CMSC 430
Introduction to Compilers
Fall 2018

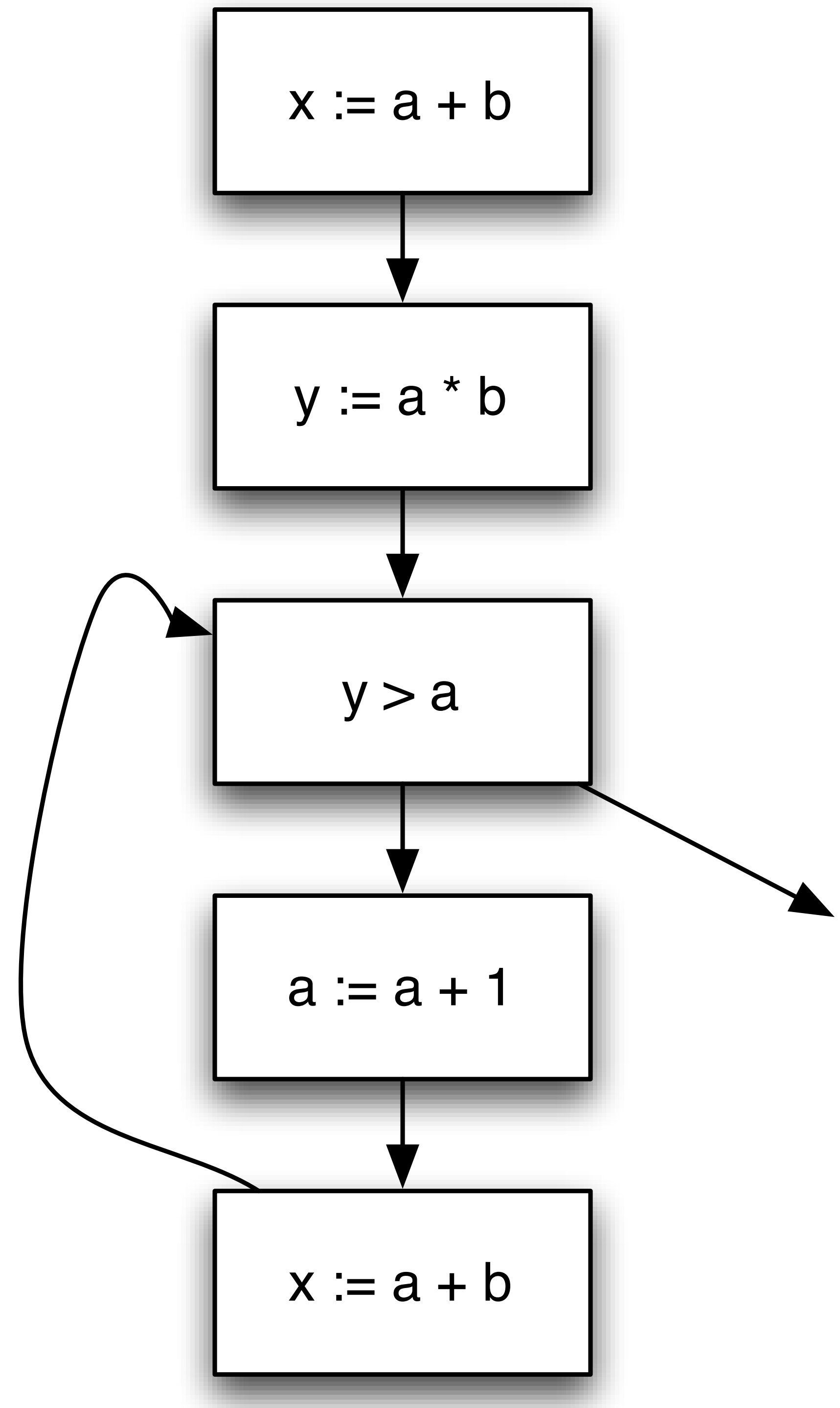
Data Flow Analysis

Data Flow Analysis

- A framework for proving facts about programs
- Reasons about lots of little facts
- Little or no interaction between facts
  - Works best on properties about *how* program computes
- Based on all paths through program
  - Including infeasible paths
- Operates on control-flow graphs, typically

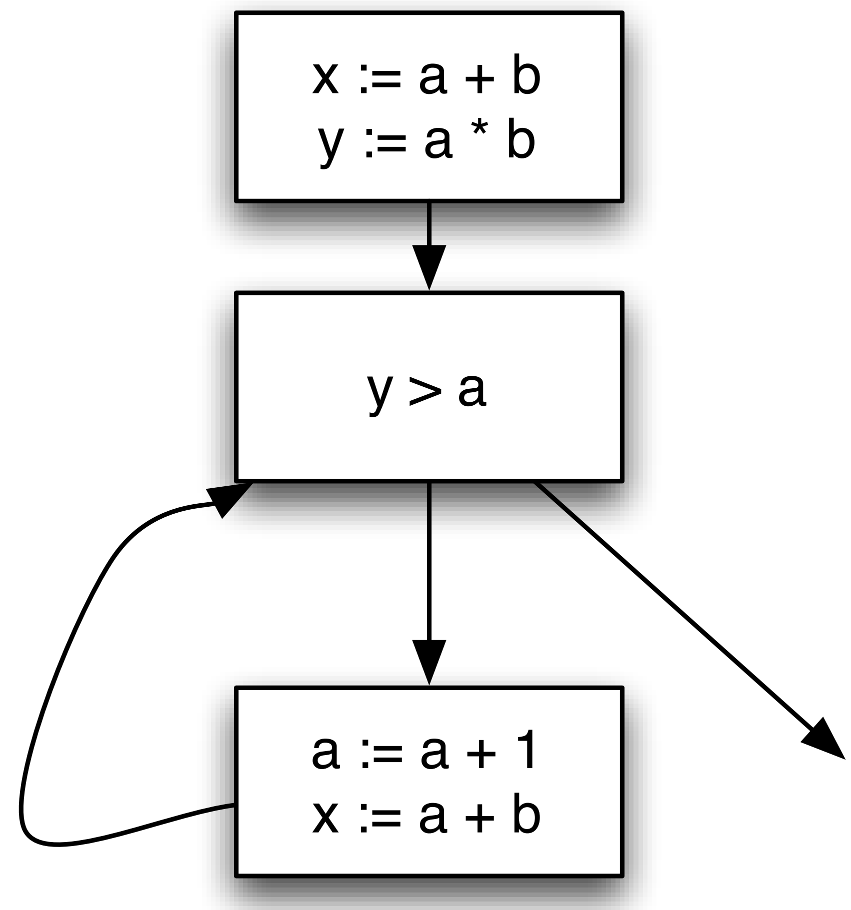
Control-Flow Graph Example

```
x := a + b;
y := a * b;
while (y > a) {
    a := a + 1;
    x := a + b
}
```



Control-Flow Graph w/Basic Blocks

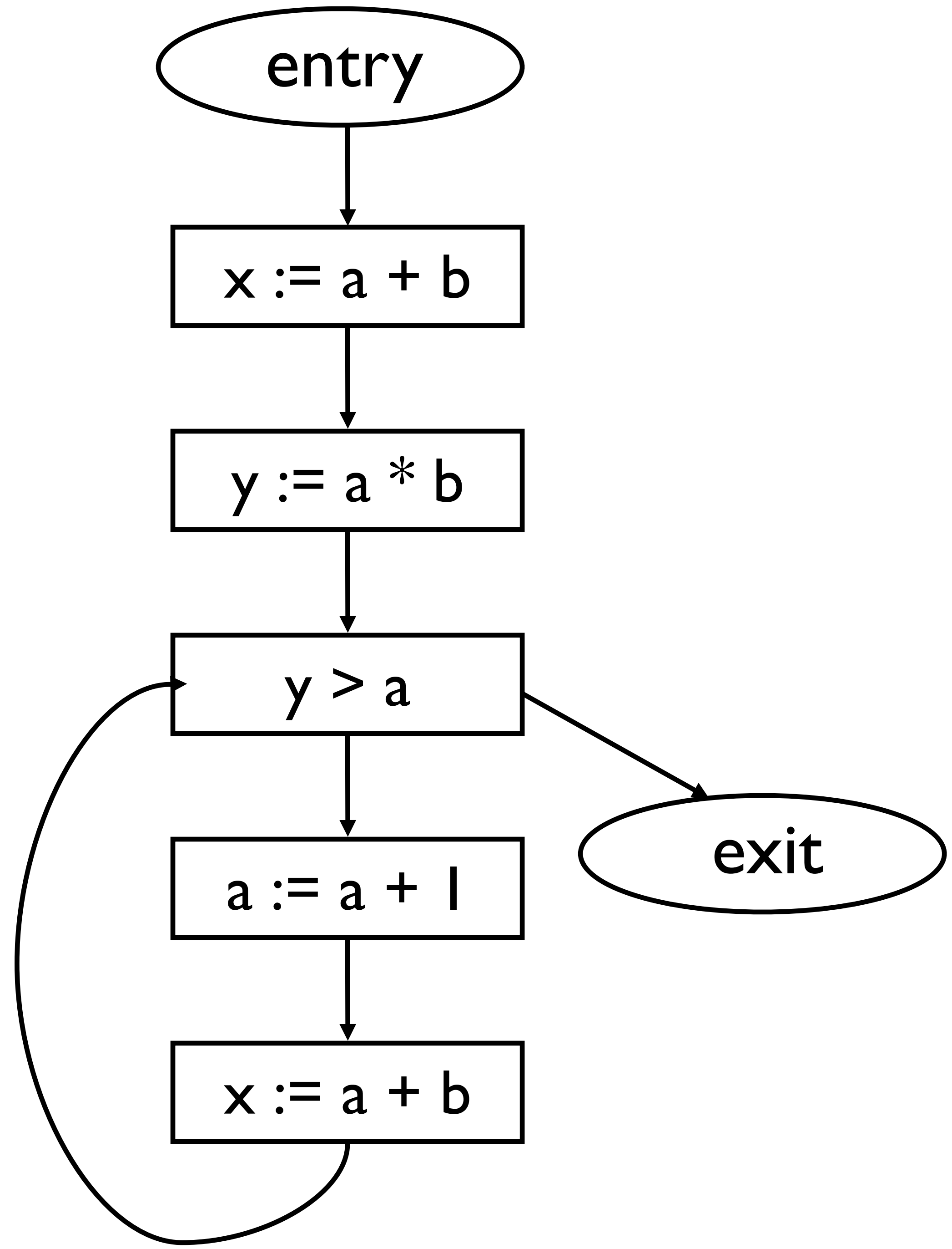
```
x := a + b;
y := a * b;
while (y > a + b) {
   a := a + 1;
   x := a + b
}
```



- Can lead to more efficient implementations
- But more complicated to explain, so...
  - We'll use single-statement blocks in lecture today

Example with Entry and Exit

```
x := a + b;
y := a * b;
while (y > a) {
    a := a + 1;
    x := a + b
}
```



- All nodes without a (normal) predecessor should be pointed to by entry
- All nodes without a successor should point to exit

Notes on Entry and Exit

- Typically, we perform data flow analysis on a function body
- Functions usually have
  - A unique entry point
  - Multiple exit points
- So in practice, there can be multiple exit nodes in the CFG
  - For the rest of these slides, we'll assume there's only one
  - In practice, just treat all exit nodes the same way as if there's only one exit node

Available Expressions

Available Expressions

- An expression e is available at program point p if
  - e is computed on every path to p, and
  - the value of e has not changed since the last time e was computed on the paths to p

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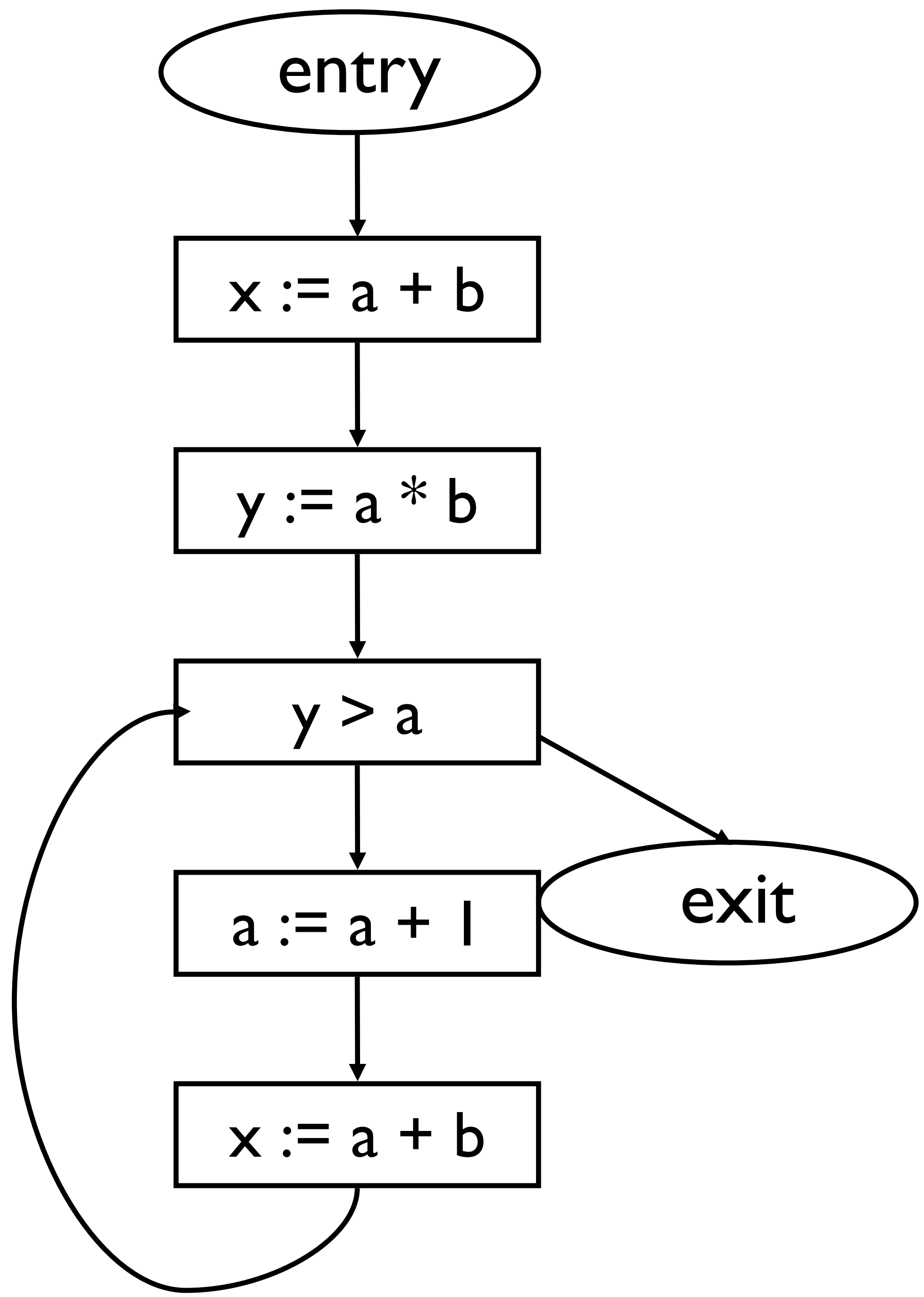
  - the value of e has not changed since the last time e was computed on the paths to p

- Optimization
  - If an expression is available, need not be recomputed
    - (At least, if it's still in a register somewhere)

Data Flow Facts

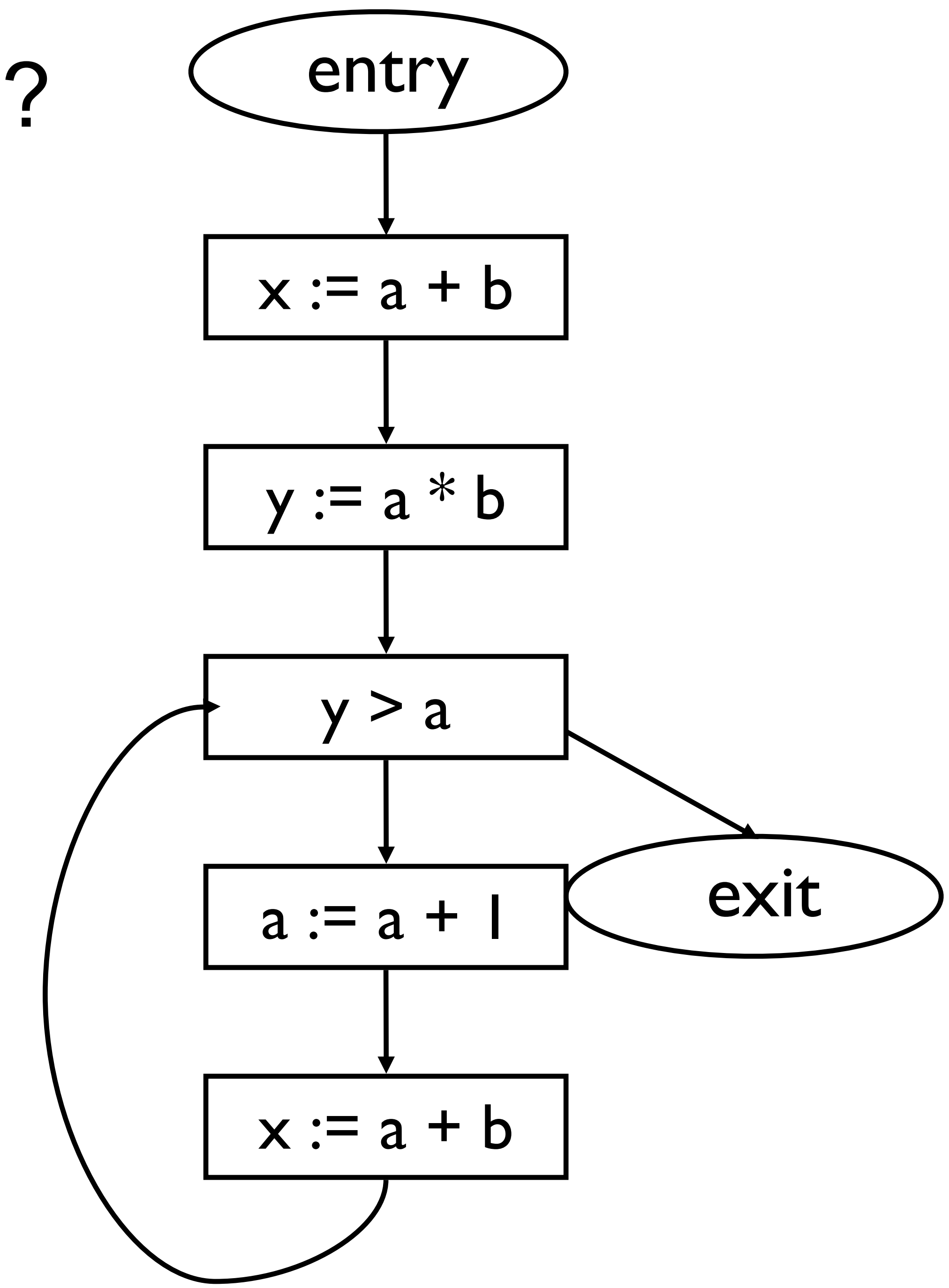
- Is expression e available?
- Facts:
  - a + b is available
  - a * b is available
  - a + 1 is available



