \Rightarrow \alpha \gamma y \Rightarrow^* x y$$

rule  $B \rightarrow \gamma$

Upper fringe: solid  
Yet to be parsed: dashed




# Bottom-up parsing, illustrated

LR(I) parsing

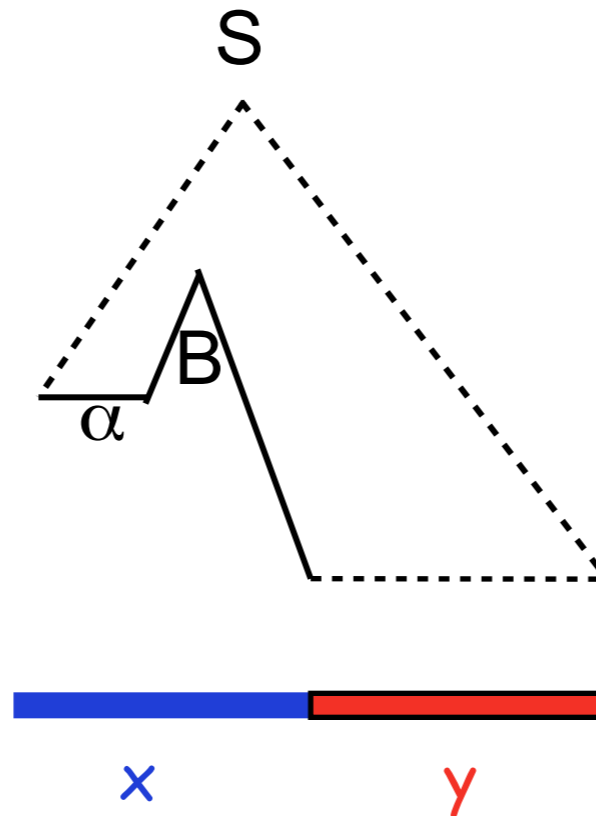
- Scan input left-to-right
- Rightmost derivation
- 1 token lookahead

$$S \Rightarrow^* \alpha B y \Rightarrow \alpha \gamma y \Rightarrow^* x y$$

rule  $B \rightarrow \gamma$



Upper fringe: solid  
Yet to be parsed: dashed



# Finding reductions

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- Consider the following grammar

1.  $S \rightarrow a A B e$

2.  $A \rightarrow A b c$

3.  $\quad | b$

4.  $B \rightarrow d$

Input: abbcde

Sentential Form	Production	Position
abbcde	3	2
aAbcde	2	4
aAde	4	3
aABe	1	4
S	N/A	N/A

- How do we find the next reduction?
  - How do we do this efficiently?

# Handles

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- Goal: Find substring  $\beta$  of tree's frontier that matches some production  $A \rightarrow \beta$ 
  - (And that occurs in the rightmost derivation)
  - Informally, we call this substring  $\beta$  a *handle*
- Formally,
  - A *handle* of a right-sentential form  $\gamma$  is a pair  $(A \rightarrow \beta, k)$  where
    - $A \rightarrow \beta$  is a production and  $k$  is the position in  $\gamma$  of  $\beta$ 's rightmost symbol.
    - If  $(A \rightarrow \beta, k)$  is a handle, then replacing  $\beta$  at  $k$  with  $A$  produces the right sentential form from which  $\gamma$  is derived in the rightmost derivation.
  - Because  $\gamma$  is a right-sentential form, the substring to the right of a handle contains only terminal symbols
    - $\Rightarrow$  the parser doesn't need to scan past the handle (only lookahead)

# Example

- Grammar

1.  $S \rightarrow E$
2.  $E \rightarrow E + T$
3.      $| E - T$
4.      $| T$
5.  $T \rightarrow T * F$
6.      $| T / F$
7.      $| F$
8.  $F \rightarrow n$
9.      $| id$
10.     $| (E)$

Production	Sentential Form	Handle (prod,k)
	S	
1	E	1,1
3	E-T	3,3
5	E-T*F	5,5
9	E-T*id	9,5
7	E-F*id	7,3
8	E-n*id	8,3
4	T-n*id	4,1
7	F-n*id	7,1
9	id-n*id	9,1

Handles for rightmost derivation of id-n\*id

# Finding reductions

---

- Theorem: If  $G$  is unambiguous, then every right-sentential form has a unique handle
  - If we can find those handles, we can build a derivation!
- Sketch of Proof:
  - $G$  is unambiguous  $\Rightarrow$  rightmost derivation is unique
  - $\Rightarrow$  a unique production  $A \rightarrow \beta$  applied to derive  $\gamma_i$  from  $\gamma_{i-1}$
  - and a unique position  $k$  at which  $A \rightarrow \beta$  is applied
  - $\Rightarrow$  a unique handle  $(A \rightarrow \beta, k)$
- This all follows from the definitions

# Bottom-up handle pruning

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- *Handle pruning*: discovering handle and reducing it
  - Handle pruning forms the basis for bottom-up parsing
- So, to construct a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{input}$$

- Apply the following simple algorithm

for  $i \leftarrow n$  to 1 by  $-1$

Find handle  $(A_i \rightarrow \beta_i, k_i)$  in  $\gamma_i$

Replace  $\beta_i$  with  $A_i$  to generate  $\gamma_{i-1}$

- This takes  $2n$  steps

# Shift-reduce parsing algorithm

---

- Maintain a stack of terminals and non-terminals matched so far
  - Rightmost terminal/non-terminal on top of stack
  - Since we're building rightmost derivation, will look at top elements of stack for reductions