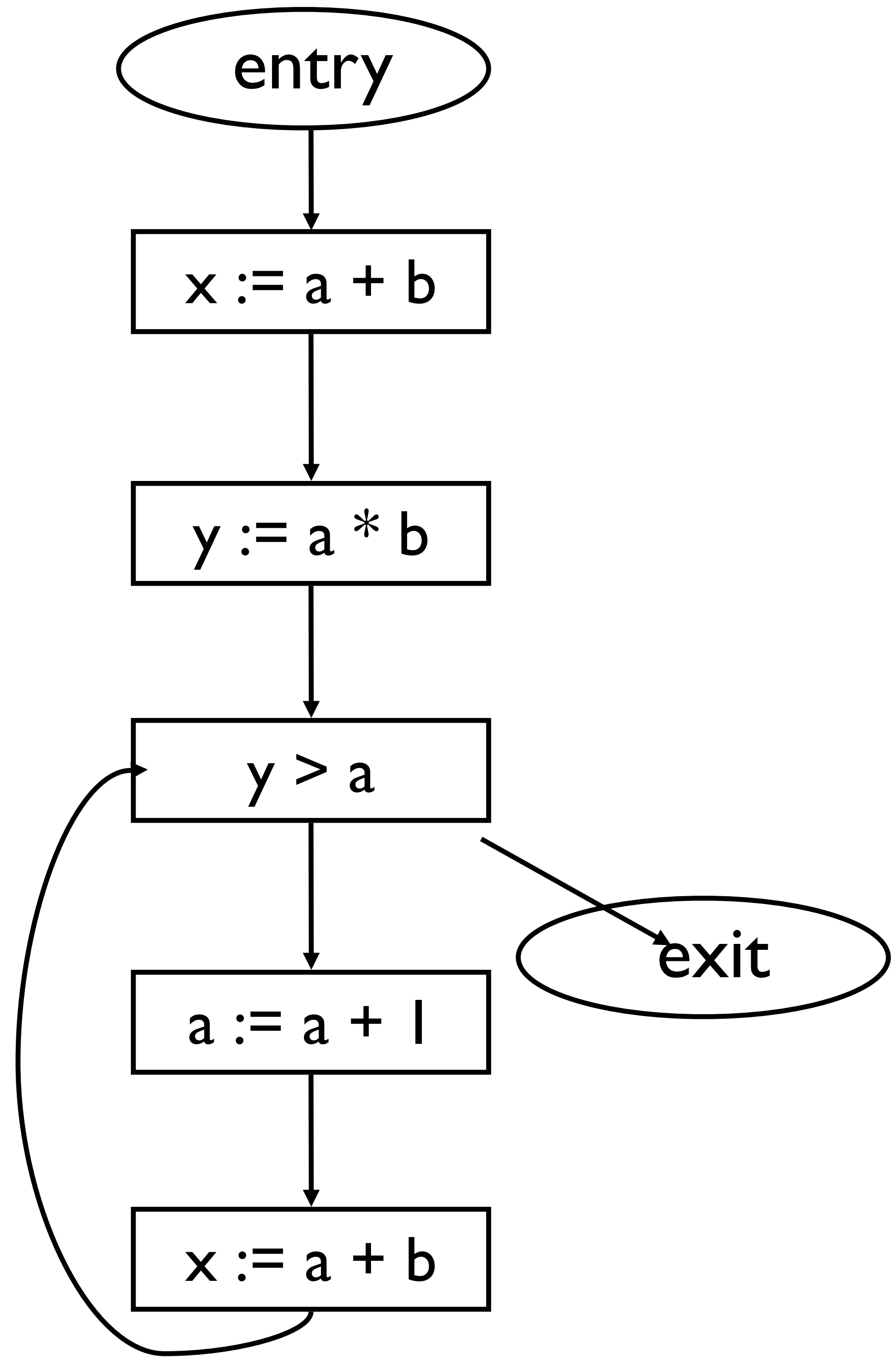
Gen and Kill

- What is the effect of each statement on the set of facts?

Stmt	Gen	Kill
x := a + b	a + b	
y := a * b	a * b	
a := a + 1		a + 1, a + b, a * b



Computing Available Expressions



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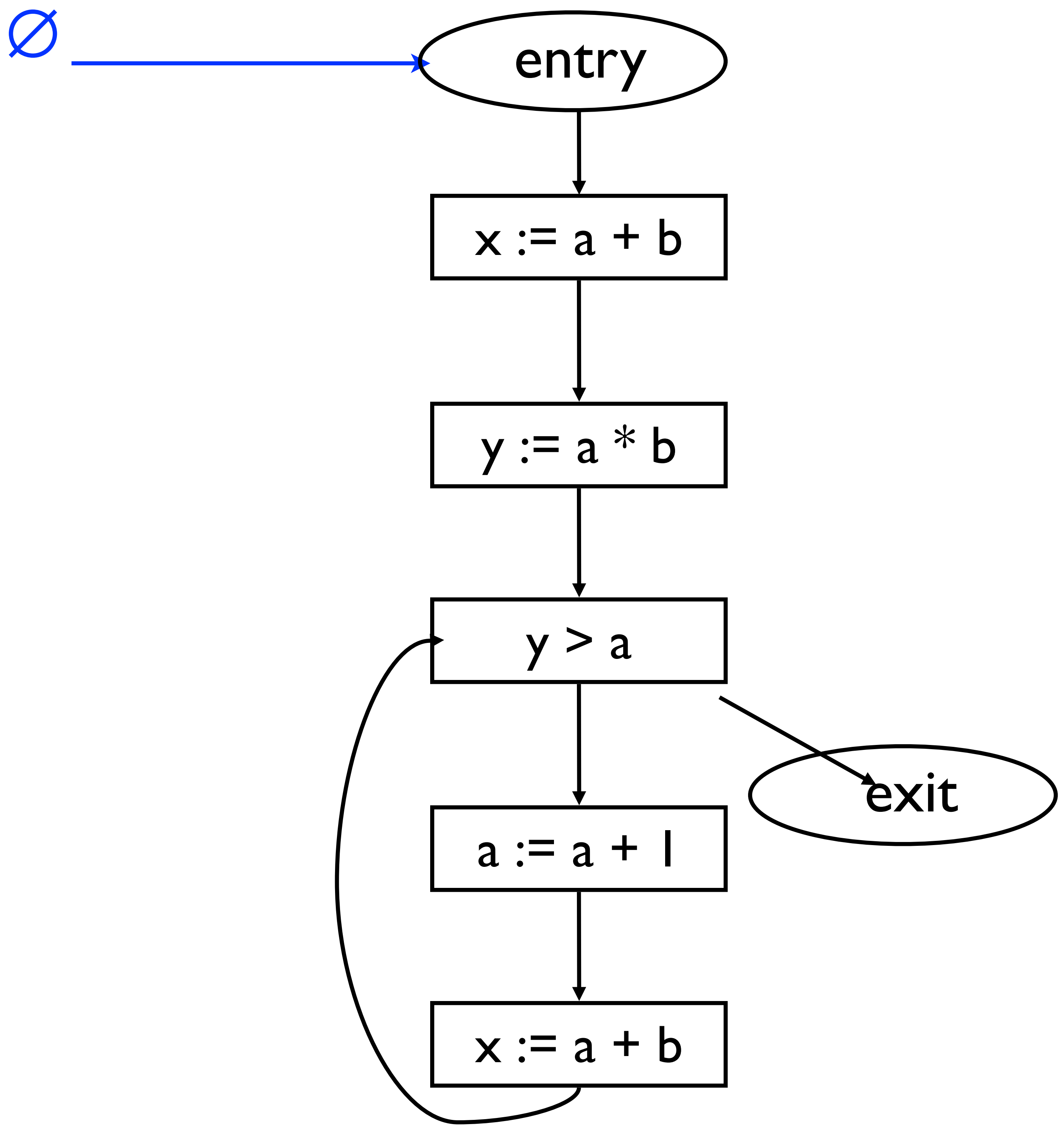

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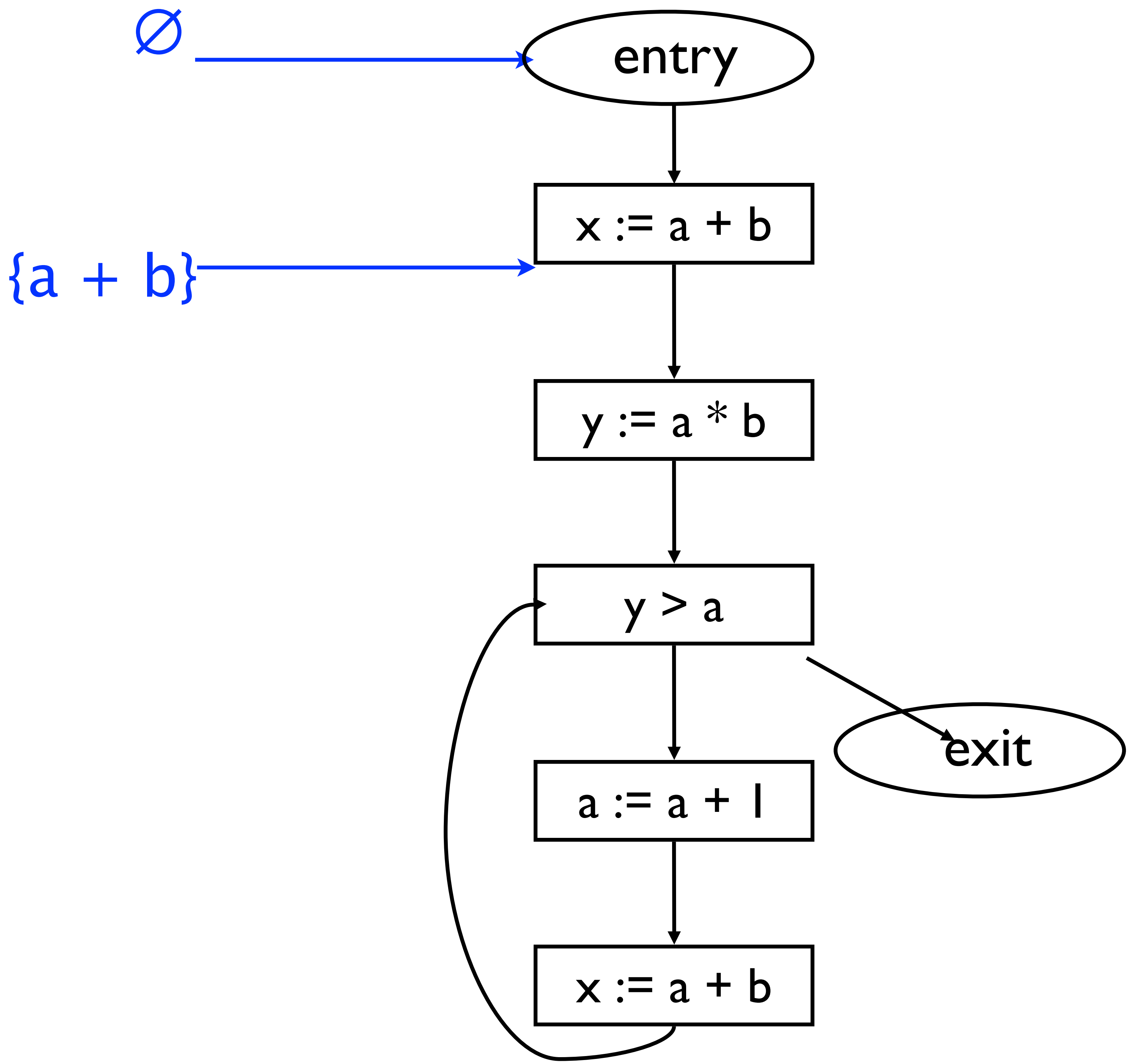

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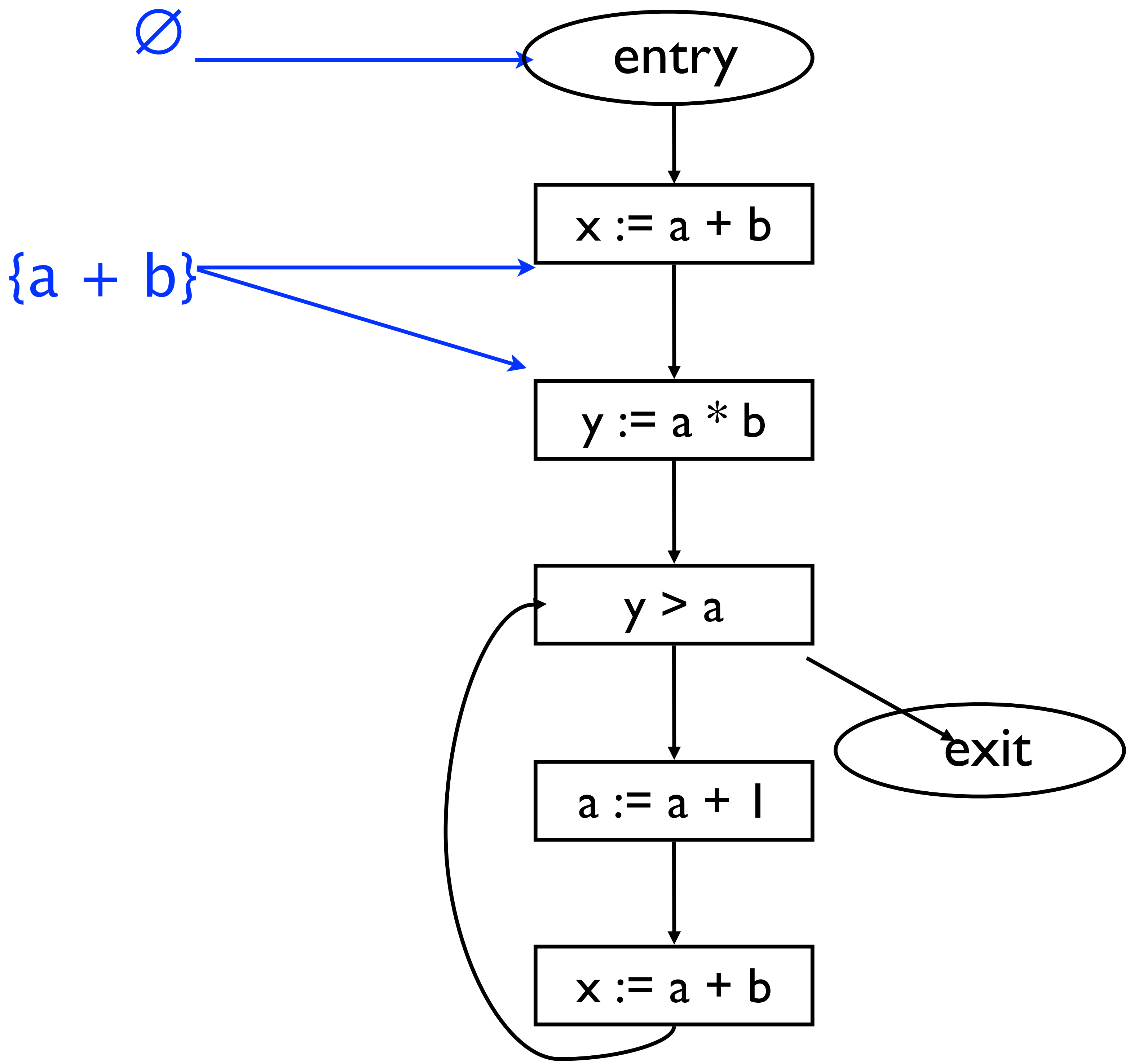