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
  if the top of the stack is a handle  $A \rightarrow \beta$ 
    then // reduce  $\beta$  to  $A$ 
      pop  $|\beta|$  symbols off the stack
      push  $A$  onto the stack
  else if (token  $\neq$  EOF)
    then // shift
      push token
      token ← next_token( )
  else // need to shift, but out of input
    report an error
```

## Potential errors

- Can't find handle
- Reach end of file

# Example

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

- Grammar

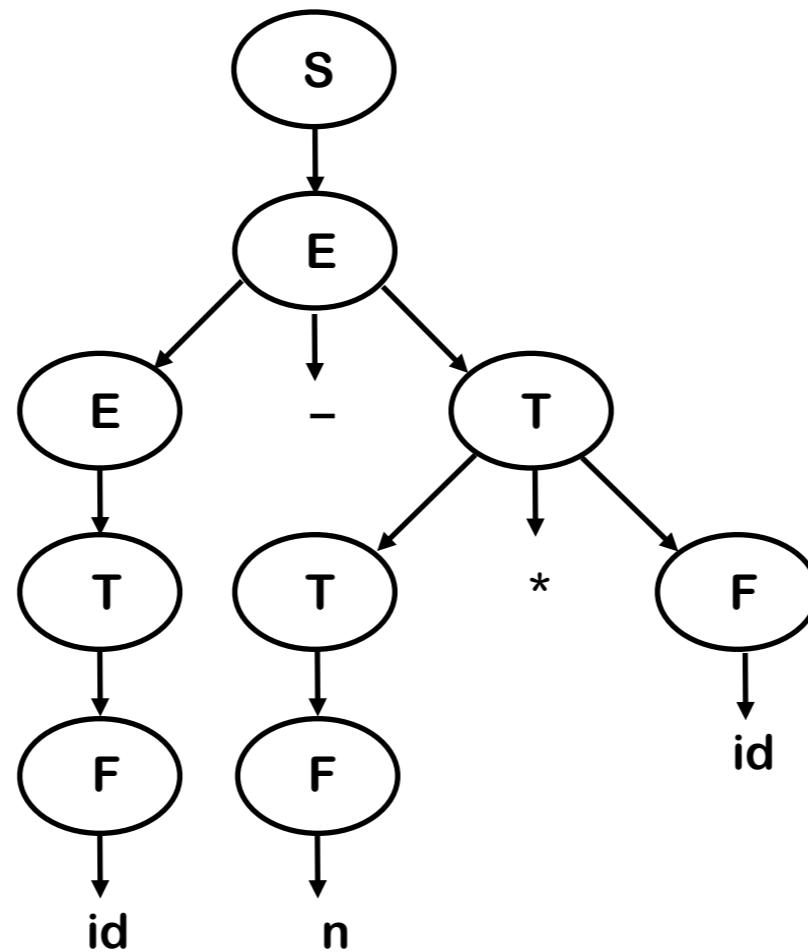
1.  $S \rightarrow E$
2.  $E \rightarrow E + T$
3.      $| E - T$
4.      $| T$
5.  $T \rightarrow T * F$
6.      $| T / F$
7.      $| F$
8.  $F \rightarrow n$
9.      $| id$
10.     $| (E)$

Shift/reduce parse of  $id-n*id$

Stack	Input	Handle (prod,k)	Action
	id-n*id	none	shift
id	-n*id	9,1	reduce 9
F	-n*id	7,1	reduce 7
T	-n*id	4,1	reduce 4
E	-n*id	none	shift
E-	n*id	none	shift
E-n	*id	8,3	reduce 8
E-F	*id	7,3	reduce 7
E-T	*id	none	shift
E-T*	id	none	shift
E-T*id		9,5	reduce 9
E-T*F		5,5	reduce 5
E-T		3,3	reduce 3
E		1,1	reduce 1
S		none	accept

# Parse tree for example

---



# Algorithm actions

---

- Shift-reduce parsers have just four actions
  - **Shift** — next word is shifted onto the stack
  - **Reduce** — right end of handle is at top of stack
    - Locate left end of handle within the stack
    - Pop handle off stack and push appropriate lhs
  - **Accept** — stop parsing and report success
  - **Error** — call an error reporting/recovery routine
- Cost of operations
  - **Accept** is constant time
  - **Shift** is just a push and a call to the scanner
  - **Reduce** takes **|rhs|** pops and 1 push
    - If handle-finding requires state, put it in the stack  $\Rightarrow$  2x work
  - **Error** depends on error recovery mechanism

# Finding handles

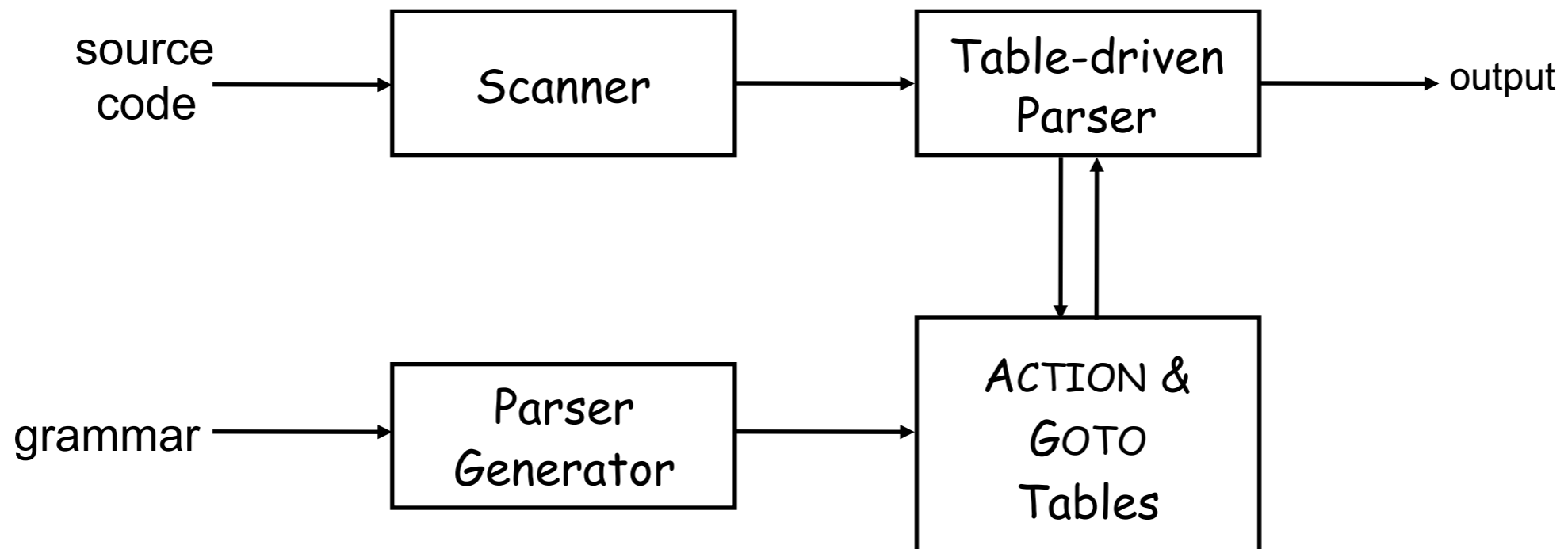
---

- To be a handle, a substring of sentential form  $\gamma$  must :
  - Match the right hand side  $\beta$  of some rule  $A \rightarrow \beta$
  - There must be some rightmost derivation from the start symbol that produces  $\gamma$  with  $A \rightarrow \beta$  as the last production applied
  - $\Rightarrow$  Looking for rhs's that match strings is not good enough
- How can we know when we have found a handle?
  - LR(1) parsers use DFA that runs over stack and finds them
    - One token look-ahead determines next action (shift or reduce) in each state of the DFA.
  - A grammar is LR(1) if we can build an LR(1) parser for it
- LR(0) parsers: no look-ahead

# LR(1) parsing

---

- Can use a set of tables to describe LR(1) parser



- ocaml yacc automates the process of building the tables
  - Standard library Parser module interprets the tables
- LR parsing invented in 1965 by Donald Knuth
- LALR parsing invented in 1969 by Frank DeRemer

# LR(1) parsing algorithm

```
stack.push(INVALID); stack.push(s0);
not_found = true;
token = scanner.next_token();
do while (not_found) {
    s = stack.top();
    if ( ACTION[s,token] == "reduce A→β" ) {
        stack.popnum(2*|β|); // pop 2*|β| symbols
        s = stack.top();
        stack.push(A);
        stack.push(GOTO[s,A]);
    }
    else if ( ACTION[s,token] == "shift si" ) {
        stack.push(token); stack.push(si);
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept" && token == EOF )
        not_found = false;
    else report a syntax error and recover;
}
report success;
```

- Two tables
  - ACTION: reduce/shift/accept
  - GOTO: state to be in after reduce
- Cost
  - |input| shifts
  - |derivation| reductions
  - One accept
- Detects errors by failure to shift, reduce, or accept

# Example parser table

- `ocamlyacc -v ex1_parser.mly` — produce `.output` file with parser table

state	action						goto			productions
	.	EOL	+	N	(	)	main	expr	term	
0										(special)
1				s3	s4		acc	6	7	entry $\rightarrow$ . main
2										(special)
3	r4									term $\rightarrow$ INT .
4				s3	s4			8	7	term $\rightarrow$ ( . expr )
5										(special)
6		s9	s10							main $\rightarrow$ expr . EOL   expr $\rightarrow$ expr . + term
7	r2									expr $\rightarrow$ term .
8			s10			s11				expr $\rightarrow$ expr . + term   term $\rightarrow$ ( expr . )
9	r1									main $\rightarrow$ expr EOL .
10				s3	s4				12	expr $\rightarrow$ expr + . term
11	r5									term $\rightarrow$ ( expr ) .
12	r3									expr $\rightarrow$ expr + term .

NB: Numbers in shift refer to state numbers

Numbers in reduction refer to production numbers

# Example parse (N+N+N)

---

Stack	Input	Action
I	N+N+N	s3
I,N,3	+N+N	r4
I,term,7	+N+N	r2
I,expr,6	+N+N	s10
I,expr,6,+,10	N+N	s3
I,expr,6,+,10,N,3	+N	r4
I,expr,6,+,10,term,12	+N	r3
I,expr,6	+N	s10
I,expr,6,+,10	N	s3
I,expr,6,+,10,N,3		r4
I,expr,6,+,10,term,12		r3
I,expr,6		s9
I,expr,6,EOL,9		r1
accept		

# Example parser table (cont'd)

---

- Notes
  - Notice derivation is built up (bottom to top)
  - Table only contains kernel of each state
    - Apply closure operation to see all the productions in the state
- LR(1) parsing requires start symbol not on any rhs
  - Thus, ocaml yacc actually adds another production
    - `%entry% → \001 main`
    - (so the `acc` in the previous table is a slight fib)
- Values returned from actions stored on the stack
  - Reduce triggers computation of action result

# Why does this work?

---

- Stack = upper fringe
  - So all possible handles on top of stack
  - Shift inputs until top elements of stack form a handle
- Build a handle-recognizing DFA
  - Language of handles is regular
  - ACTION and GOTO tables encode the DFA
    - Shift = DFA transition
    - Reduce = DFA accept
      - New state = GOTO[state at top of stack (after pop), lhs]
- If we can build these tables, grammar is LR(1)

# LR(k) items

---

- An  $LR(k)$  item is a pair  $[P, \delta]$ , where
  - $P$  is a production  $A \rightarrow \beta$  with a  $\bullet$  at some position in the rhs
  - $\delta$  is a lookahead string of length  $\leq k$  (words or \$)
  - The  $\bullet$  in an item indicates the position of the top of the stack
- LR(1):
  - $[A \rightarrow \bullet \beta \gamma, a]$  — input so far consistent with using  $A \rightarrow \beta \gamma$  immediately after symbol on top of stack
  - $[A \rightarrow \beta \bullet \gamma, a]$  — input so far consistent with using  $A \rightarrow \beta \gamma$  at this point in the parse, and parser has already recognized  $\beta$
  - $[A \rightarrow \beta \gamma \bullet, a]$  — parser has seen  $\beta \gamma$ , and lookahead of  $a$  consistent with reducing to  $A$
- LR(1) items represent valid configurations of an LR(1) parser; DFA states are sets of LR(1) items

# LR(k) items, cont'd

---

- Ex:  $A \rightarrow BCD$  with lookahead  $a$  can yield 4 items
  - $[A \rightarrow \bullet BCD, a]$ ,  $[A \rightarrow B \bullet CD, a]$ ,  $[A \rightarrow BC \bullet D, a]$ ,  $[A \rightarrow BCD \bullet, a]$
  - Notice: set of LR(1) items for a grammar is finite
- Carry lookaheads along to choose correct reduction
  - Lookahead has no direct use in  $[A \rightarrow \beta \bullet \gamma, a]$
  - In  $[A \rightarrow \beta \bullet, a]$ , a lookahead of  $a \Rightarrow$  reduction by  $A \rightarrow \beta$
  - For  $\{ [A \rightarrow \beta \bullet, a], [B \rightarrow \gamma \bullet \delta, b] \}$ 
    - Lookahead of  $a \Rightarrow$  reduce to  $A$
    - $\text{FIRST}(\delta) \Rightarrow$  shift
    - (else error)

# LR(1) table construction

---

- States of LR(1) parser contain sets of LR(1) items
  - Initial state  $s_0$ 
    - Assume  $S'$  is the start symbol of grammar, does not appear in rhs
      - (Extend grammar if necessary to ensure this)
    - $s_0 = \text{closure}([S' \rightarrow \bullet S, \$])$  ( $\$ = \text{EOF}$ )
  - For each  $s_k$  and each terminal/non-terminal  $X$ , compute new state  $\text{goto}(s_k, X)$ 
    - Use  $\text{closure}()$  to “fill out” kernel of new state
    - If the new state is not already in the collection, add it
    - Record all the transitions created by  $\text{goto}()$ 
      - These become ACTION and GOTO tables
      - i.e., the handle-finding DFA
  - This process eventually reaches a fixpoint

# Closure()

- $[A \rightarrow \beta \bullet B \delta, a]$  implies  $[B \rightarrow \bullet \gamma, x]$  for each production with  $B$  on lhs and each  $x \in \text{FIRST}(\delta a)$ 
  - (If you're about to see a  $B$ , you may also see a  $\gamma$ )