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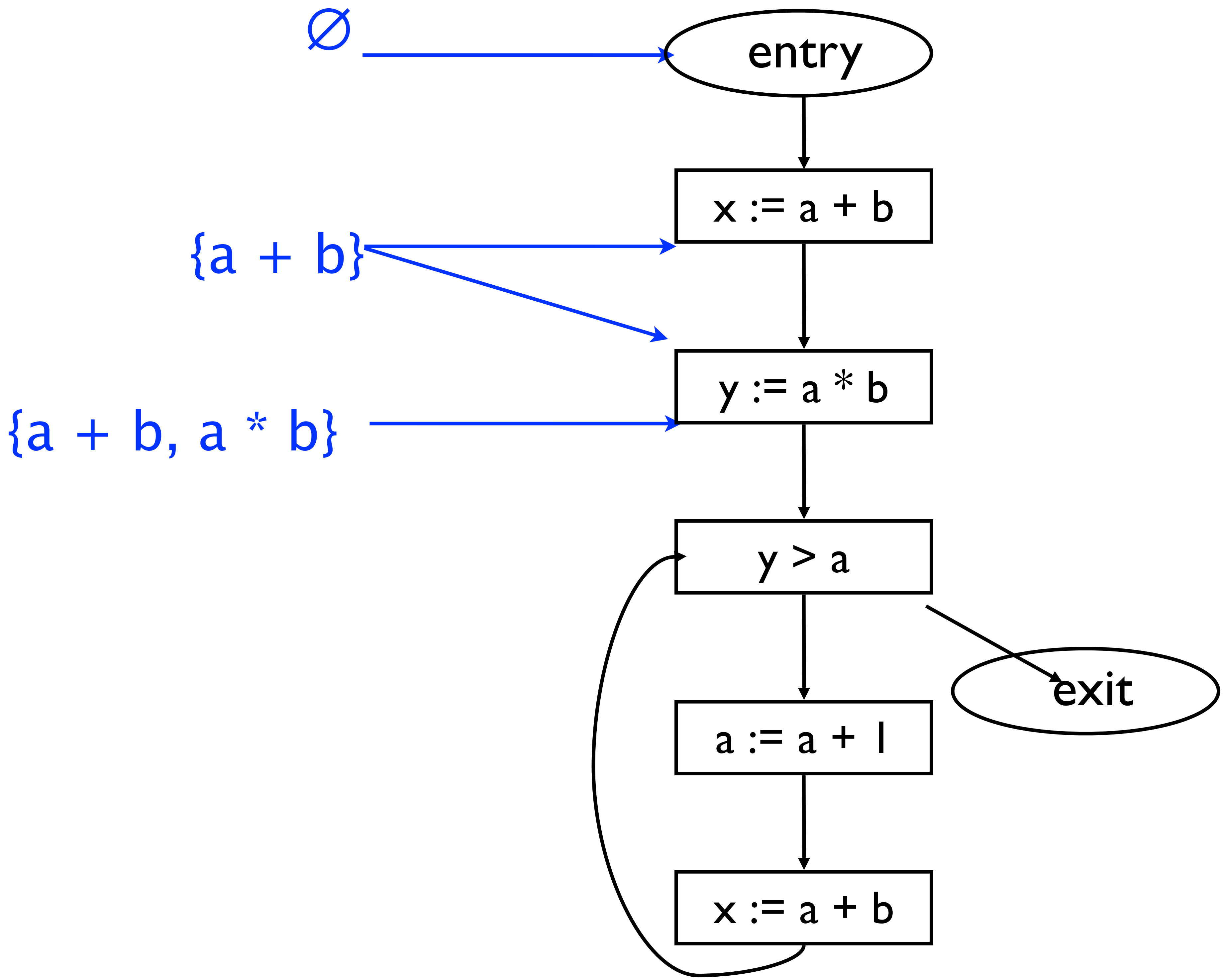

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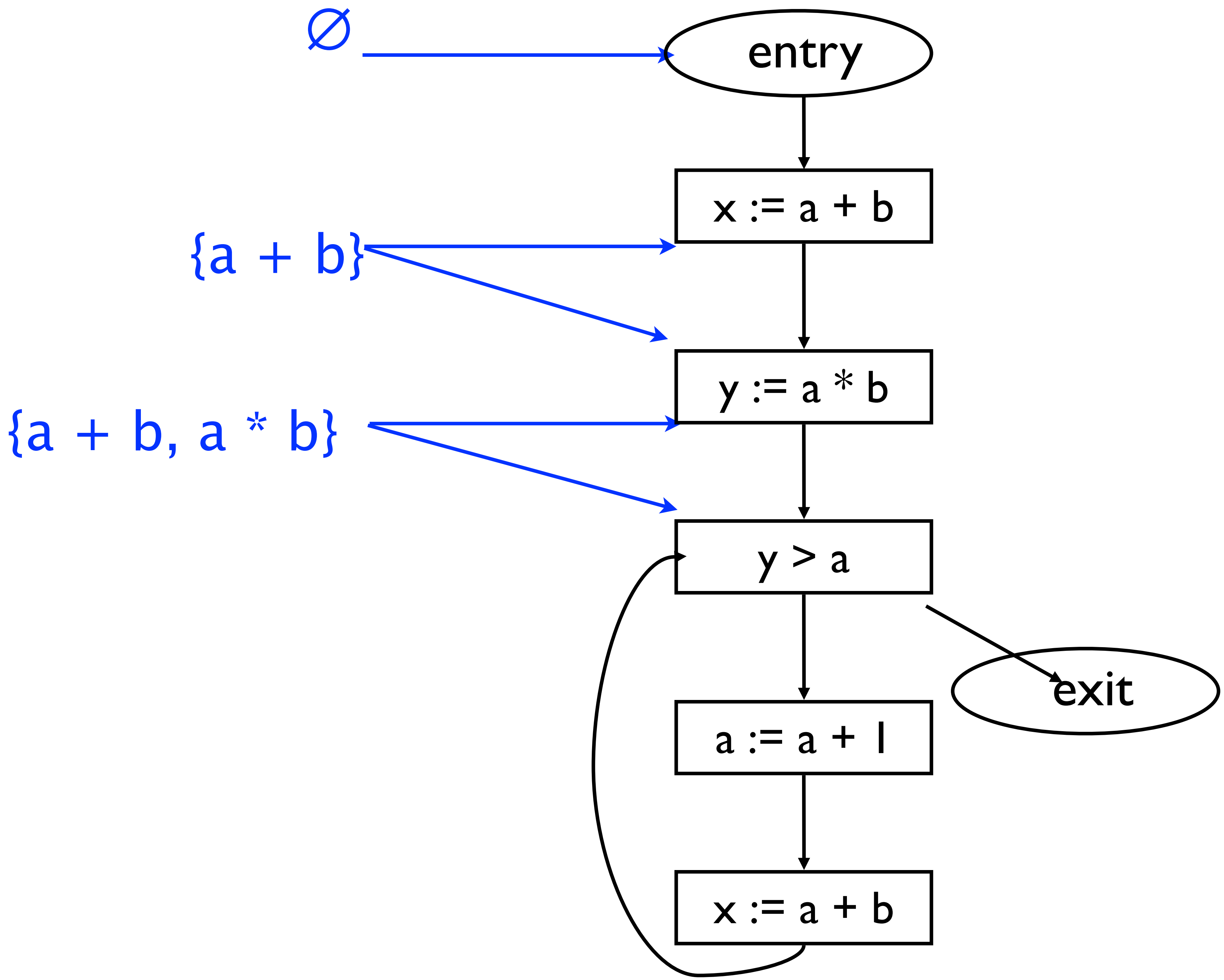

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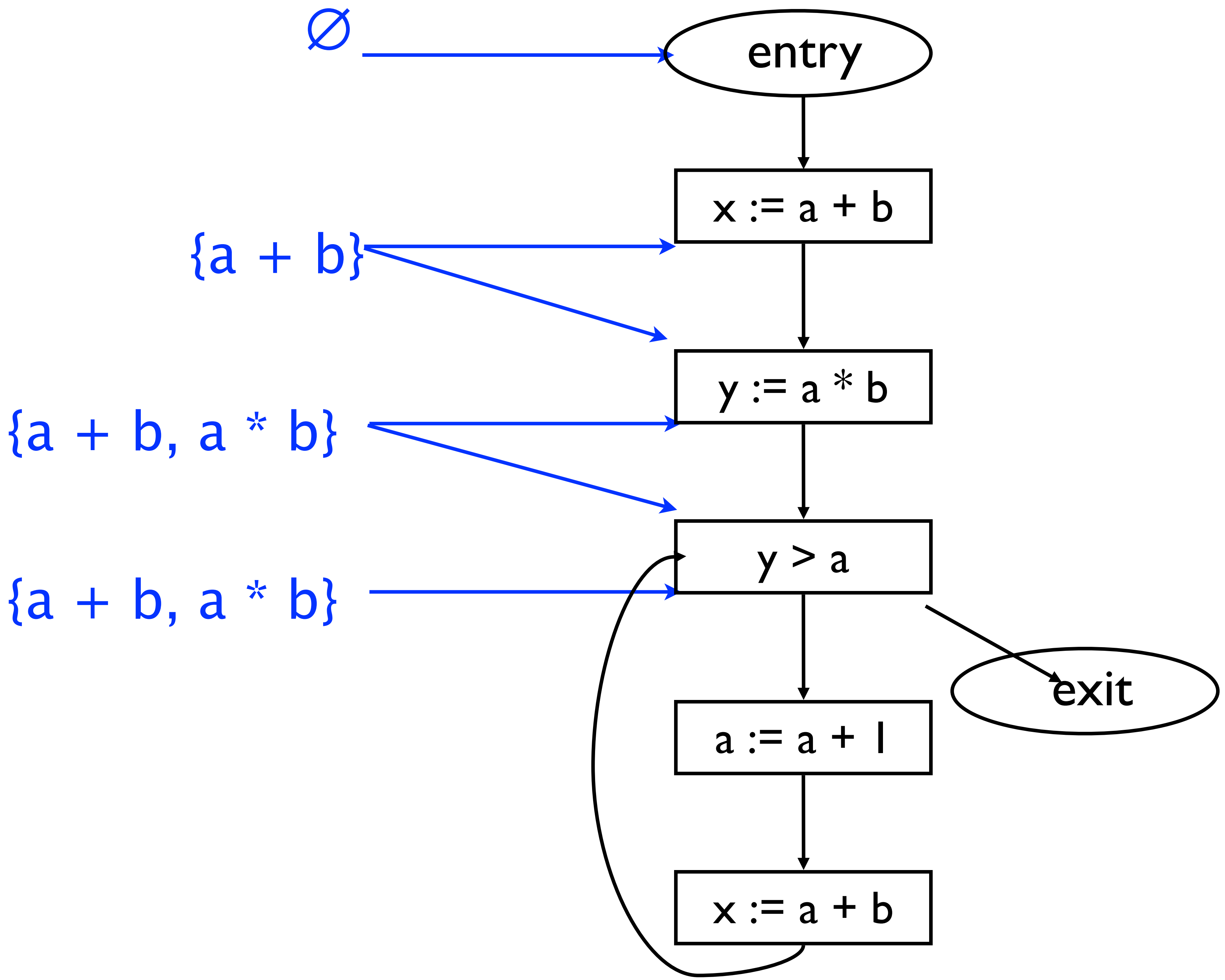

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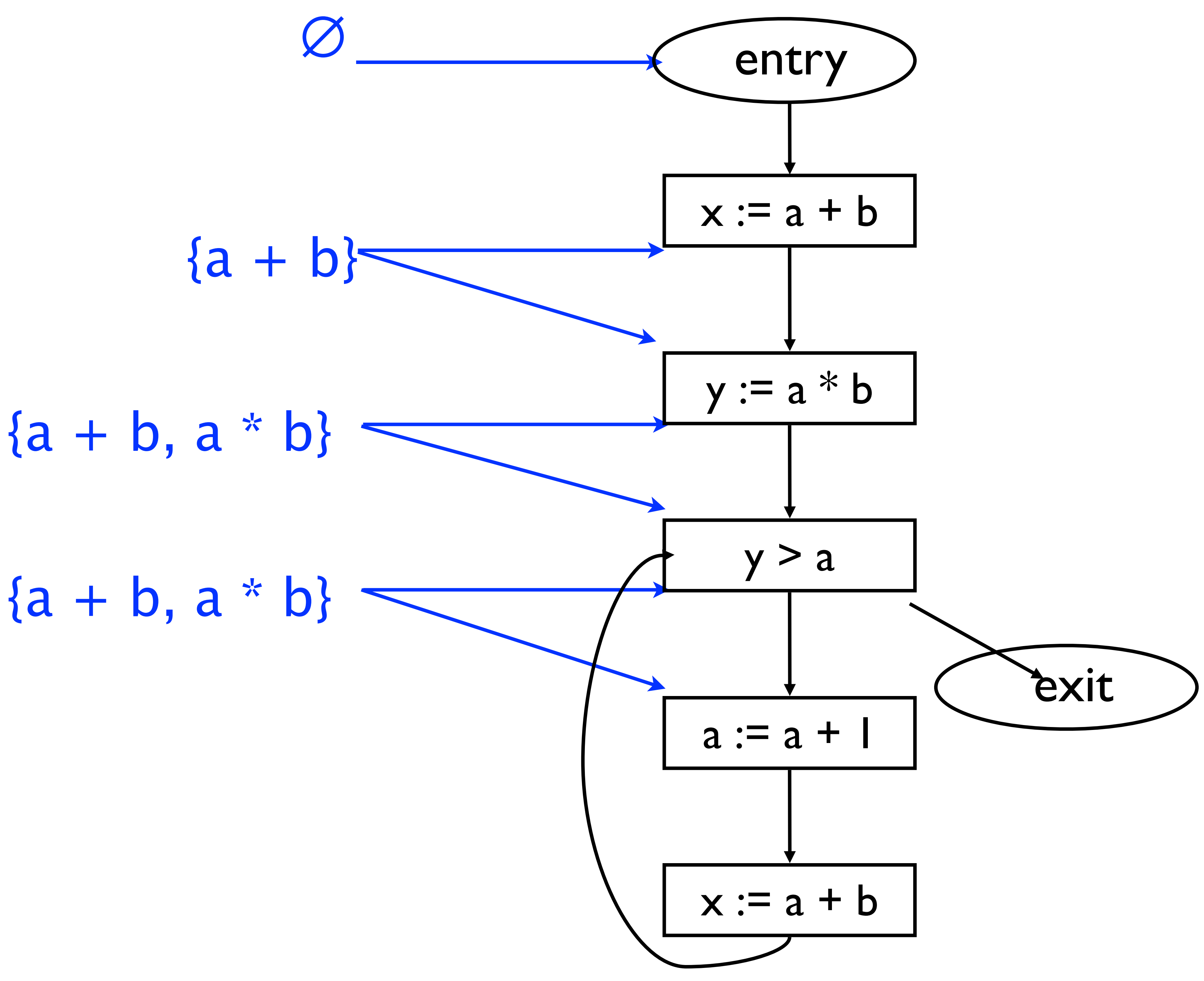

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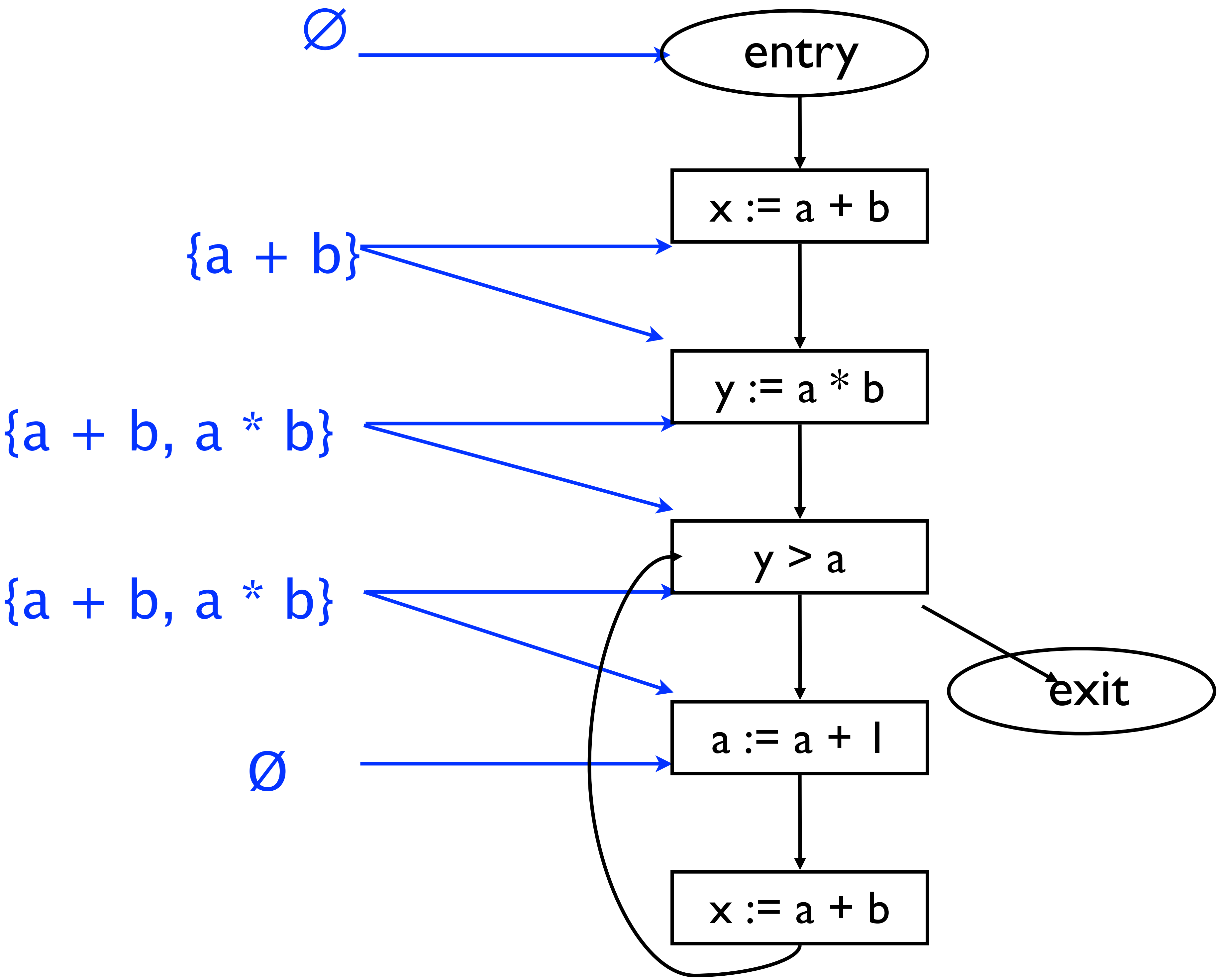