```
Closure( s )
while ( s is still changing )
   $\forall$  items  $[A \rightarrow \beta \bullet B \delta, \underline{a}] \in s$            // item with  $\bullet$  to left of nonterminal B
   $\forall$  productions  $B \rightarrow \gamma \in P$                  // all productions for B
   $\forall \underline{b} \in \text{FIRST}(\delta \underline{a})$                // tokens appearing after B
    if  $[B \rightarrow \bullet \gamma, \underline{b}] \notin s$       // form LR(1) item w/ new lookahead
      then add  $[B \rightarrow \bullet \gamma, \underline{b}]$  to s    // add item to s if new
```

- Classic fixed-point method
- Halts because  $s \subset \text{ITEMS}$  (worklist version is faster)
  - Closure “fills out” a state

# Example — closure with LR(0)

---

$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow \text{id}$

[kernel item]  
[derived item]

[ $S \rightarrow \bullet E$ ]  
[ $E \rightarrow \bullet T + E$ ]  
[ $E \rightarrow \bullet T$ ]  
[ $T \rightarrow \bullet \text{id}$ ]

[ $E \rightarrow T + \bullet E$ ]  
[ $E \rightarrow \bullet T + E$ ]  
[ $E \rightarrow \bullet T$ ]  
[ $T \rightarrow \bullet \text{id}$ ]

# Example — closure with LR(1)

---

$S \rightarrow E$

$E \rightarrow T+E$

$\mid T$

$T \rightarrow \text{id}$

[kernel item]  
[derived item]

[ $S \rightarrow \bullet E, \$$ ]  
[ $E \rightarrow \bullet T+E, \$$ ]  
[ $E \rightarrow \bullet T, \$$ ]  
[ $T \rightarrow \bullet \text{id}, +$ ]  
[ $T \rightarrow \bullet \text{id}, \$$ ]

[ $E \rightarrow T+ \bullet E, \$$ ]  
[ $E \rightarrow \bullet T+E, \$$ ]  
[ $E \rightarrow \bullet T, \$$ ]  
[ $T \rightarrow \bullet \text{id}, +$ ]  
[ $T \rightarrow \bullet \text{id}, \$$ ]

# Goto

- $\text{Goto}(s, x)$  computes the state that the parser would reach if it recognized an  $x$  while in state  $s$ 
  - $\text{Goto}(\{ [A \rightarrow \beta \bullet X \delta, a] \}, X)$  produces  $[A \rightarrow \beta X \bullet \delta, a]$
  - Should also include  $\text{closure}([A \rightarrow \beta X \bullet \delta, a])$

```
Goto( s, X )  
  new  $\leftarrow \emptyset$   
   $\forall$  items  $[A \rightarrow \beta \bullet X \delta, \underline{a}] \in s$            // for each item with  $\bullet$  to left of X  
    new  $\leftarrow$  new  $\cup [A \rightarrow \beta X \bullet \delta, \underline{a}]$  // add item with  $\bullet$  to right of X  
  return closure(new)                               // remember to compute closure!
```

- Not a fixed-point method!
- Straightforward computation
- Uses  $\text{closure}()$ 
  - $\text{Goto}()$  moves forward

# Example — goto with LR(0)

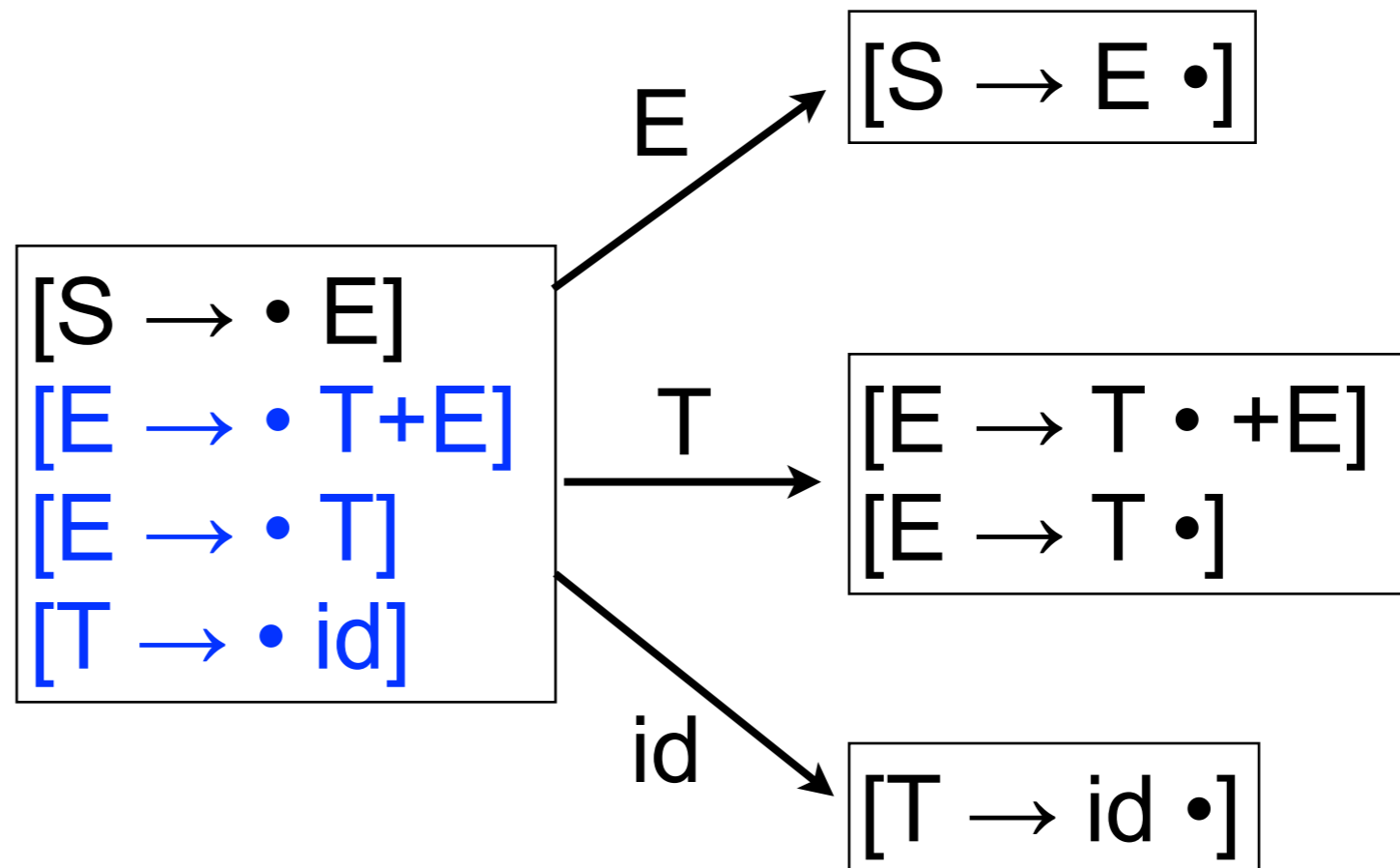
$S \rightarrow E$

$E \rightarrow T + E$

$| T$

$T \rightarrow id$

[kernel item]  
[derived item]



# Example — goto with LR(1)

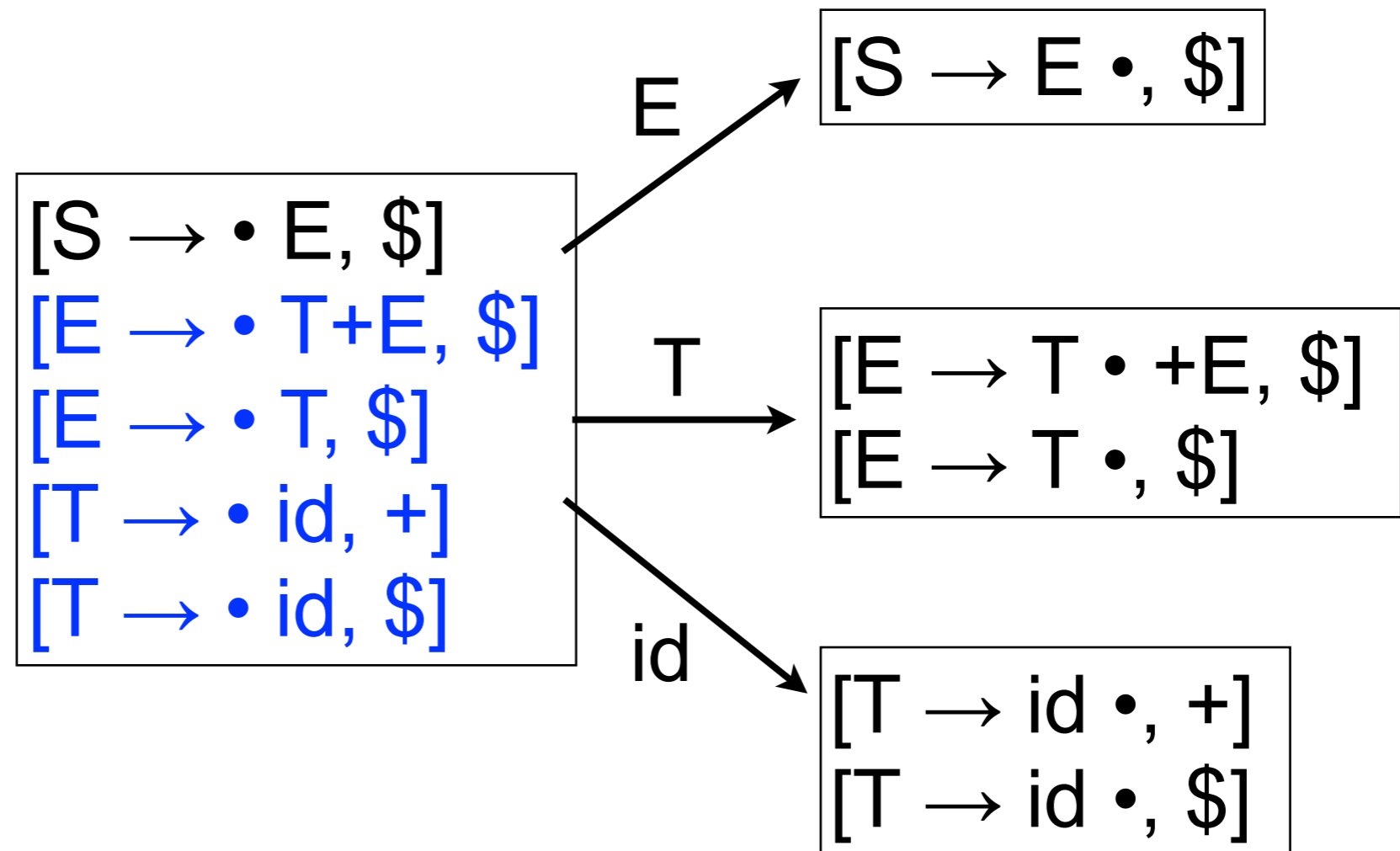
$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow \text{id}$

[kernel item]  
[derived item]



# Building parser states

---

```
 $CC_0 \leftarrow \text{closure}([S' \rightarrow \bullet S, \$])$   
 $CC \leftarrow \{ CC_0 \}$   
while ( new sets are still being added to  $CC$  )  
  for each unmarked set  $CC_j \in CC$   
    mark  $CC_j$  as processed  
    for each  $x$  following a  $\bullet$  in an item in  $CC_j$   
      temp  $\leftarrow \text{goto}(CC_j, x)$   
      if temp  $\notin CC$   
        then  $CC \leftarrow CC \cup \{ \text{temp} \}$   
      record transitions from  $CC_j$  to temp on  $x$ 
```

- $CC$  = canonical collection (of LR(k) items)
- Fixpoint computation (worklist version)
- Loop adds to  $CC$ 
  - $CC \subseteq 2^{\text{ITEMS}}$ , so  $CC$  is finite

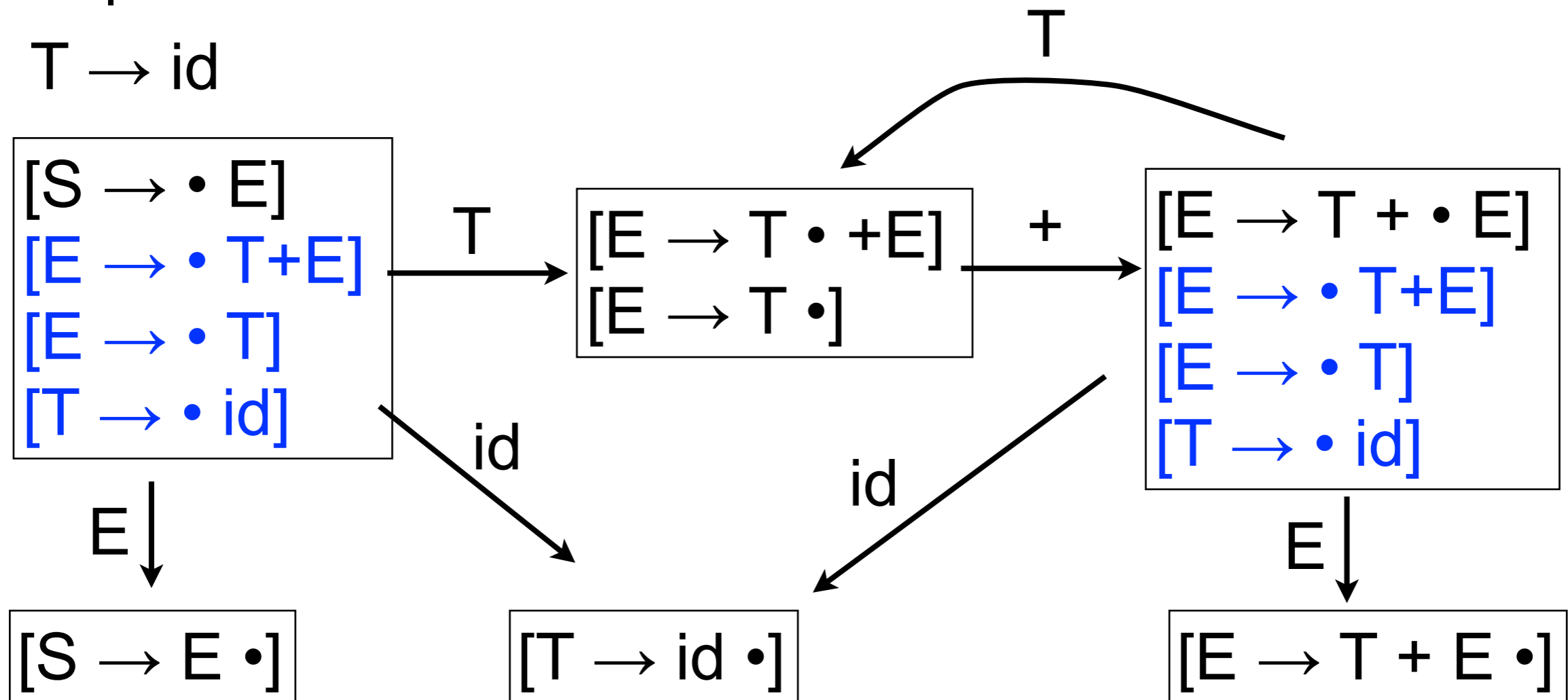
# Example LR(0) states

$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow id$



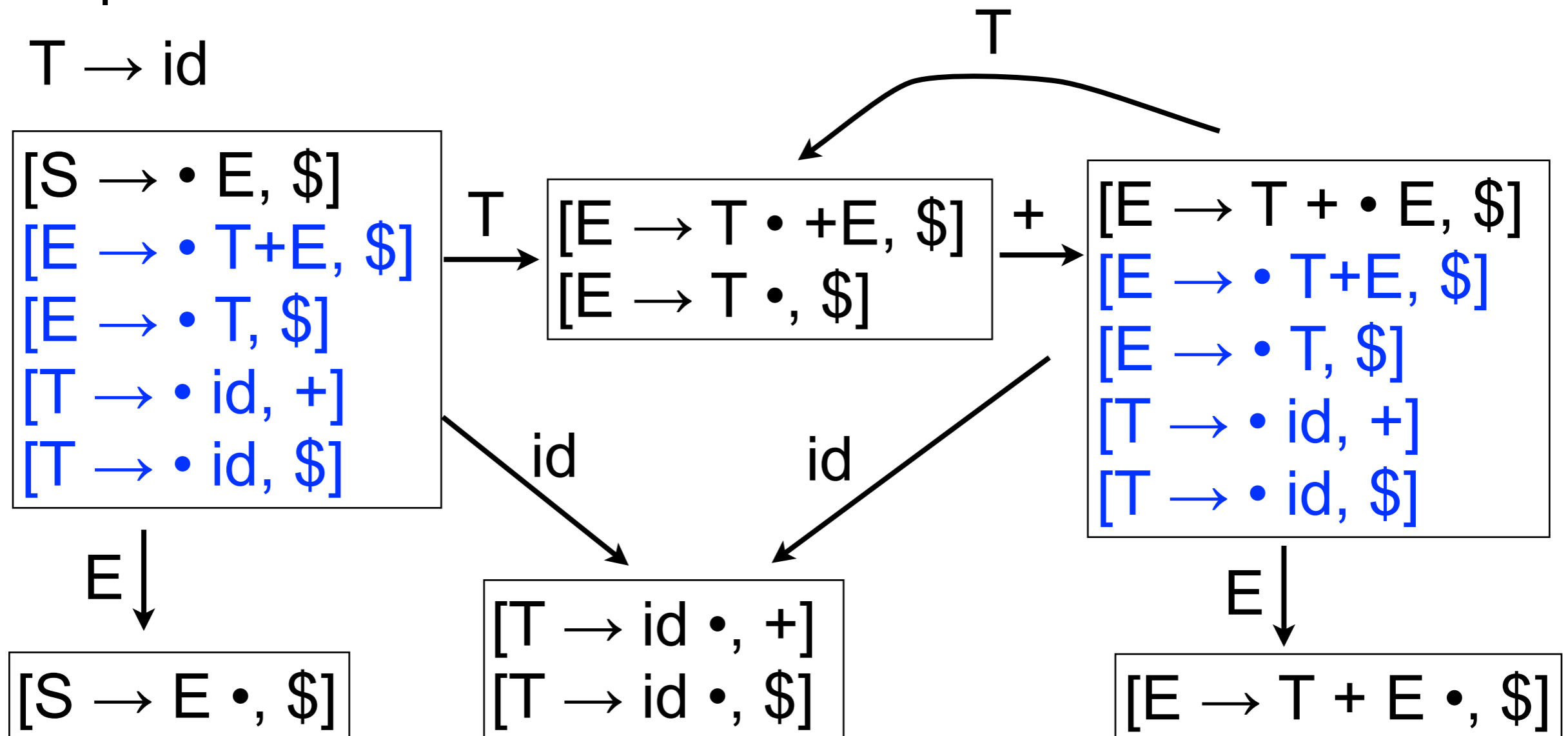
# Example LR(1) states

$S \rightarrow E$

$E \rightarrow T + E$

$| T$

$T \rightarrow id$



# Building ACTION and GOTO tables