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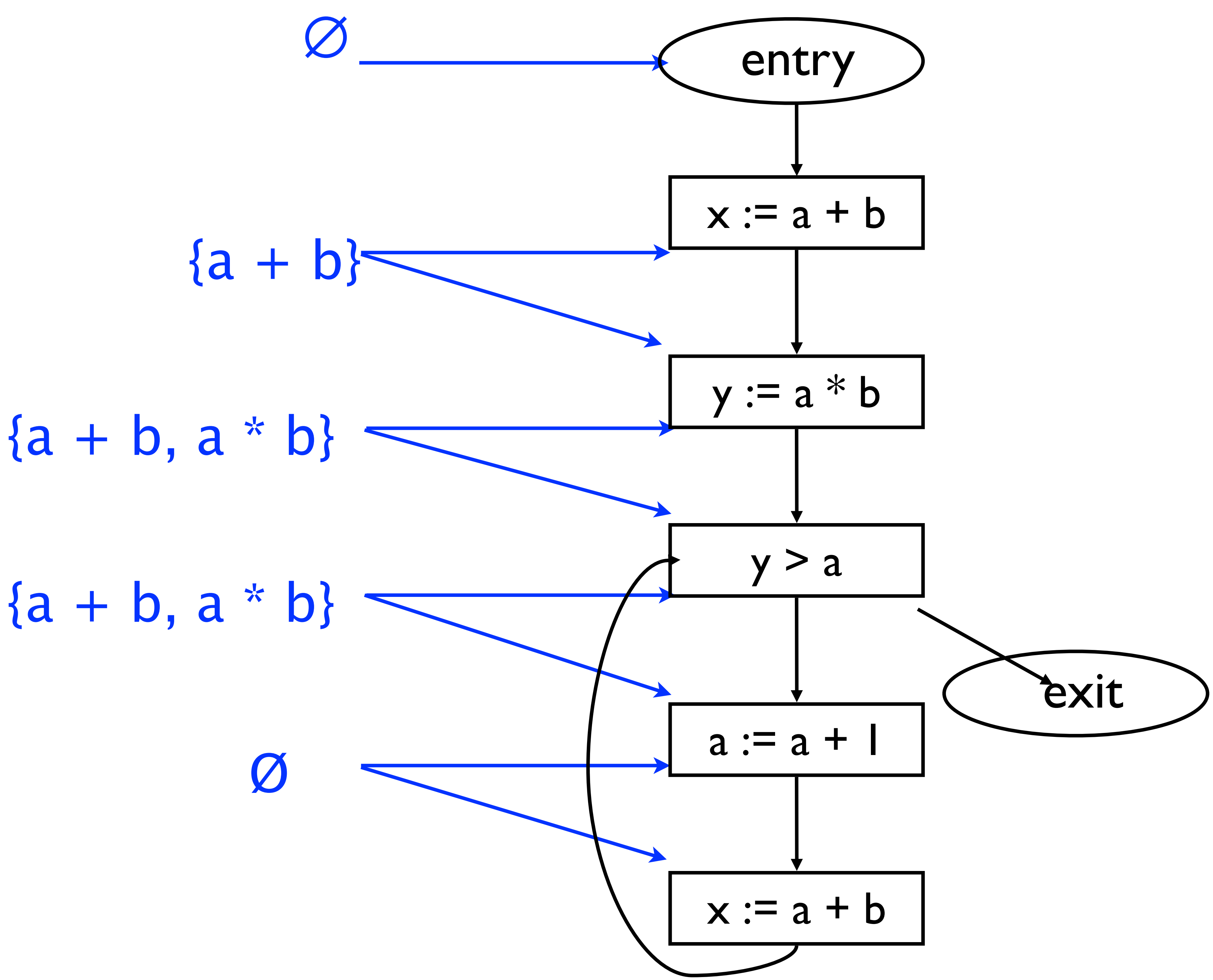

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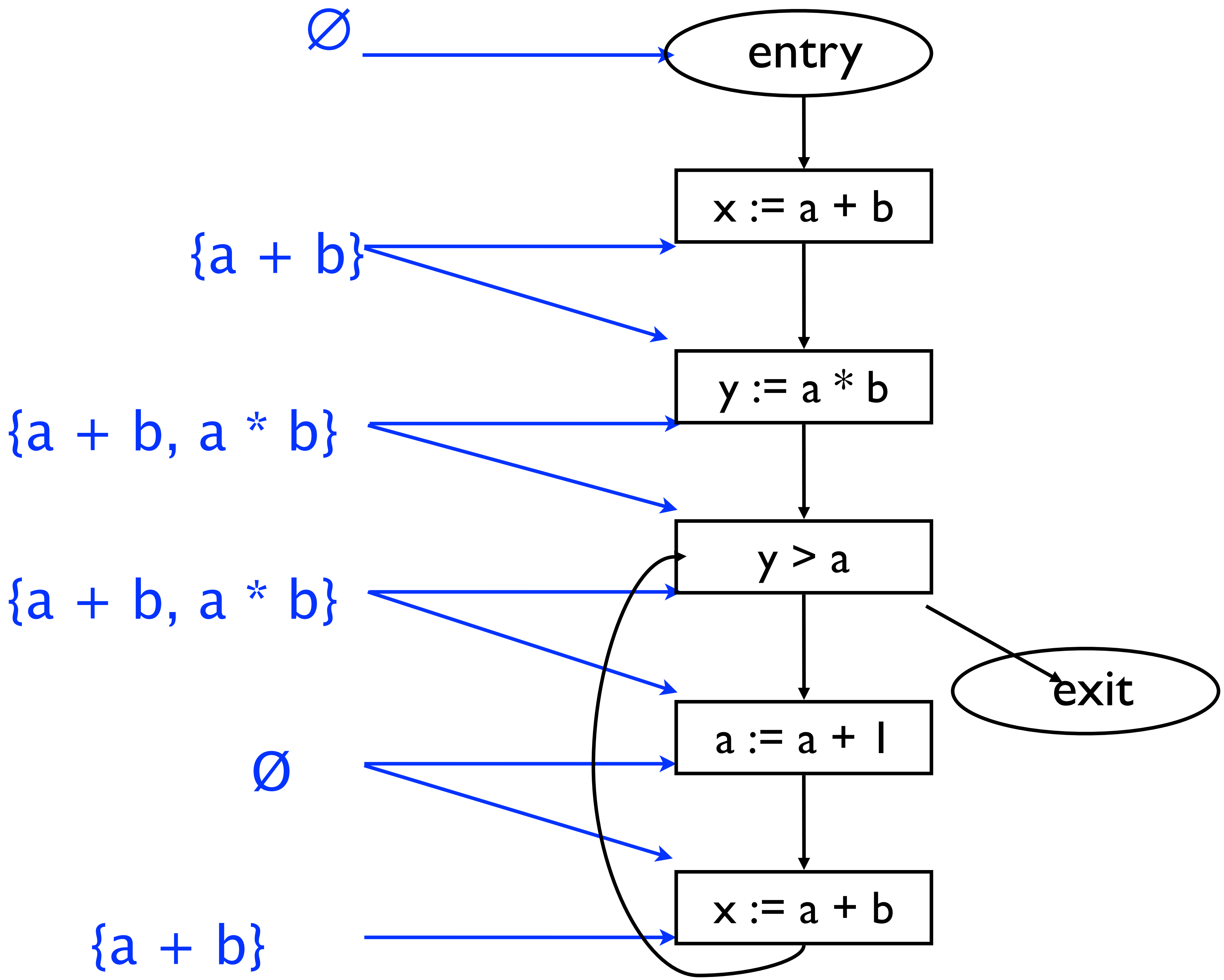

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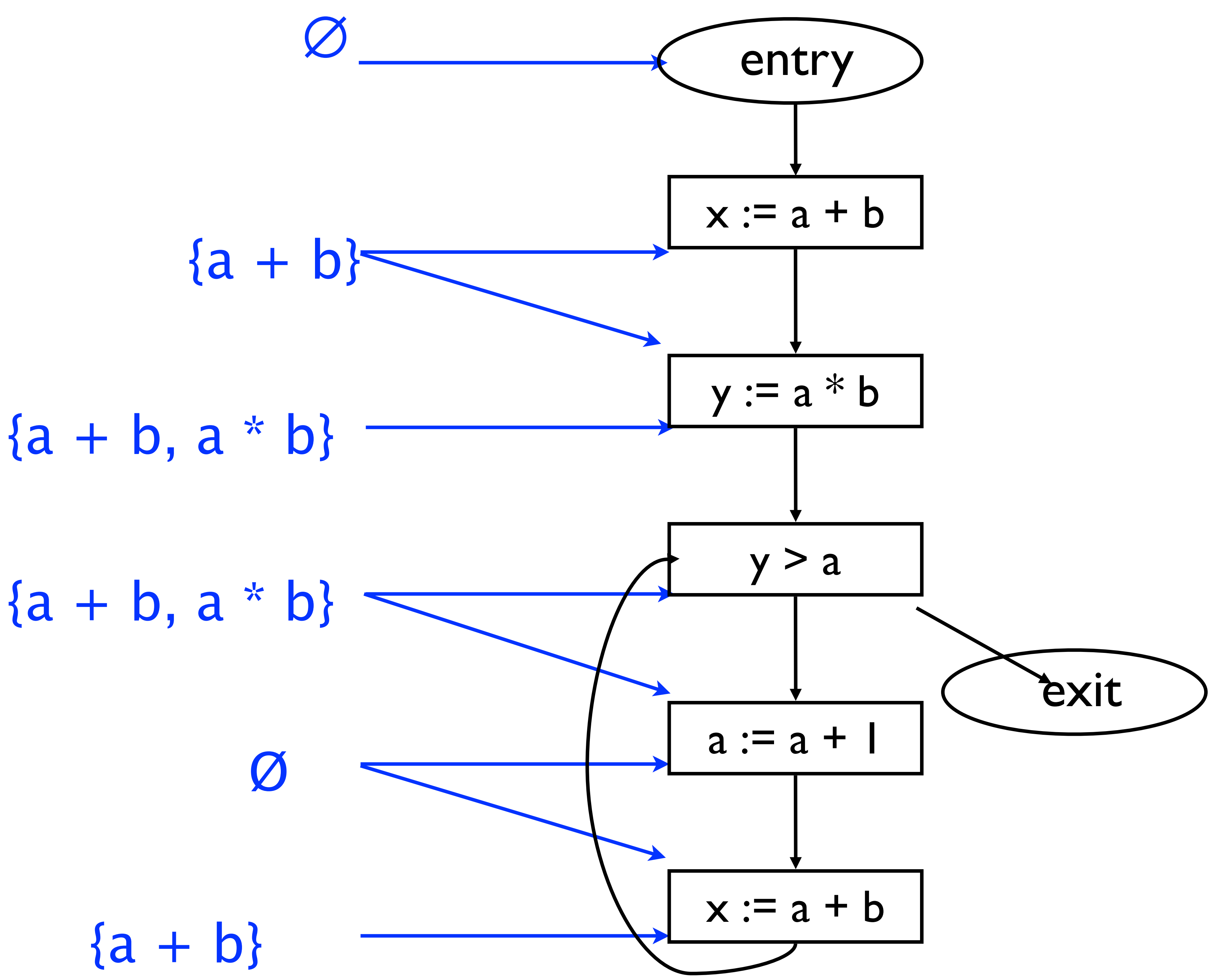

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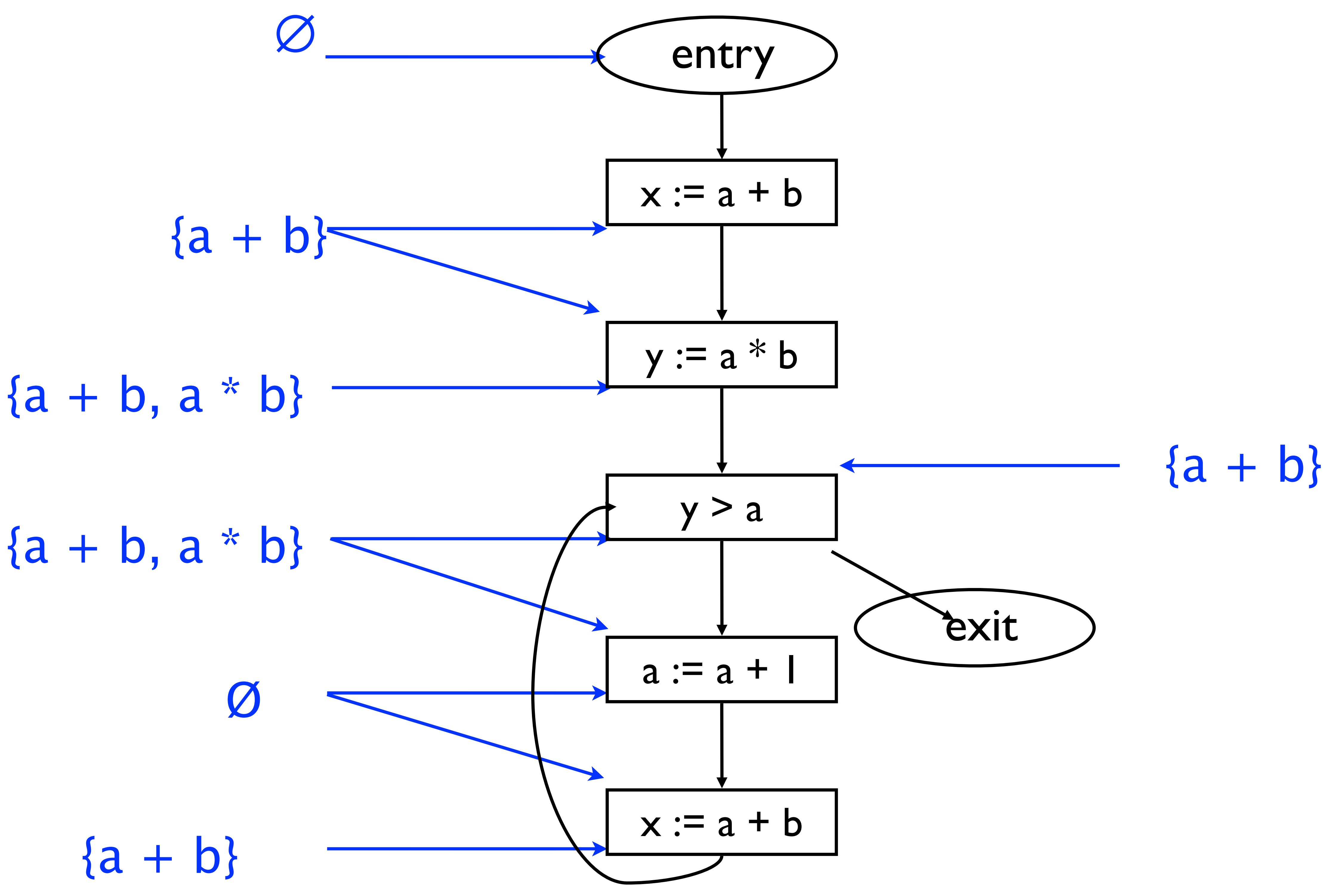

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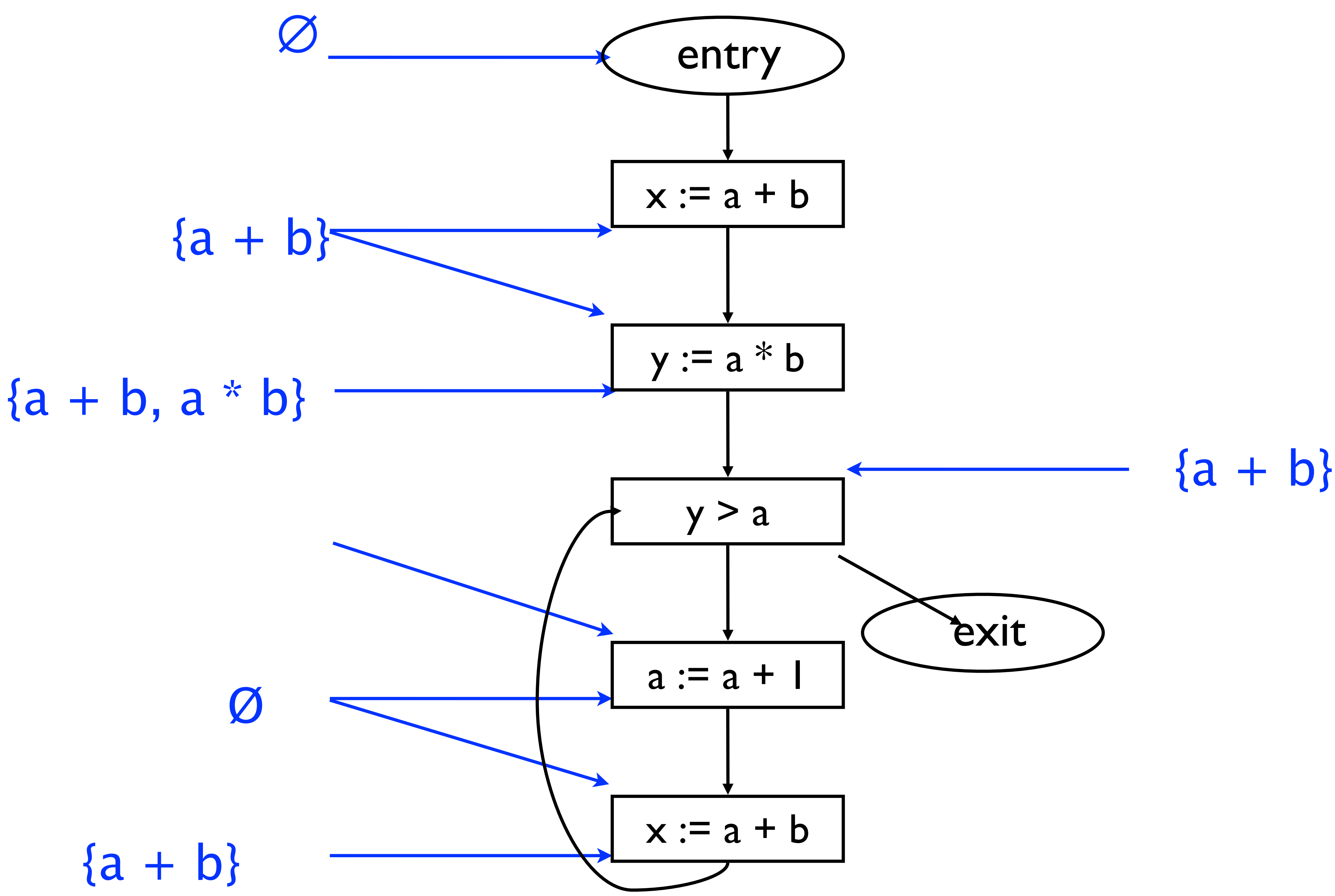