```

 $\forall$  set  $s_x \in S$ 
   $\forall$  item  $i \in s_x$ 
    if  $i$  is  $[A \rightarrow \beta \cdot \underline{a} \gamma, \underline{b}]$  and  $\text{goto}(s_x, \underline{a}) = s_k, \underline{a} \in \text{terminals}$  // • to left of terminal a
      then  $\text{ACTION}[x, \underline{a}] \leftarrow \text{"shift k"}$  //  $\Rightarrow$  shift if lookahead = a
    else if  $i$  is  $[S' \rightarrow S \cdot, \$]$  // start production done,
      then  $\text{ACTION}[x, \$] \leftarrow \text{"accept"}$  //  $\Rightarrow$  accept if lookahead = $
    else if  $i$  is  $[A \rightarrow \beta \cdot, \underline{a}]$  // • all the way to right
      then  $\text{ACTION}[x, \underline{a}] \leftarrow \text{"reduce } A \rightarrow \beta"$  //  $\rightarrow$  production done
   $\forall n \in \text{nonterminals}$  // reduce if lookahead = a
    if  $\text{goto}(s_x, n) = s_k$ 
      then  $\text{GOTO}[x, n] \leftarrow k$  // store transitions for nonterminals

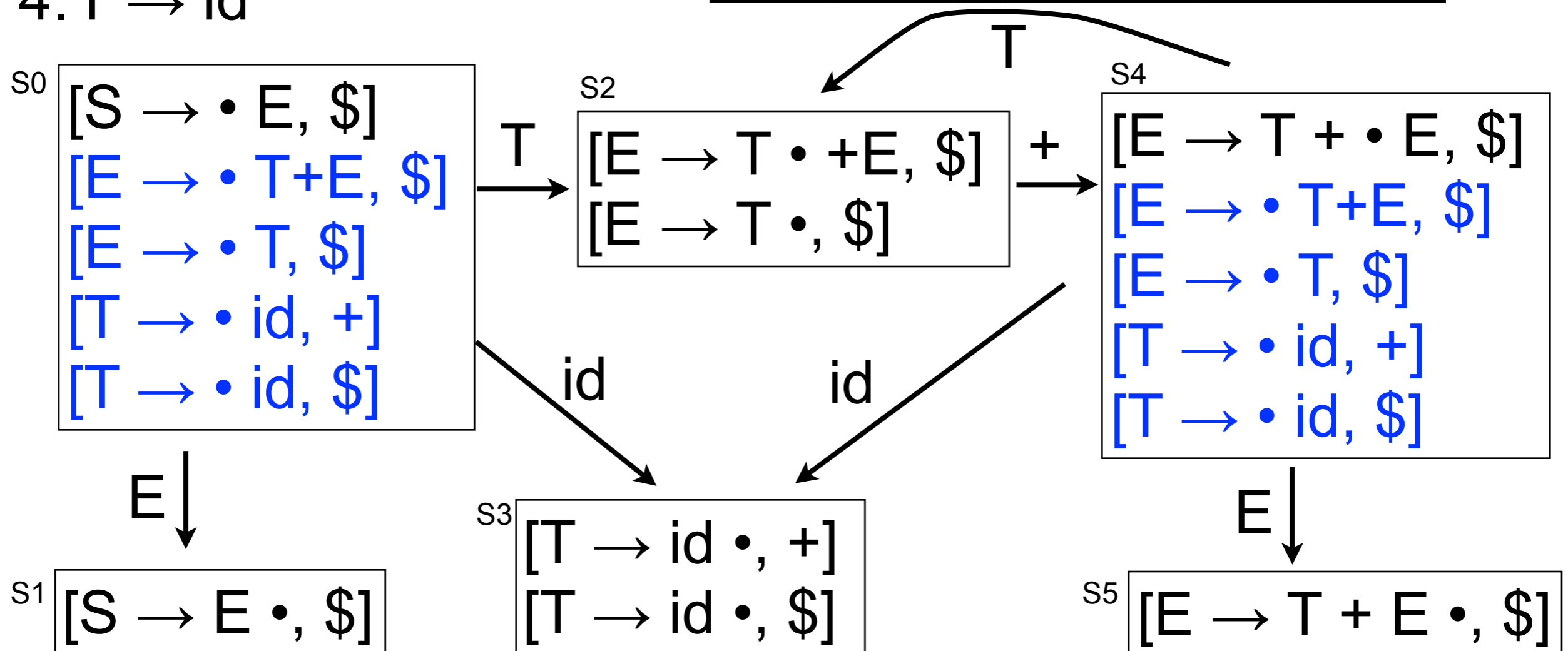
```

- Many items generate no table entry
  - e.g.,  $[A \rightarrow \beta \cdot B \alpha, a]$  does not, but closure ensures that all the rhs's for  $B$  are in  $s_x$

# Ex ACTION and GOTO tables

1.  $S \rightarrow E$
2.  $E \rightarrow T+E$
3.  $\mid T$
4.  $T \rightarrow id$

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

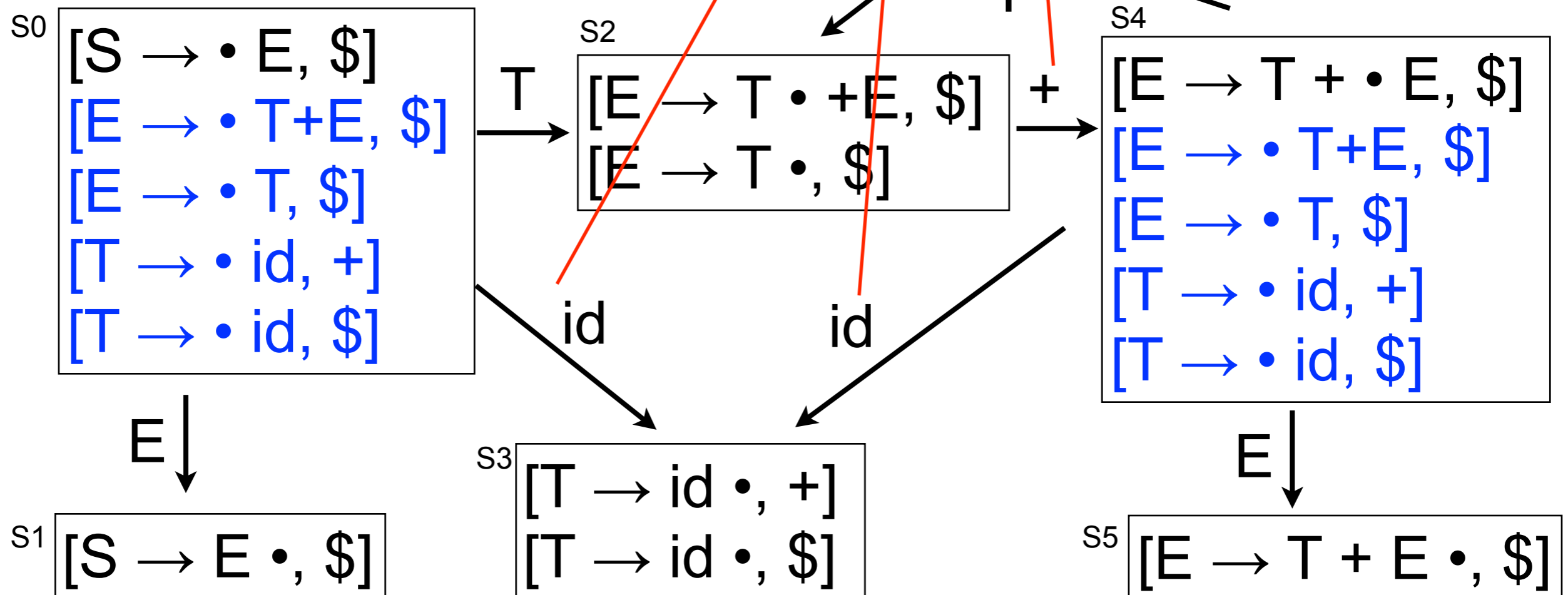


# Ex ACTION and GOTO tables

1.  $S \rightarrow E$
2.  $E \rightarrow T + E$
3.  $\mid T$
4.  $T \rightarrow id$

Entries  
for  
shift

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

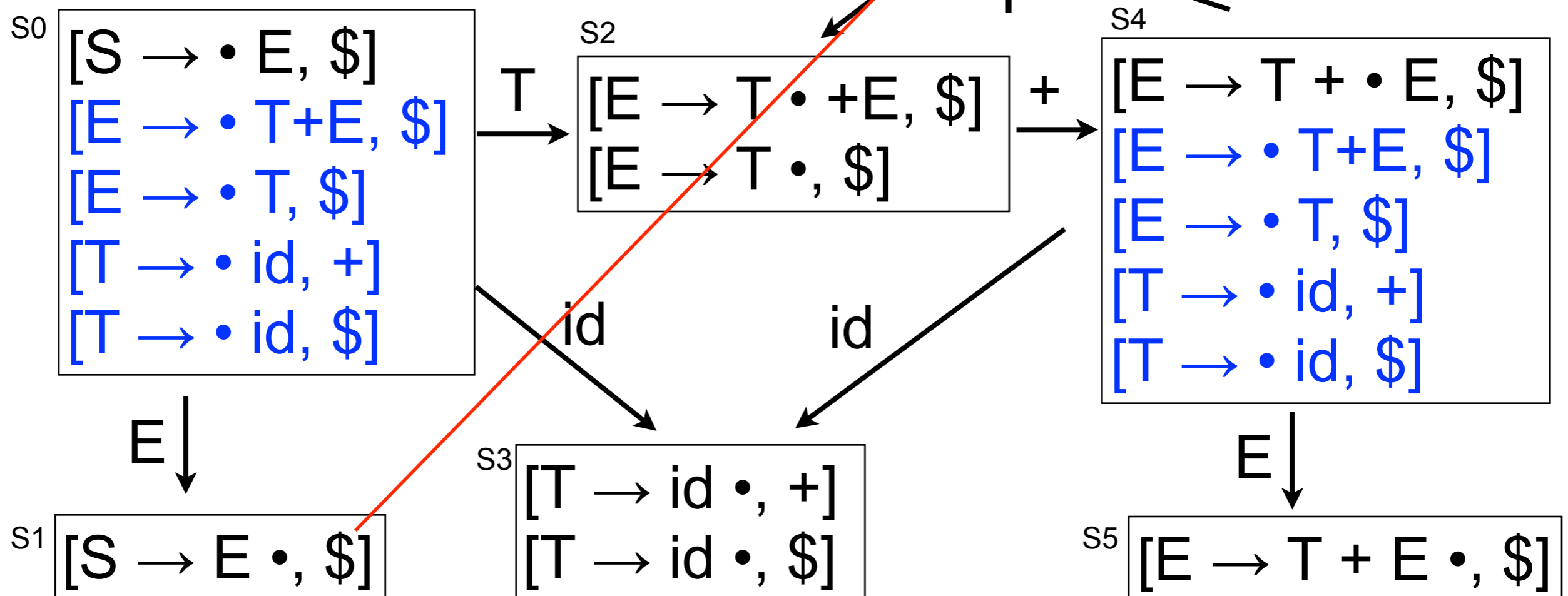


# Ex ACTION and GOTO tables

1.  $S \rightarrow E$
2.  $E \rightarrow T + E$
3.  $\mid T$
4.  $T \rightarrow id$

Entry  
for  
accept

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

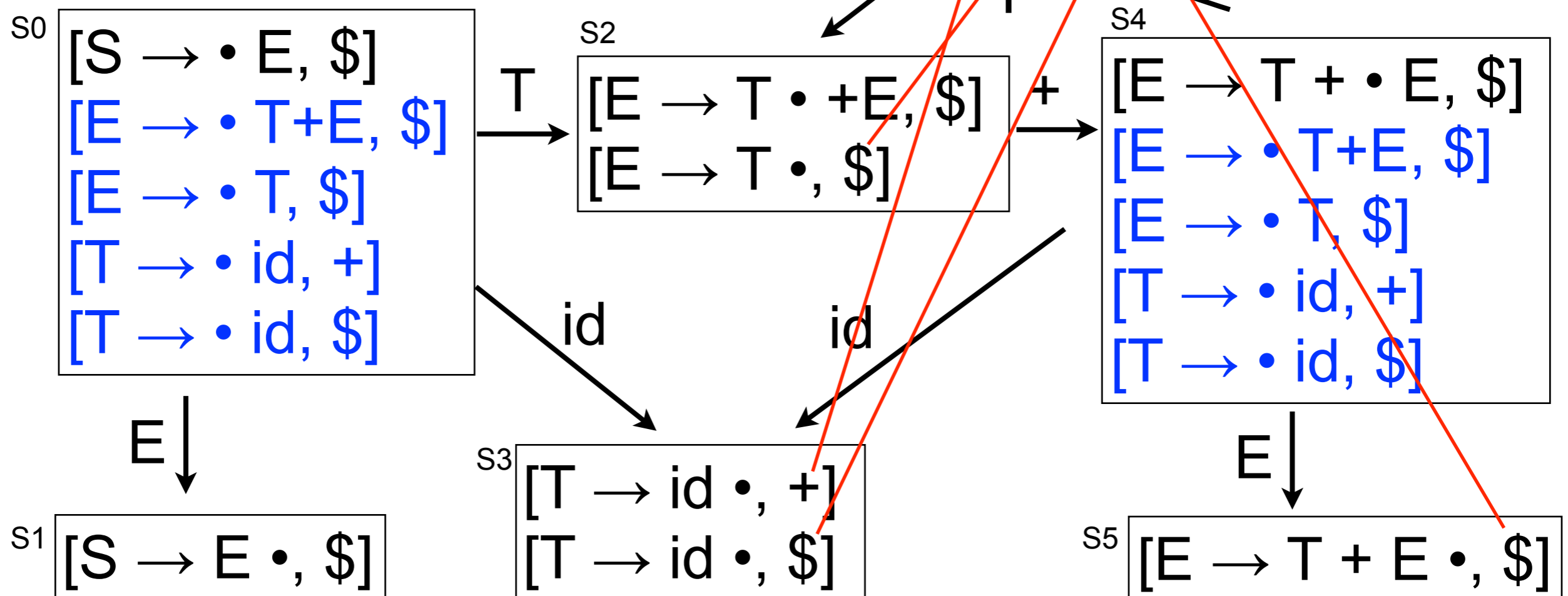


# Ex ACTION and GOTO tables

1.  $S \rightarrow E$
2.  $E \rightarrow T + E$
3.  $\mid T$
4.  $T \rightarrow id$

Entries  
for  
reduce

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

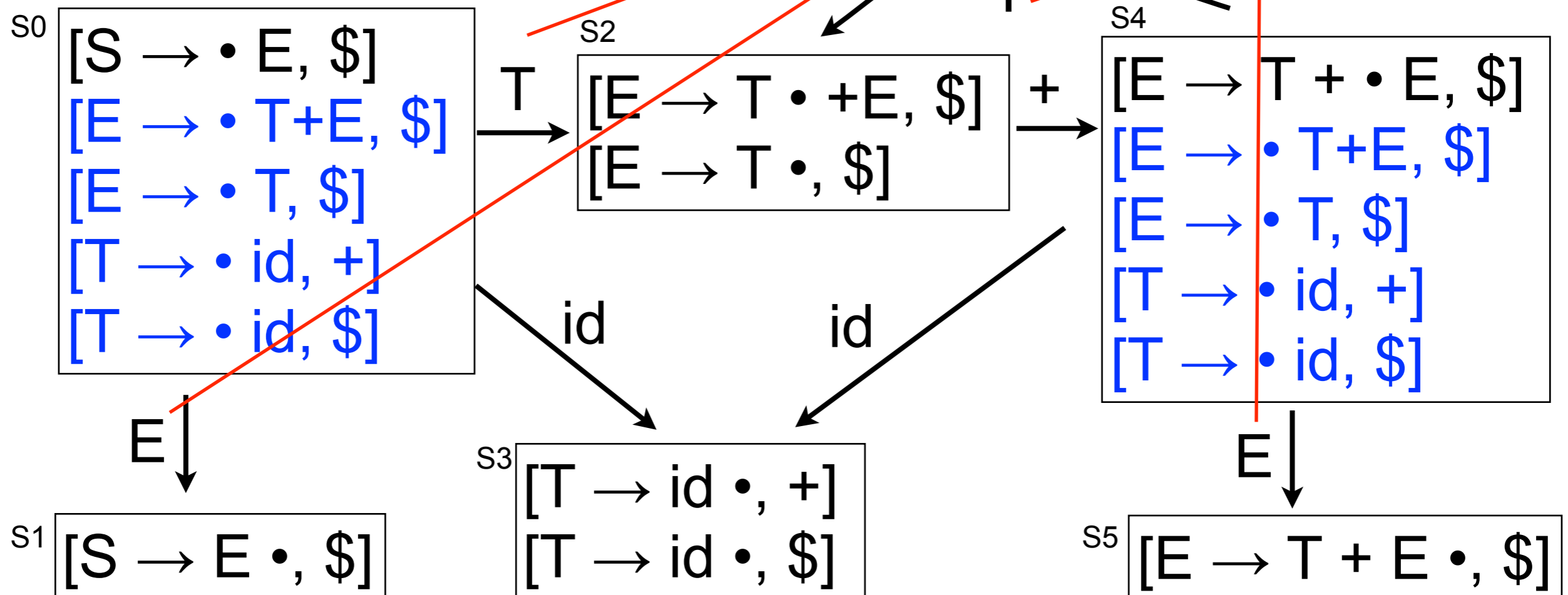


# Ex ACTION and GOTO tables

1.  $S \rightarrow E$
2.  $E \rightarrow T + E$
3.  $\mid T$
4.  $T \rightarrow id$

Entries  
for  
GOTO

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		



# What can go wrong?

---

- What if set  $s$  contains  $[A \rightarrow \beta \cdot a \gamma, b]$  and  $[B \rightarrow \beta \cdot, a]$  ?
  - First item generates “shift”, second generates “reduce”
  - Both define  $\text{ACTION}[s, a]$  — cannot do both actions
  - This is a *shift/reduce conflict*
- What if set  $s$  contains  $[A \rightarrow \gamma \cdot, a]$  and  $[B \rightarrow \gamma \cdot, a]$  ?
  - Each generates “reduce”, but with a different production
  - Both define  $\text{ACTION}[s, a]$  — cannot do both reductions
  - This is called a reduce/reduce conflict
- In either case, the grammar is not LR(1)

# Shift/reduce conflict

---

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }
expr:
| INT                { $1 }
| expr PLUS expr      { $1 + $3 }
| LPAREN expr RPAREN { $2 }
```