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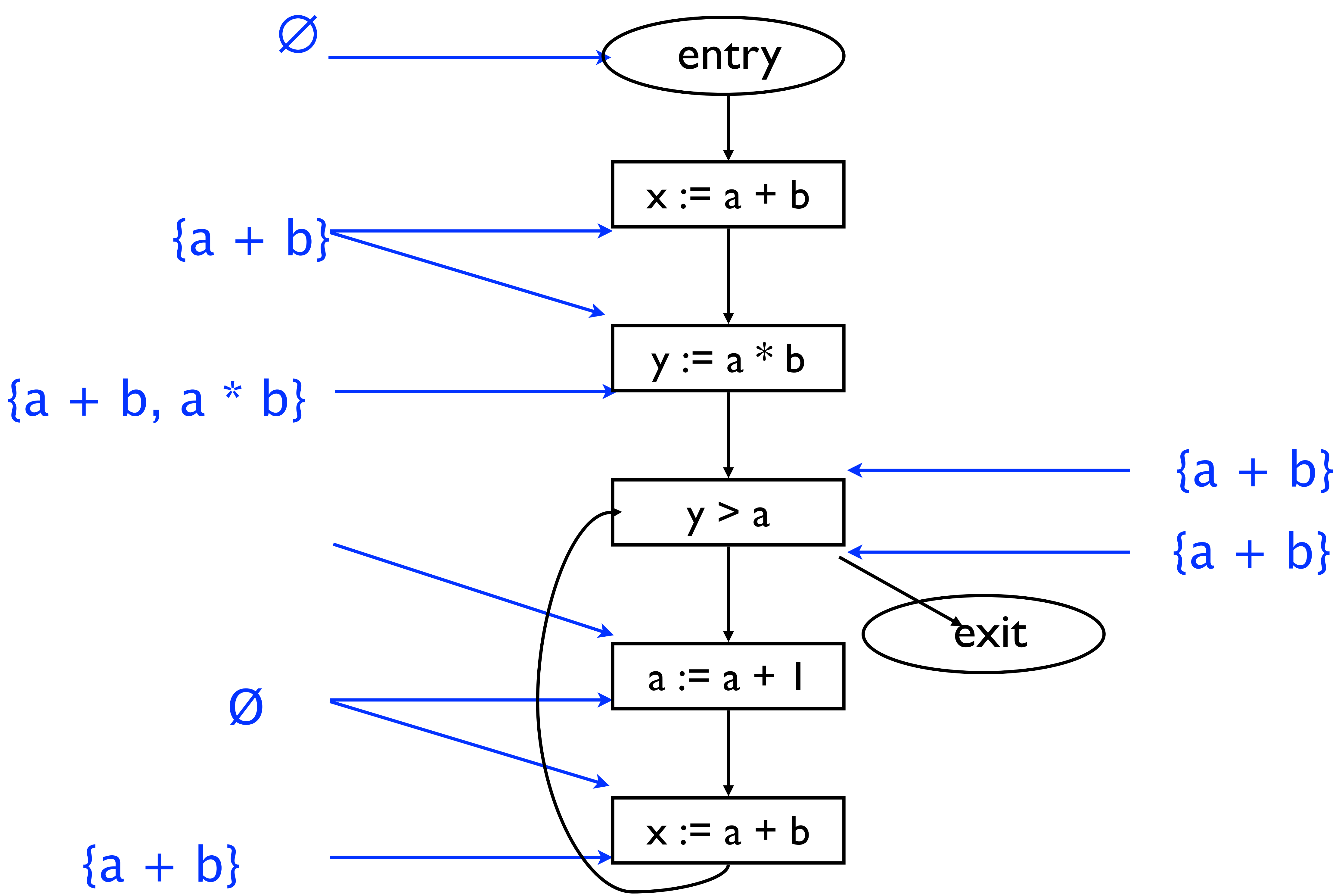

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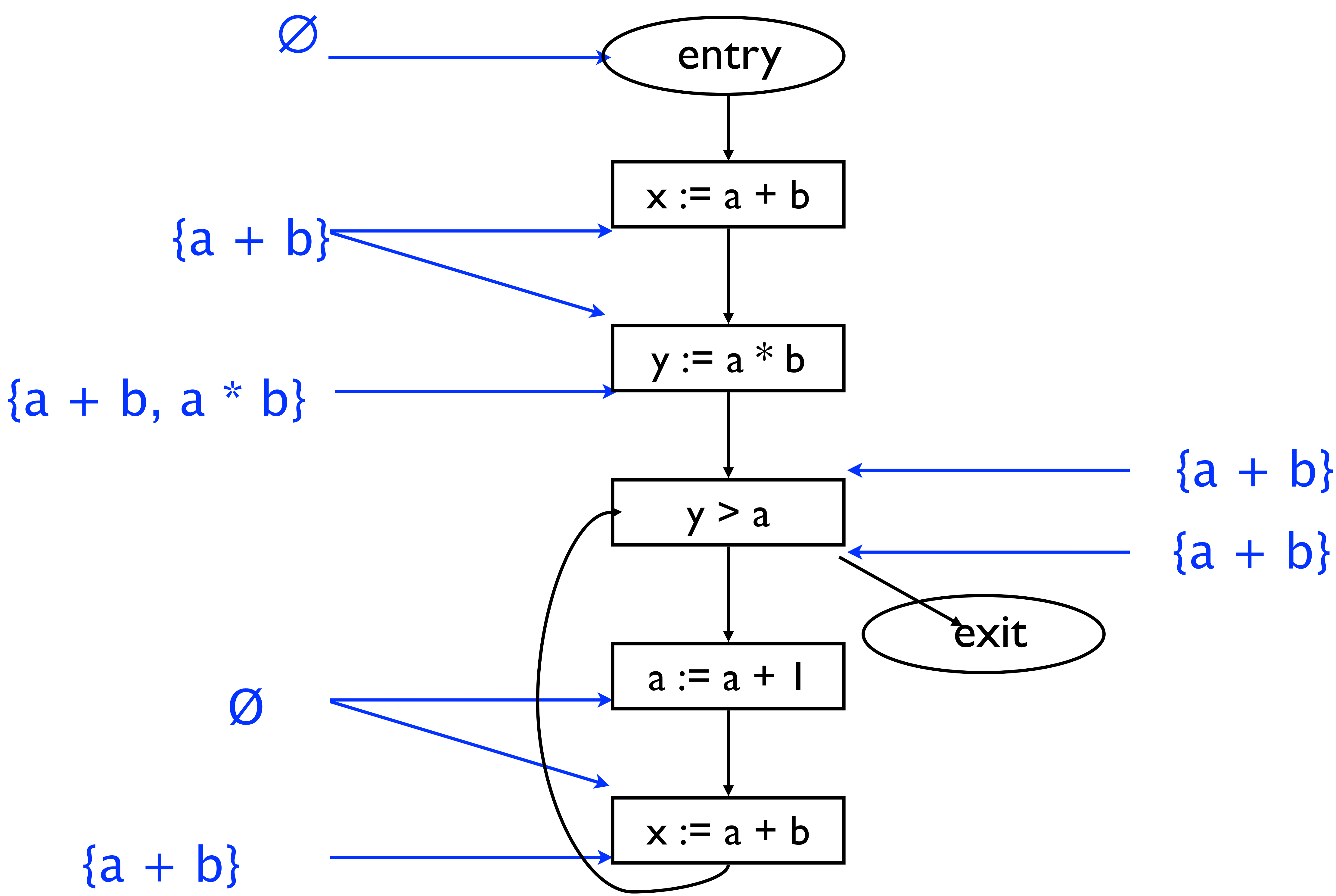

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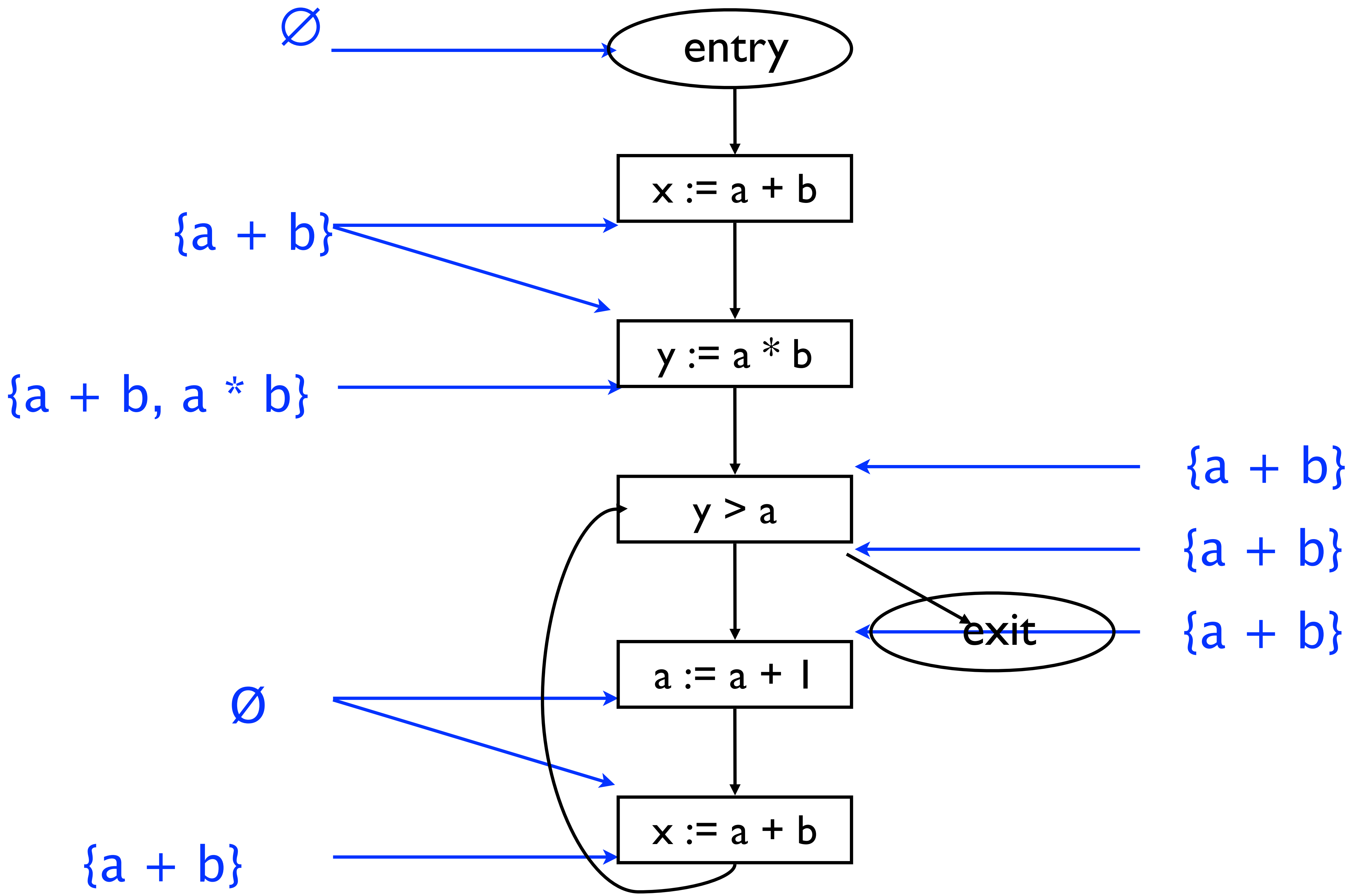

Terminology

- A *joint point* is a program point where two branches meet

- Available expressions is a *forward must* problem
  - Forward = Data flow from in to out
  - Must = At join point, property must hold on all paths that are joined

Data Flow Equations

- Let s be a statement
  - succ(s) = { immediate successor statements of s }
  - pred(s) = { immediate predecessor statements of s}
  - in(s) = program point just before executing s
  - out(s) = program point just after executing s

- $\text{in}(s) = \bigcap_{s' \in \text{pred}(s)} \text{out}(s')$

- $\text{out}(s) = \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$
  - Note: These are also called *transfer functions*

Liveness Analysis

Liveness Analysis

- A variable v is *live* at program point p if
  - v will be used on some execution path originating from p...
  - before v is overwritten

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- Optimization
  - If a variable is not live, no need to keep it in a register
  - If variable is dead at assignment, can eliminate assignment

Data Flow Equations

- Available expressions is a forward must analysis
  - Data flow propagate in same dir as CFG edges
  - Expr is available only if available on all paths

- Liveness is a *backward may* problem
  - To know if variable live, need to look at future uses
  - Variable is live if used on some path

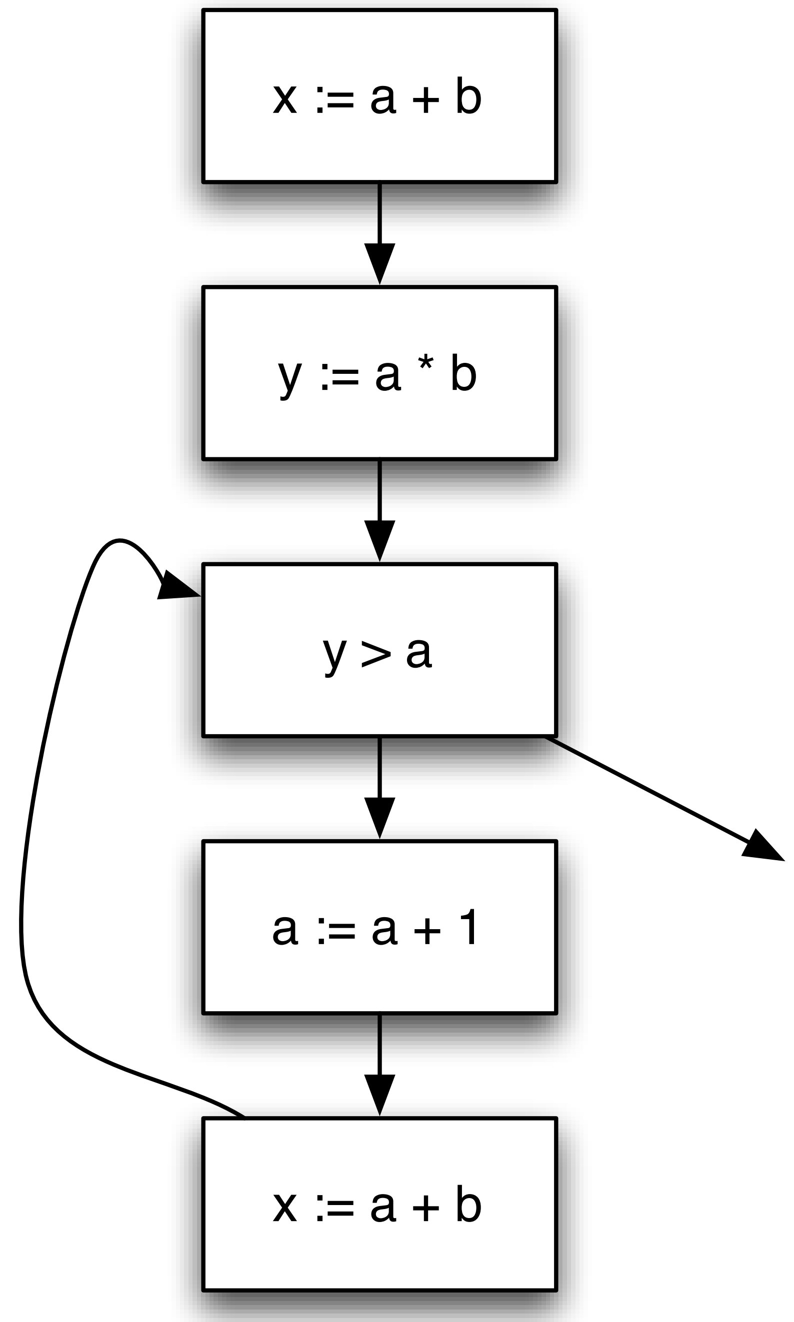
- $\text{out}(s) = \bigcup_{s' \in \text{succ}(s)} \text{in}(s')$