- Associativity unspecified
  - Ambiguous grammars always have conflicts
  - But, some non-ambiguous grammars also have conflicts

# Solving conflicts

---

- Refactor grammar
- Specify operator precedence and associativity

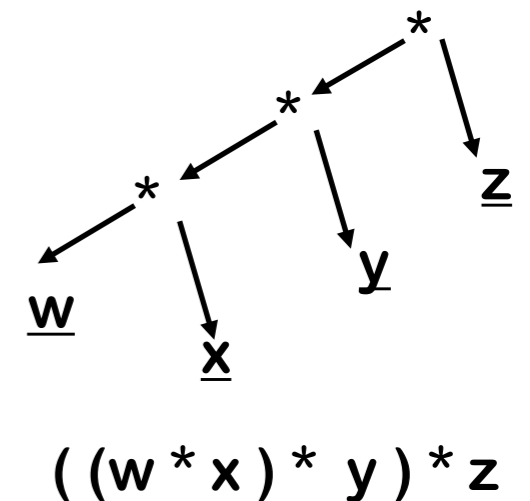
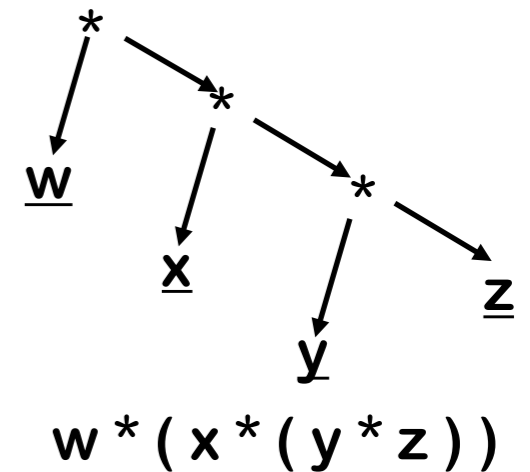
```
%left PLUS MINUS      /* lowest precedence */  
%left TIMES DIV        /* medium precedence */  
%nonassoc UMINUS       /* highest precedence */
```

- Lots of details here
  - See “12.4.2 Declarations” at
  - <http://caml.inria.fr/pub/docs/manual-ocaml/manual026.html#htoc151>
- When comparing operator on stack with lookahead
  - Shift if lookahead has higher prec OR same prec, right assoc
  - Reduce if lookahead has lower prec OR same prec, left assoc
- Can use smaller, simpler (ambiguous) grammars
  - Like the one we just saw

# Left vs. right recursion

---

- Right recursion
  - Required for termination in top-down parsers
  - Produces right-associative operators
- Left recursion
  - Works fine in bottom-up parsers
  - Limits required stack space
  - Produces left-associative operators
- Rule of thumb
  - Left recursion for bottom-up parsers
  - Right recursion for top-down parsers



# Reduce/reduce conflict (1)

---

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }
expr:
| INT                { $1 }
| term               { $1 }
| term PLUS expr     { $1 + $3 }
term :
| INT                { $1 }
| LPAREN expr RPAREN { $2 }
```

- Often these conflicts suggest a serious problem
  - Here, there's a deep ambiguity

# Reduce/reduce conflict (2)

---

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }
expr:
| term1              { $1 }
| term1 PLUS PLUS expr { $1 + $4 }
| term2 PLUS expr     { $1 + $3 }
term1 :
| INT                { $1 }
| LPAREN expr RPAREN { $2 }
term2 :
| INT                { $1 }
```

- Grammar not ambiguous, but not enough lookahead to distinguish last two `expr` productions

# Shrinking the tables

---

- Combine terminals
  - E.g., number and identifier, or + and -, or \* and /
    - Directly removes a column, may remove a row
- Combine rows or columns (*table compression*)
  - Implement identical rows once and remap states
  - Requires extra indirection on each lookup
  - Use separate mapping for ACTION and for GOTO
- Use another construction algorithm
  - LALR(1) used by ocamllyacc

# LALR(1) parser

---

- Define the *core* of a set of LR(1) items as
  - Set of LR(0) items derived by ignoring lookahead symbols

$[E \rightarrow a \bullet, b]$ $[A \rightarrow a \bullet, c]$
------------------------------------------------------------------

LR(1) state

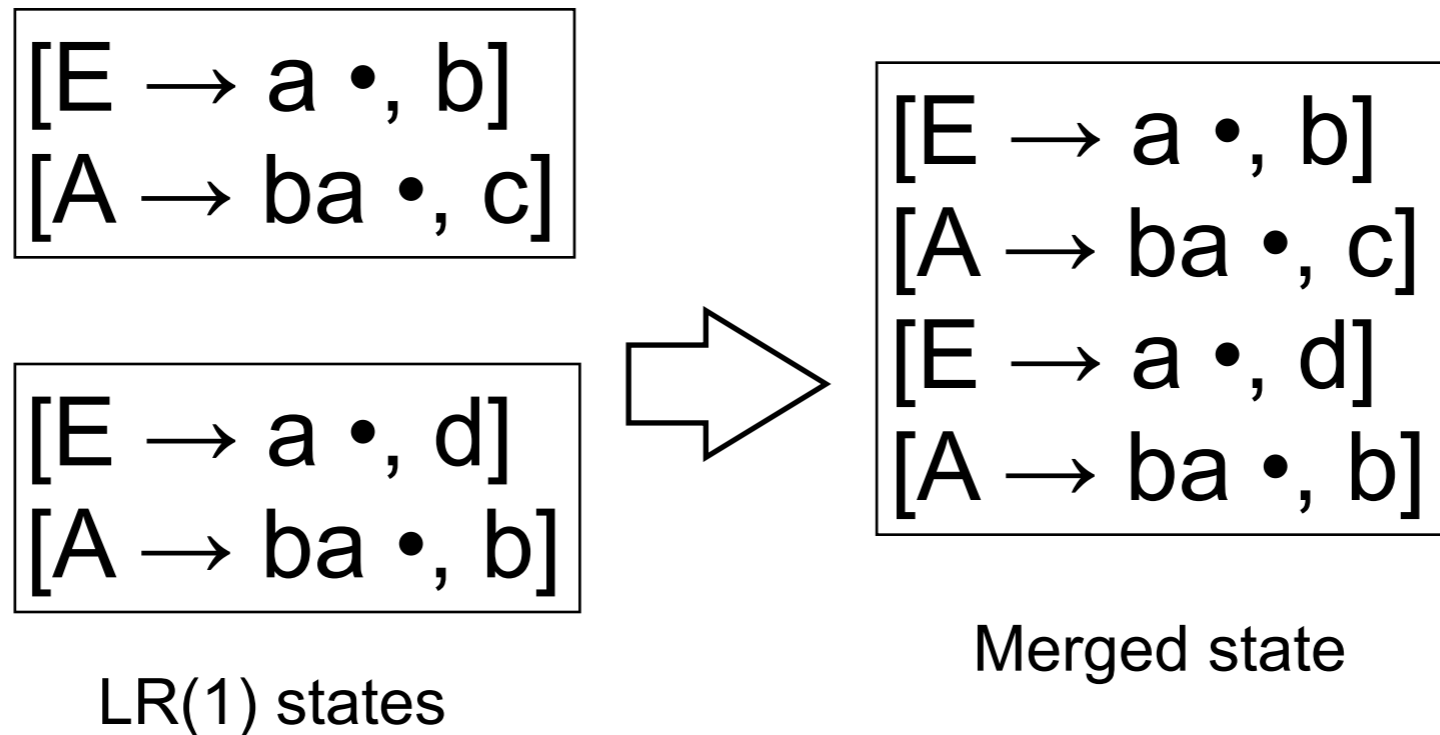
$[E \rightarrow a \bullet]$ $[A \rightarrow a \bullet]$
------------------------------------------------------------

Core

- LALR(1) parser merges two states if they have the same core
- Result
  - Potentially much smaller set of states
  - May introduce reduce/reduce conflicts
  - Will not introduce shift/reduce conflicts

# LALR(1) example

---



- Introduces reduce/reduce conflict
  - Can reduce either  $E \rightarrow a$  or  $A \rightarrow ba$  for lookahead =  $b$

# LALR(1) vs. LR(1)

---

- Example grammar

$$S' \rightarrow S$$

$$S \rightarrow aAd \mid bBd \mid aBe \mid bAe$$

$$A \rightarrow c$$

$$B \rightarrow c$$

- LR(0) ?
- LR(1) ?
- LALR(1) ?

# LR(k) Parsers

---

- Properties
  - Strictly more powerful than LL(k) parsers
  - Most general non-backtracking shift-reduce parser
  - Detects error as soon as possible in left-to-right scan of input
    - Contents of stack are **viable prefixes**
      - Possible for remaining input to lead to successful parse

# Error handling (lexing)

---

- What happens when input not handled by any lexing rule?
  - An exception gets raised
  - Better to provide more information, e.g.,

```
rule token = parse
...

| _ as lxm { Printf.printf "Illegal character %c" lxm;
              failwith "Bad input" }
```

- Even better, keep track of line numbers
  - Store in a global-ish variable (oh no!)
  - Increment as a side effect whenever `\n` recognized

# Error handling (parsing)

---

- What happens when parsing a string not in the grammar?
  - Reject the input
  - Do we keep going, parsing more characters?
    - May cause a cascade of error messages
    - Could be more useful to programmer, if they don't need to stop at the first error message (what do you do, in practice?)
- Ocaml yacc includes a basic error recovery mechanism
  - Special token **error** may appear in rhs of production
  - Matches erroneous input, allowing recovery

# Error example (1)

---

```
...  
expr:  
| term                { $1 }  
| expr PLUS term      { $1 + $3 }  
| error                { Printf.printf "invalid expression"; 0 }  
term: ...
```

- If unexpected input appears while trying to match **expr**, match token to **error**
  - Effectively treats token as if it is produced from **expr**
  - Triggers error action

# Error example (2)

---

```
...  
term:  
| INT          { $1 }  
| LPAREN expr RPAREN { $2 }  
| LPAREN error RPAREN {Printf.printf "Syntax error!\n"; 0}
```

- If unexpected input appears while trying to match **term**, match tokens to **error**
  - Pop every state off the stack until **LPAREN** on top
  - Scan tokens up to **RPAREN**, and discard those, also
  - Then match **error** production

# Error recovery in practice

---

- A very hard thing to get right!
  - Necessarily involves guessing at what malformed inputs you may see
- How useful is recovery?
  - Compilers are very fast today, so not so bad to stop at first error message, fix it, and go on
  - On the other hand, that does involve some delay
- Perhaps the most important feature is *good error messages*
  - Error recovery features useful for this, as well
  - Some compilers are better at this than others

# OCamlyacc tip

---

- Setting OCAMLRUNPARAM=p will cause the parsing steps to be printed out as the parser runs
- (And setting OCAMLRUNPARAM=b will tell OCaml to print a stack backtrace for any thrown exceptions.)

# Real programming languages

---

- Essentially all real programming languages don't quite work with parser generators
  - Even Java is not quite LALR(1)
- Thus, real implementations play tricks with parsing actions to resolve conflicts
- In-class exercise: C typedefs and identifier declarations/definitions

# Additional Parsing Technologies

---

- For a long time, parsing was a “dead” field
  - Considered solved a long time ago
- Recently, people have come back to it
  - LALR parsing can have unnecessary parsing conflicts
  - LALR parsing tradeoffs more important when computers were slower and memory was smaller
- Many recent new (or new-old) parsing techniques
  - GLR — generalized LR parsing, for ambiguous grammars
  - LL(\*) — ANTLR
  - Packrat parsing — for *parsing expression grammars*
  - etc...
- The input syntax to many of these looks like yacc/lex

# Designing language syntax

---

- Idea 1: Make it look like other, popular languages
  - Java did this (OO with C syntax)
- Idea 2: Make it look like the domain
  - There may be well-established notation in the domain (e.g., mathematics)
  - Domain experts already know that notation
- Idea 3: Measure design choices
  - E.g., ask users to perform programming (or related) task with various choices of syntax, evaluate performance, survey them on understanding
    - This is very hard to do!
- Idea 4: Make your users adapt
  - People are really good at learning...