- $\text{in}(s) = \text{gen}(s) \cup (\text{out}(s) - \text{kill}(s))$

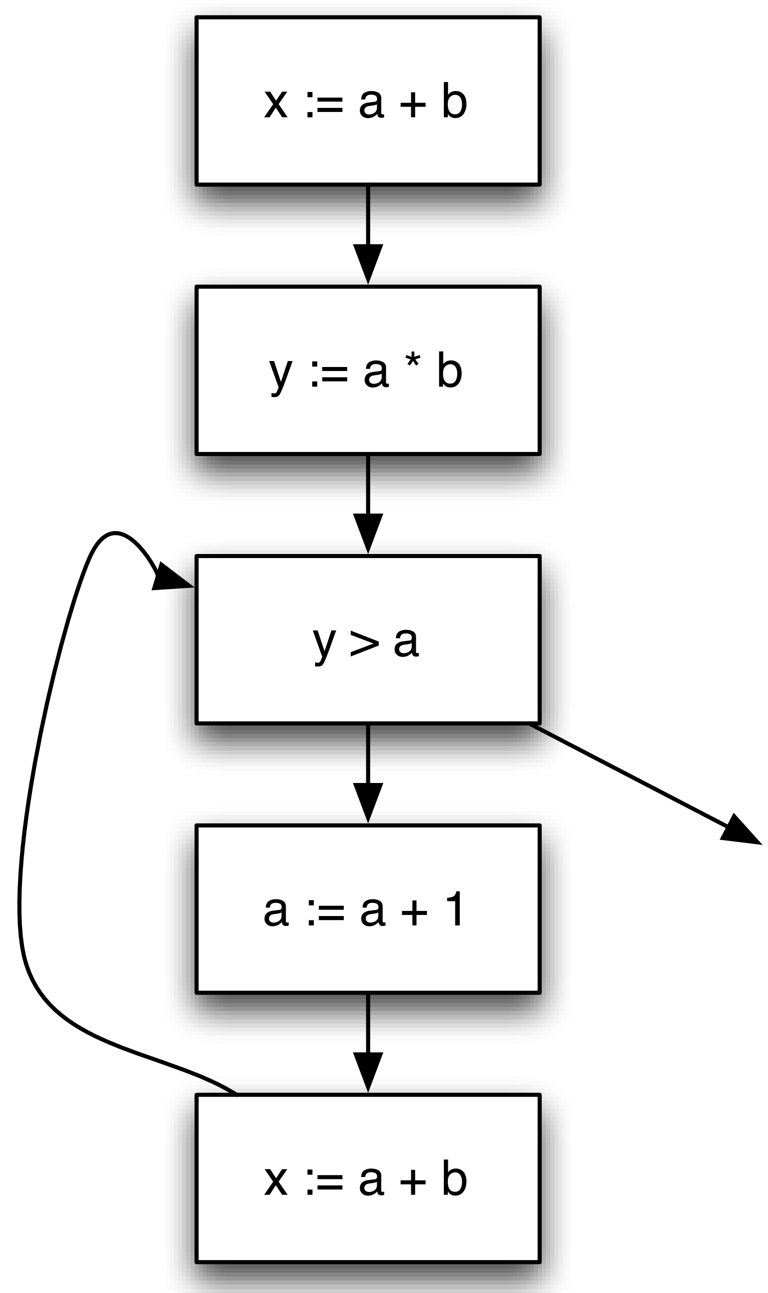
Gen and Kill

- What is the effect of each statement on the set of facts?

Stmt	Gen	Kill
x := a + b	a, b	x
y := a * b	a, b	y
y > a	a, y	
a := a + 1	a	a



Computing Live Variables



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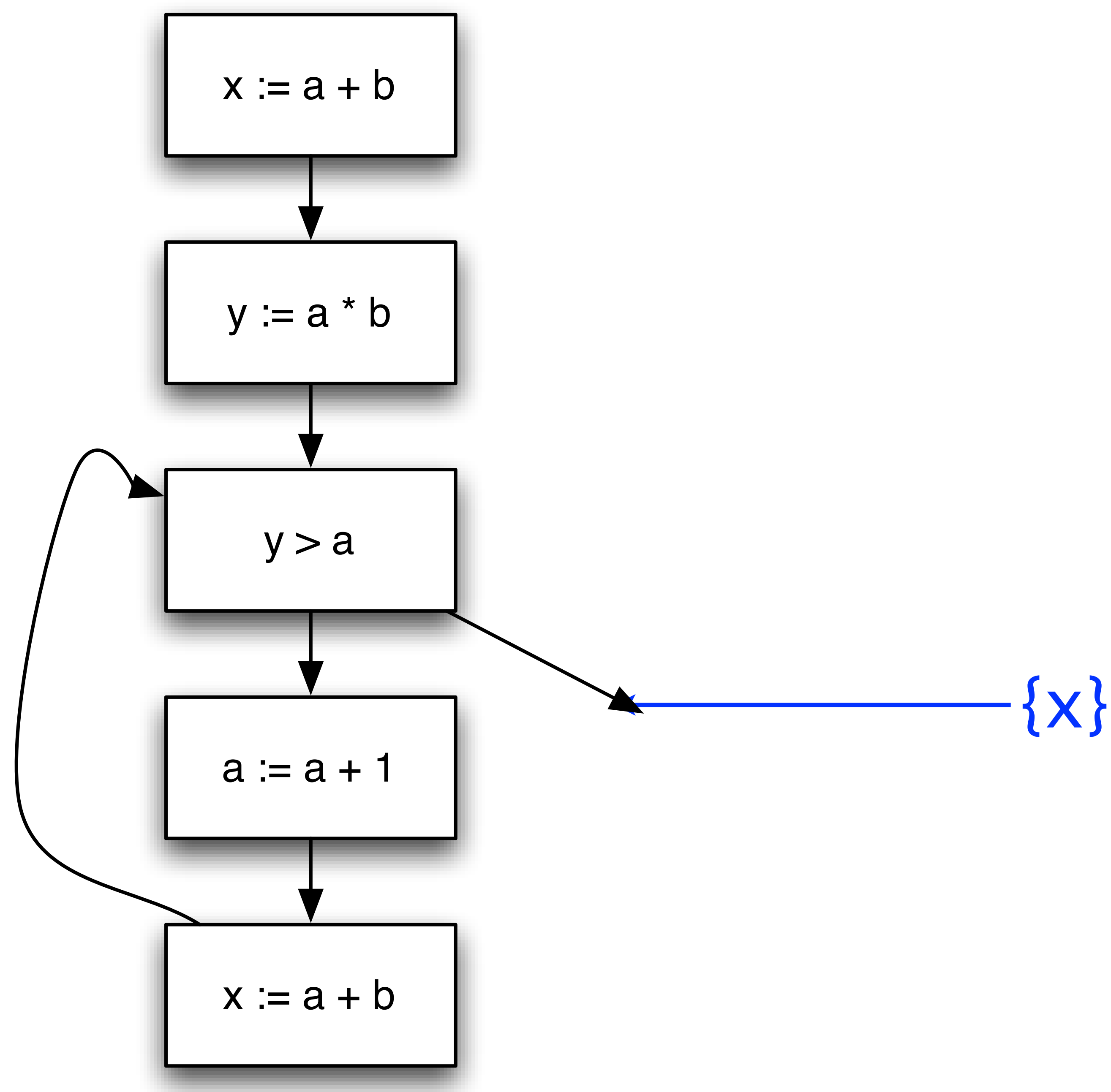

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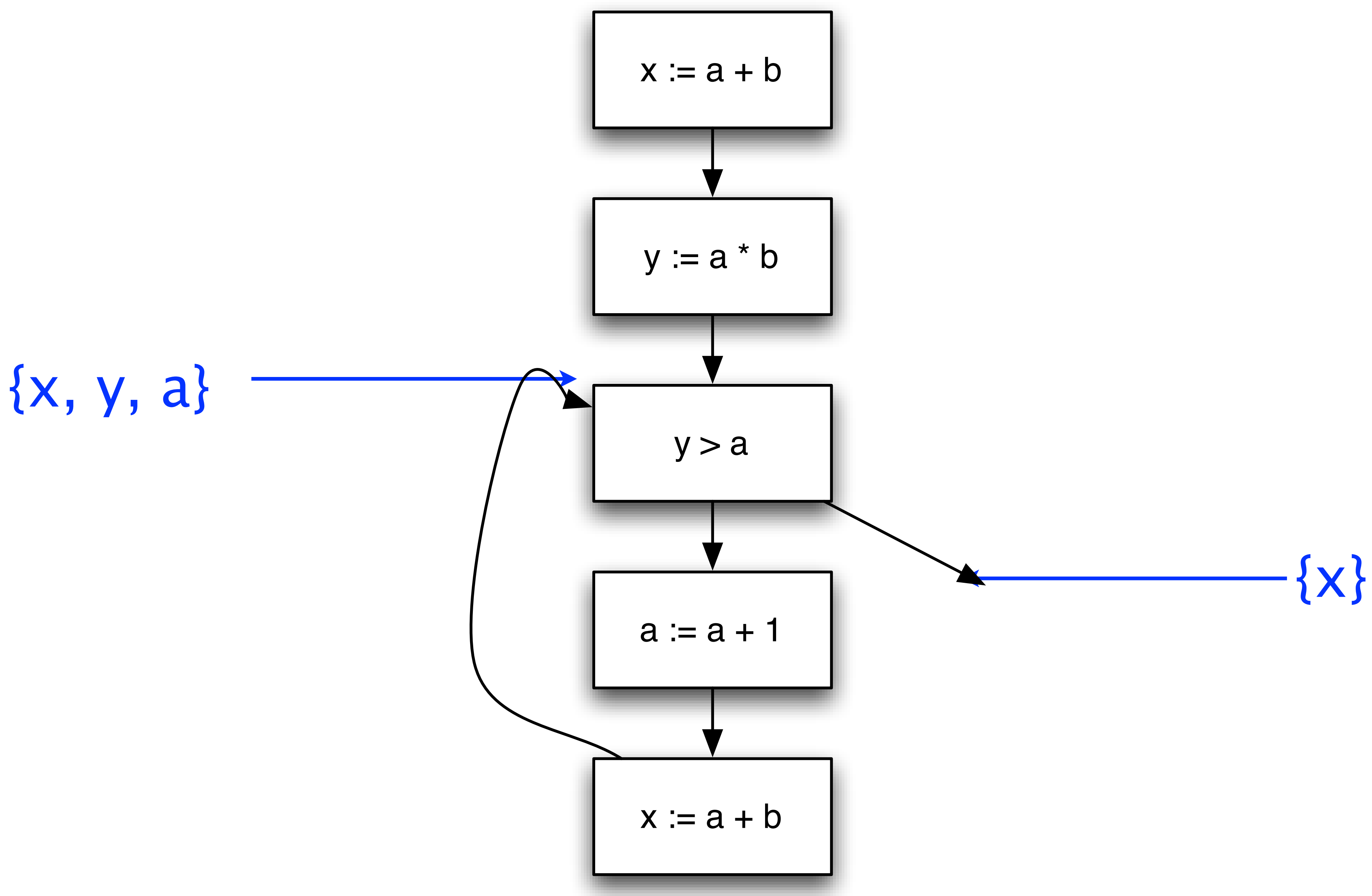

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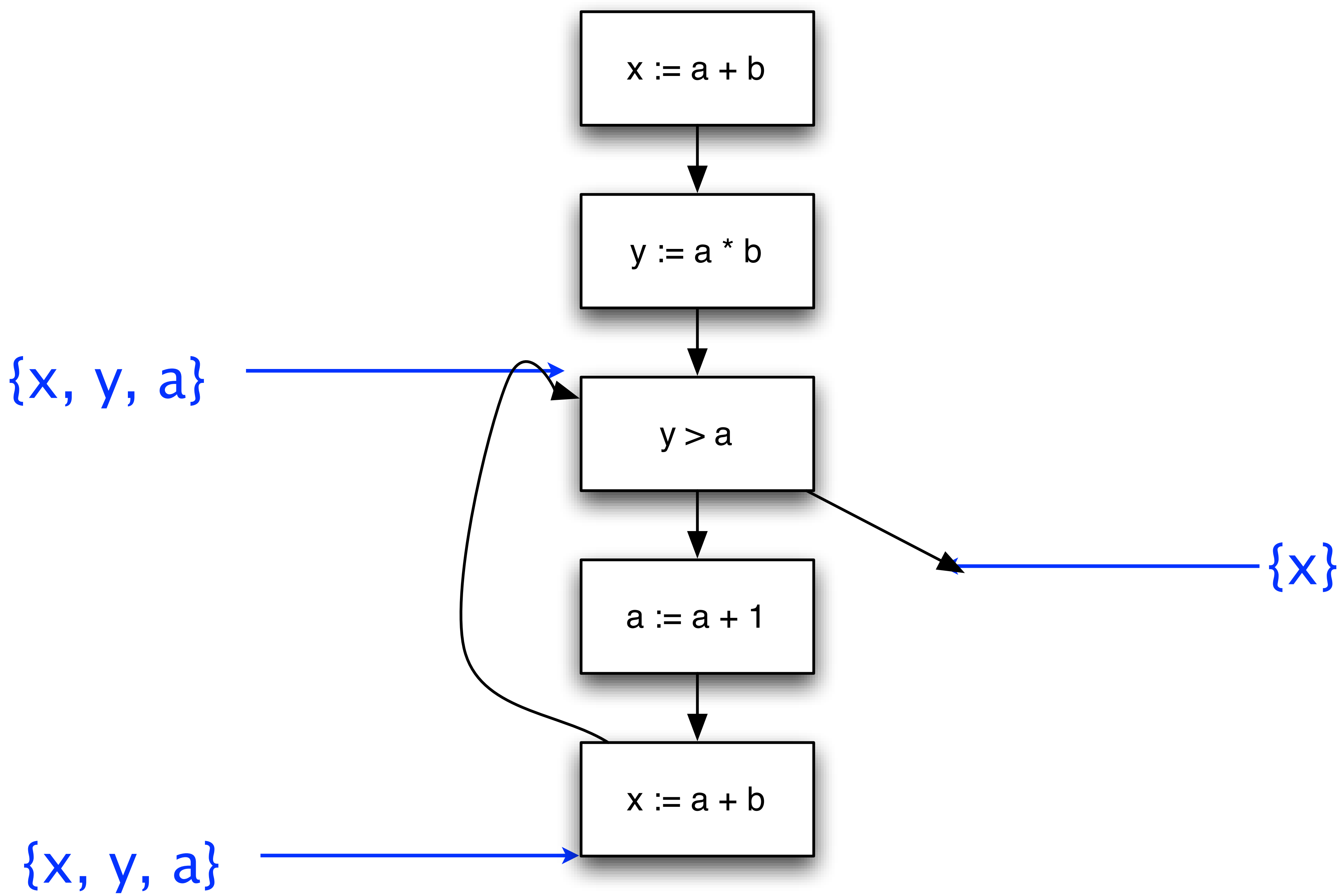

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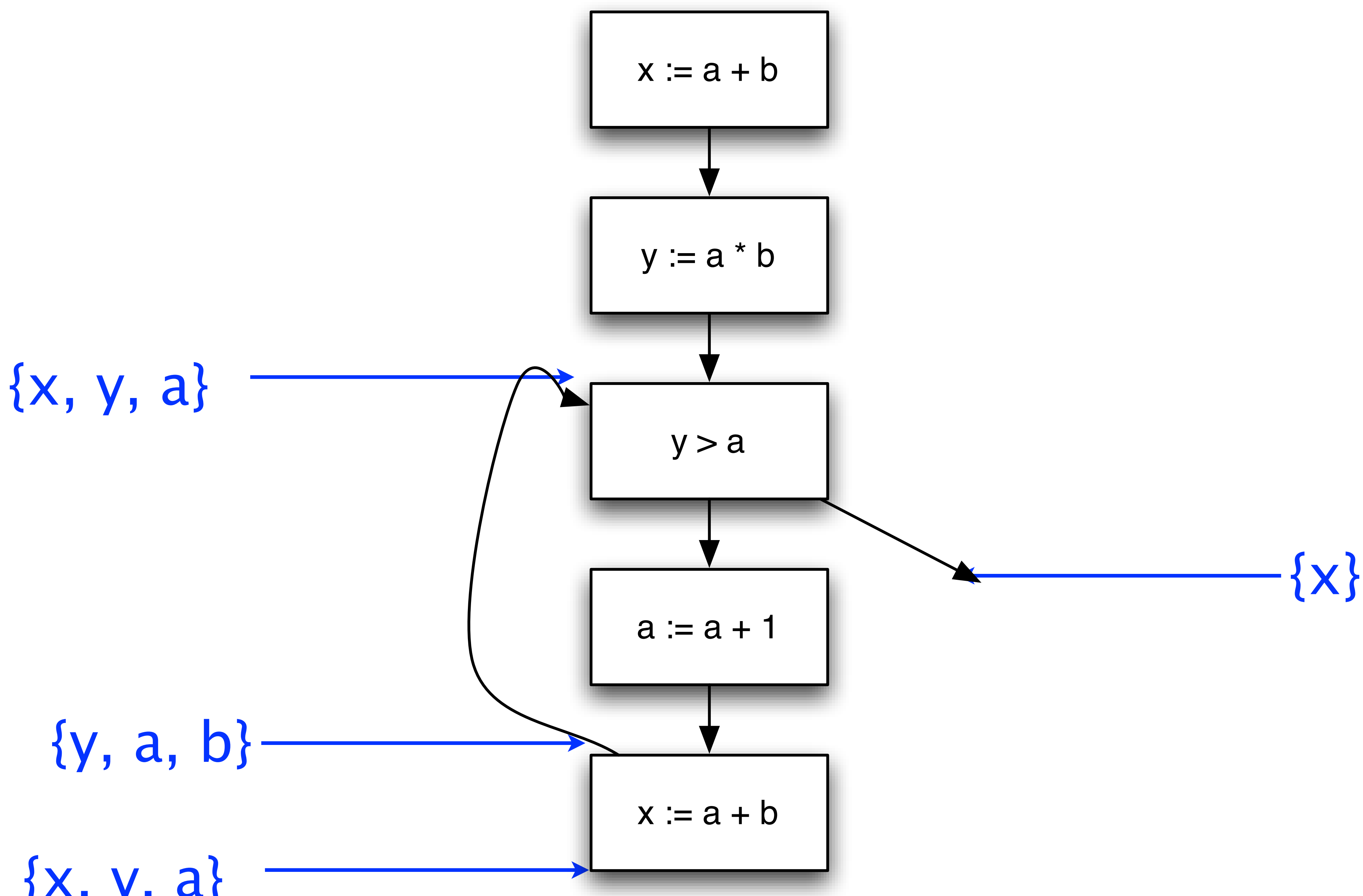

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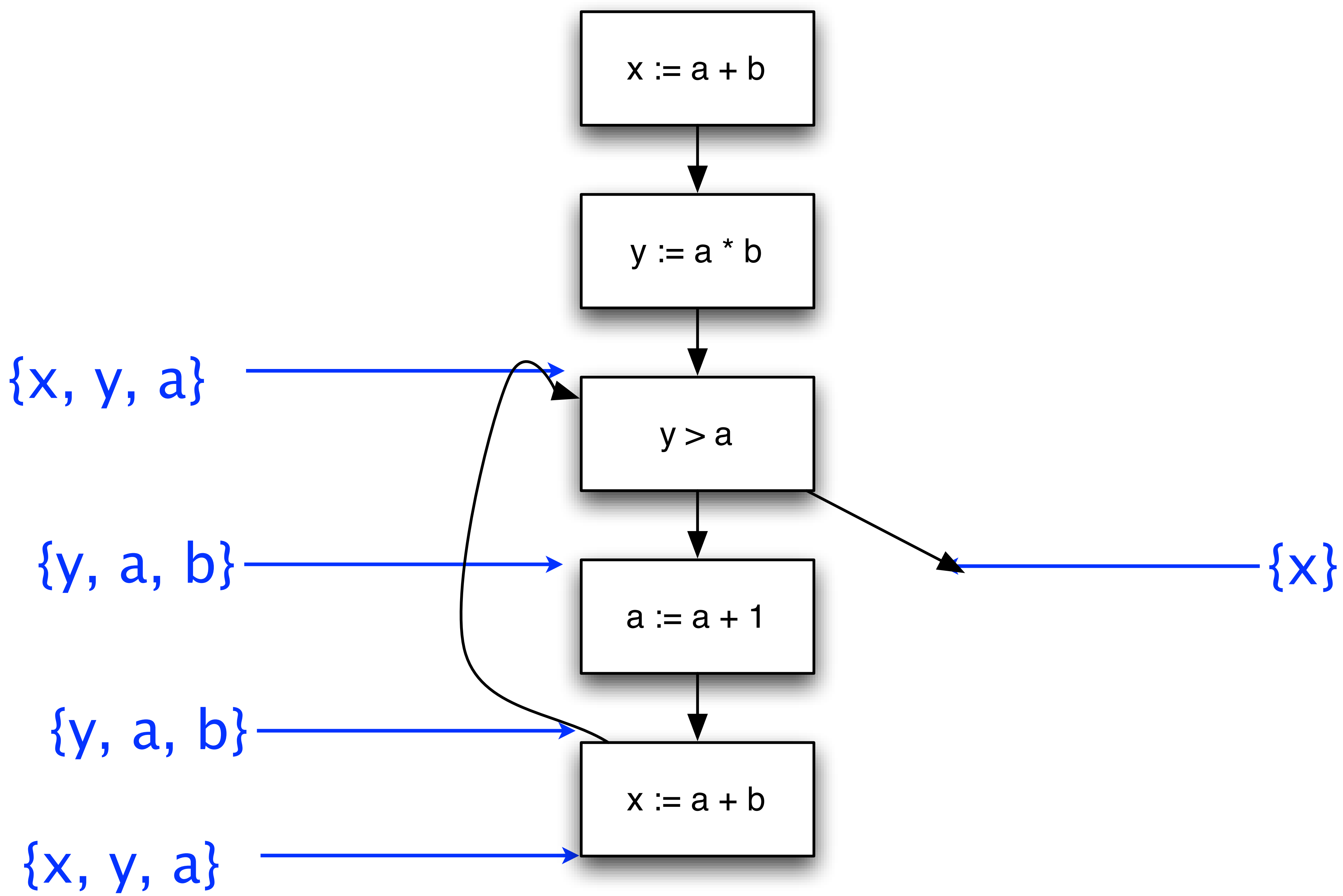

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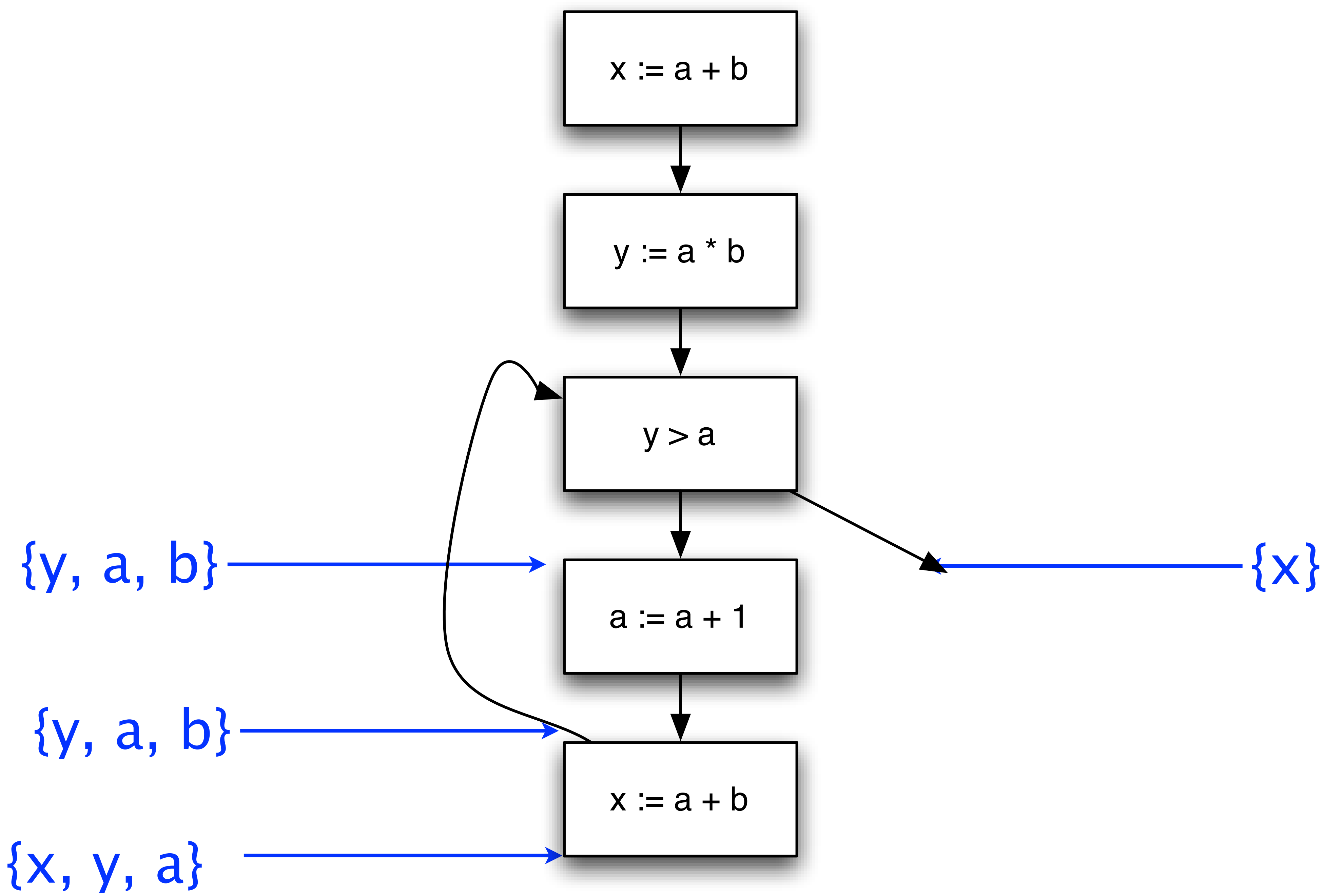

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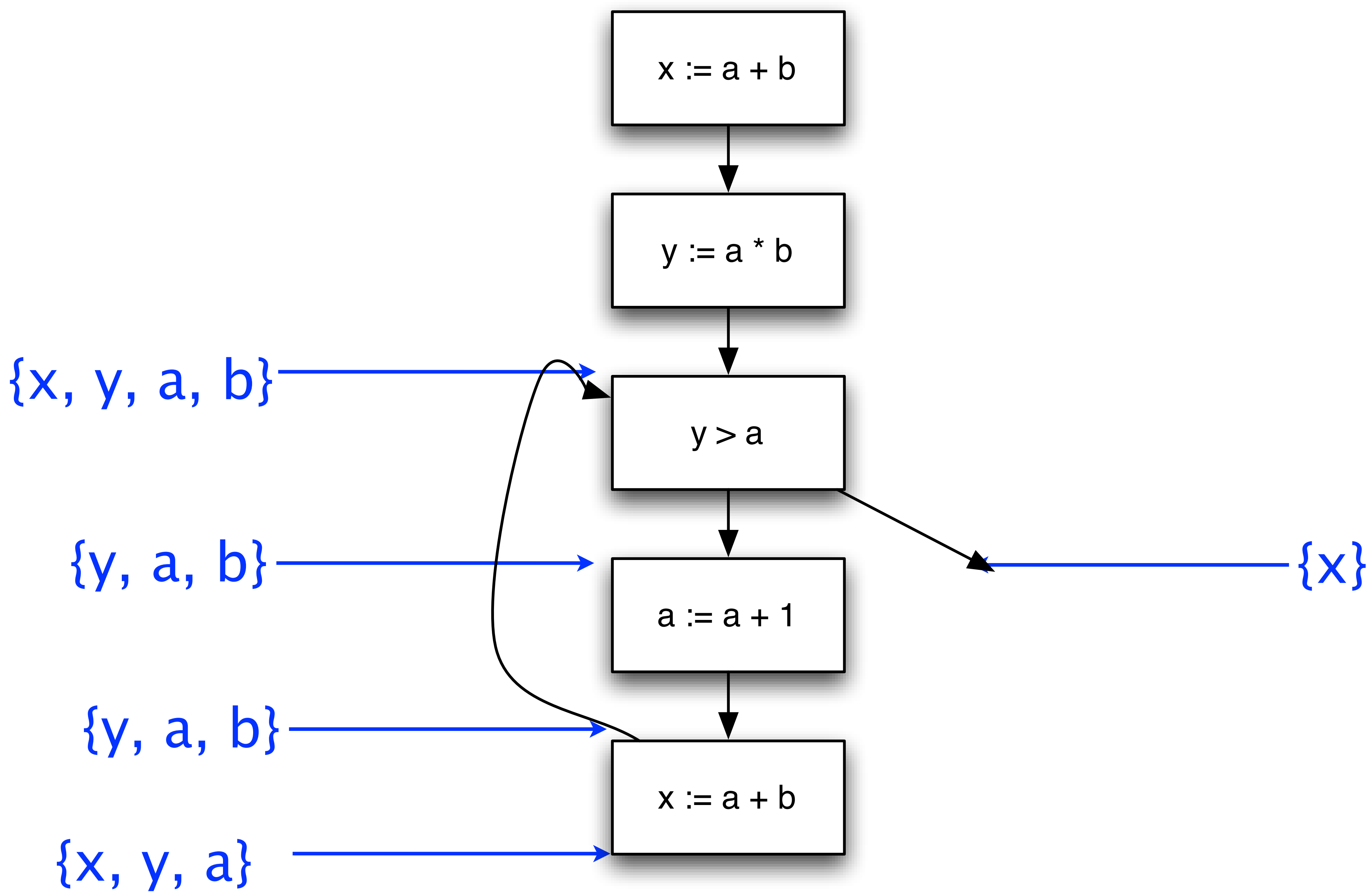

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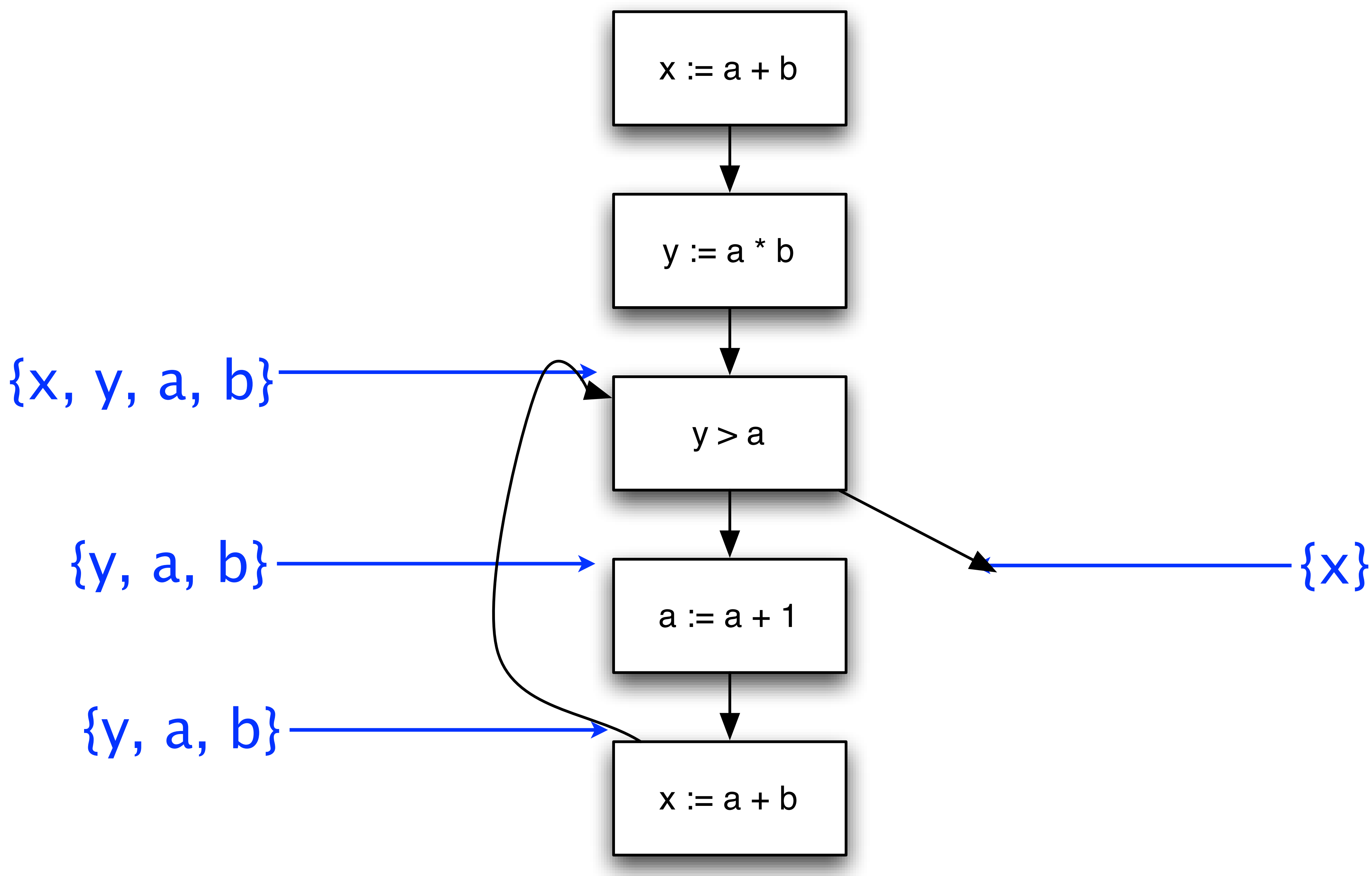

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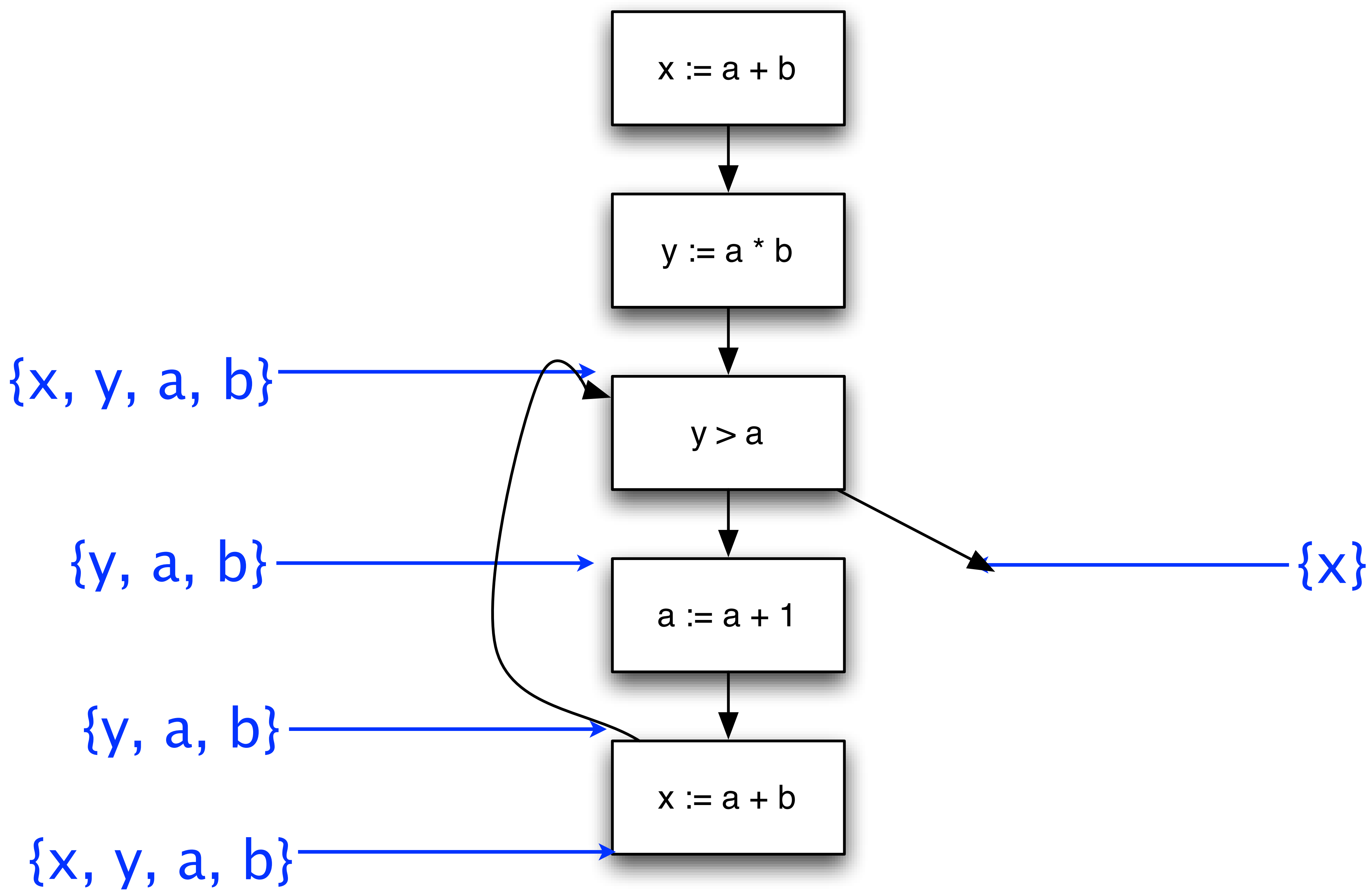

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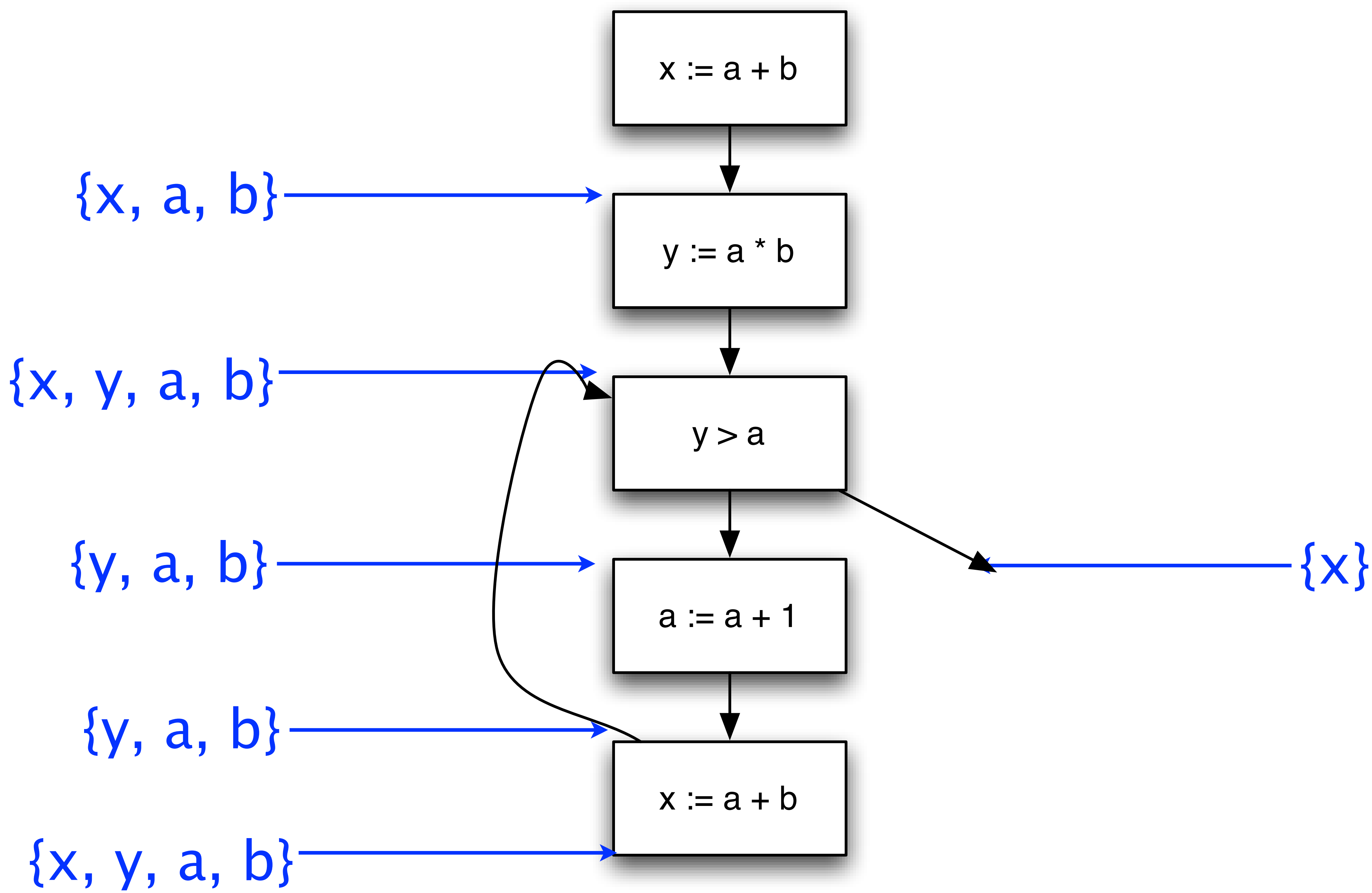

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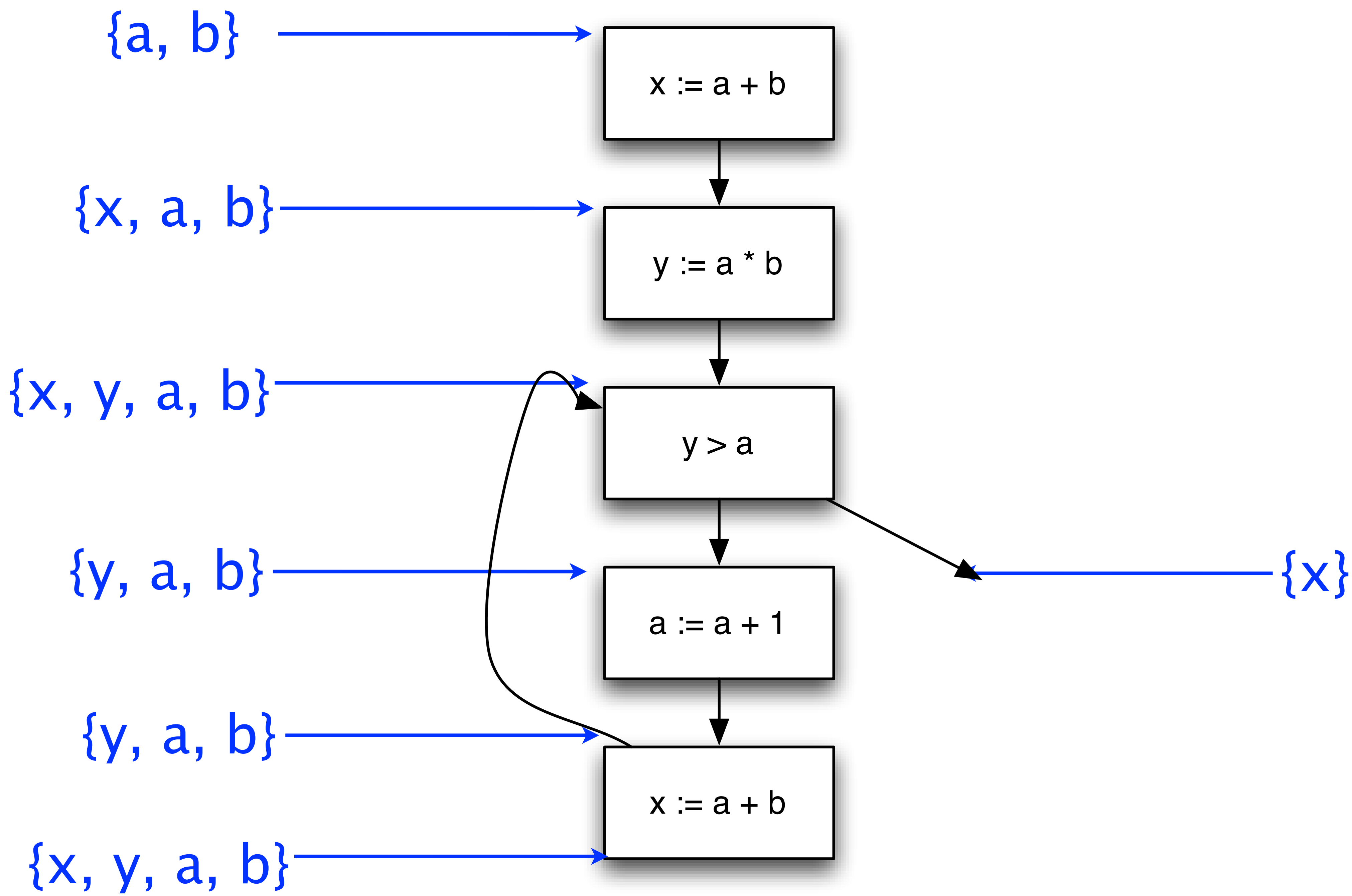

Very Busy Expressions

- An expression e is *very busy* at point p if
  - On every path from p, expression e is evaluated before the value of e is changed
- Optimization
  - Can hoist very busy expression computation
- What kind of problem?
  - Forward or backward?
  - May or must?

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  - Forward or backward? backward
  - May or must? must

Reaching Definitions

- A *definition* of a variable v is an assignment to v
- A definition of variable v reaches point p if
  - There is some path from the definition to p such that there is no intervening assignment to v on the path
- Also called def-use information
- What kind of problem?
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- What kind of problem?
  - Forward or backward?
  - May or must?

forward

may

Space of Data Flow Analyses

	May	Must
Forward	Reaching definitions	Available expressions
Backward	Live variables	Very busy expressions

- Most data flow analyses can be classified this way
  - A few don't fit: bidirectional analysis
- Lots of literature on data flow analysis

Solving data flow equations

- Let's start with forward may analysis
  - Dataflow equations:
    - $in(s) = \bigcup_{s' \in pred(s)} out(s')$
    - $out(s) = gen(s) \cup (in(s) - kill(s))$
- Need algorithm to compute in and out at each stmt
- Key observation: out(s) is *monotonic* in in(s)
  - gen(s) and kill(s) are fixed for a given s
  - If, during our algorithm, in(s) grows, then out(s) grows
  - Furthermore, out(s) and in(s) have max size
- Same with in(s)
  - in terms of out(s') for precedessors s'

Solving data flow equations (cont'd)

- Idea: fixpoint algorithm
  - Set out(entry) to emptyset
    - E.g., we know no definitions reach the entry of the program
  - Initially, assume in(s), out(s) empty everywhere else, also
  - Pick a statement s
    - Compute in(s) from predecessors' out's
    - Compute new out(s) for s
  - Repeat until nothing changes
- Improvement: use a worklist
  - Add statements to worklist if their in(s) might change
  - Fixpoint reached when worklist is empty

Forward May Data Flow Algorithm

```
out(entry) = ∅
for all other statements s
  out(s) = ∅
W = all statements     // worklist
while W not empty
  take s from W
    in(s) = ∪s′∈pred(s) out(s′)
    temp = gen(s) ∪ (in(s) - kill(s))
    if temp ≠ out(s) then
      out(s) = temp
      W := W ∪ succ(s)
    end
end
```

Generalizing

	May	Must
Forward	$in(s) = \bigcup_{s' \in pred(s)} out(s')$ $out(s) = gen(s) \cup (in(s) - kill(s))$ $out(entry) = \varnothing$ initial out elsewhere $= \varnothing$	$in(s) = \bigcap_{s' \in pred(s)} out(s')$ $out(s) = gen(s) \cup (in(s) - kill(s))$ $out(entry) = \varnothing$ initial out elsewhere = {all facts}
Backward	$out(s) = \bigcup_{s' \in succ(s)} in(s')$ $in(s) = gen(s) \cup (out(s) - kill(s))$ $in(exit) = \varnothing$ initial in elsewhere $= \varnothing$	$out(s) = \bigcap_{s' \in succ(s)} in(s')$ $in(s) = gen(s) \cup (out(s) - kill(s))$ $in(exit) = \varnothing$ initial in elsewhere = {all facts}

Forward Analysis

```
out(entry) = ∅
for all other statements s
  out(s) = ∅
W = all statements      // worklist
while W not empty
  take s from W
    in(s) = ∪s′∈pred(s) out(s′)
    temp = gen(s) ∪ (in(s) - kill(s))
    if temp ≠ out(s) then
      out(s) = temp
      W := W ∪ succ(s)
    end
end
```

May

```
out(entry) = ∅
for all other statements s
  out(s) = all facts
W = all statements
while W not empty
  take s from W
    in(s) = ∩s′∈pred(s) out(s′)
    temp = gen(s) ∪ (in(s) - kill(s))
    if temp ≠ out(s) then
      out(s) = temp
      W := W ∪ succ(s)
    end
end
```

Must

Backward Analysis

May

$in(exit) = \varnothing$
for all other statements s
  $in(s) = \varnothing$
W = *all statements*
while W not empty
  take s from W
    $out(s) = \cup_{s' \in succ(s)} in(s')$
    temp = $gen(s) \cup (out(s) - kill(s))$
    if temp ≠ $in(s)$ then
      $in(s)$ = temp
      W := W ∪ pred(s)
    end
end

Must

$in(exit) = \varnothing$
for all other statements s
  $in(s)$ = *all facts*
W = *all statements*
while W not empty
  take s from W
    $out(s) = \cap_{s' \in succ(s)} in(s')$
    temp = $gen(s) \cup (out(s) - kill(s))$
    if temp ≠ $in(s)$ then
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      W := W ∪ pred(s)
    end
end

Practical Implementation

- Represent set of facts as bit vector
  - $Fact_i$ represented by bit i
  - Intersection = bitwise and, union = bitwise or, etc

- "Only" a constant factor speedup
  - But very useful in practice

Basic Blocks

- Recall a *basic block* is a sequence of statements s.t.
  - No statement except the last in a branch
  - There are no branches to any statement in the block except the first

- In some data flow implementations,
  - Compute gen/kill for each basic block as a whole
    - Compose transfer functions
  - Store only in/out for each basic block
  - Typical basic block ~5 statements
    - At least, this used to be the case...

Order Matters

- Assume forward data flow problem
  - Let $G = (V, E)$ be the CFG
  - Let k be the height of the lattice

- If G acyclic, visit in topological order
  - Visit head before tail of edge
- Running time $O(|E|)$
  - No matter what size the lattice

Order Matters — Cycles

- If G has cycles, visit in reverse postorder
  - Order from depth-first search
  - (Reverse for backward analysis)
- Let Q = max # back edges on cycle-free path
  - Nesting depth
  - Back edge is from node to ancestor in DFS tree
- In common cases, running time can be shown to be O((Q+1)|E|)
  - Proportional to structure of CFG rather than lattice

Flow-Sensitivity

- Data flow analysis is *flow-sensitive*
  - The order of statements is taken into account
  - I.e., we keep track of facts per program point

- Alternative: *Flow-insensitive* analysis
  - Analysis the same regardless of statement order
  - Standard example: types
    - /* x : int */ x := ... /* x : int */

Data Flow Analysis and Functions

- What happens at a function call?
  - Lots of proposed solutions in data flow analysis literature
- In practice, only analyze one procedure at a time
- Consequences
  - Call to function kills all data flow facts
  - May be able to improve depending on language, e.g., function call may not affect locals

More Terminology

- An analysis that models only a single function at a time is *intraprocedural*
- An analysis that takes multiple functions into account is *interprocedural*
- An analysis that takes the whole program into account is *whole program*

- Note: *global* analysis means “more than one basic block,” but still within a function
  - Old terminology from when computers were slow...

Data Flow Analysis and The Heap

- Data Flow is good at analyzing local variables
  - But what about values stored in the heap?
  - Not modeled in traditional data flow

- In practice: *x := e
  - Assume all data flow facts killed (!)
  - Or, assume write through x may affect any variable whose address has been taken

- In general, hard to analyze pointers

Proebsting's Law

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- Moore's Law: Hardware advances double computing power every 18 months.

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- Proebsting's Law: Compiler advances double computing power every 18 *years*.

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  - Not so much bang for the buck!

DFA and Defect Detection

- LCLint - Evans et al. (UVa)
- METAL - Engler et al. (Stanford, now Coverity)
- ESP - Das et al. (MSR)
- FindBugs - Hovemeyer, Pugh (Maryland)
  - For Java. The first three are for C.

- Many other one-shot projects
  - Memory leak detection
  - Security vulnerability checking (tainting, info. leaks)