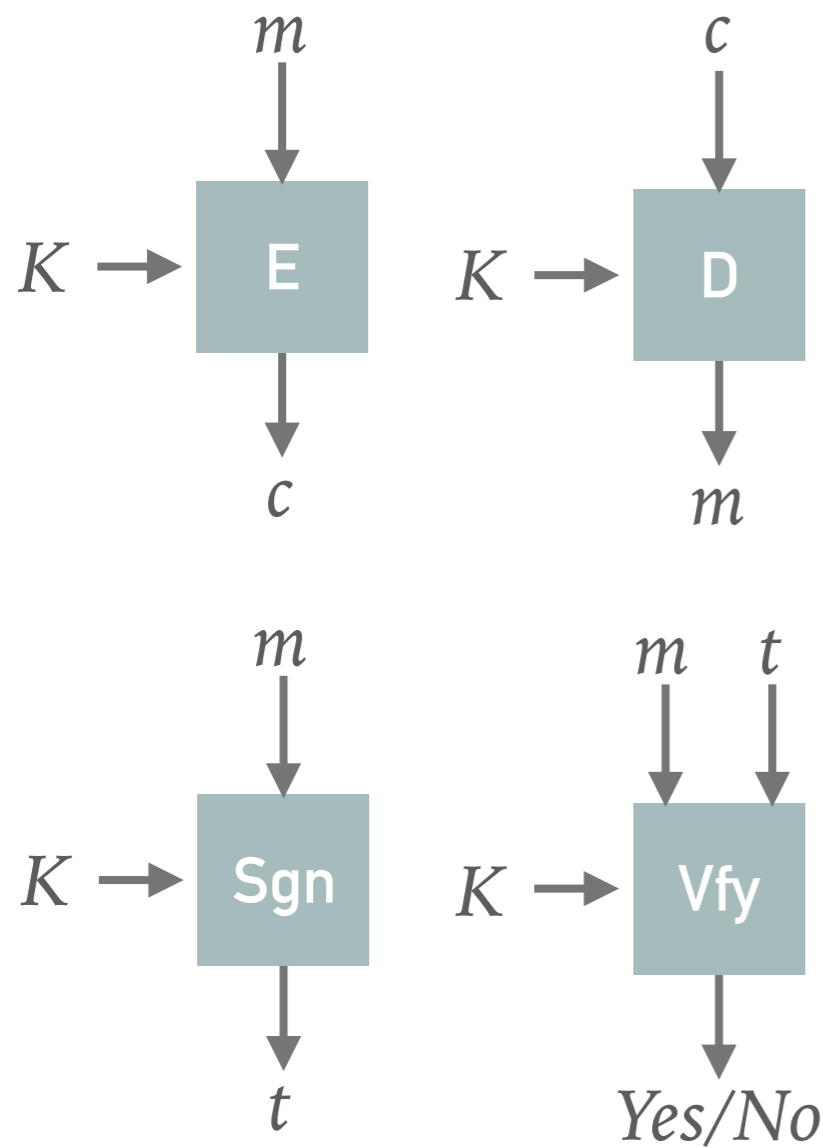


PUBLIC KEY CRYPTO

GRAD SEC
OCT 26 2017



RECAP



CONFIDENTIALITY

Block ciphers

Deterministic \Rightarrow use IVs

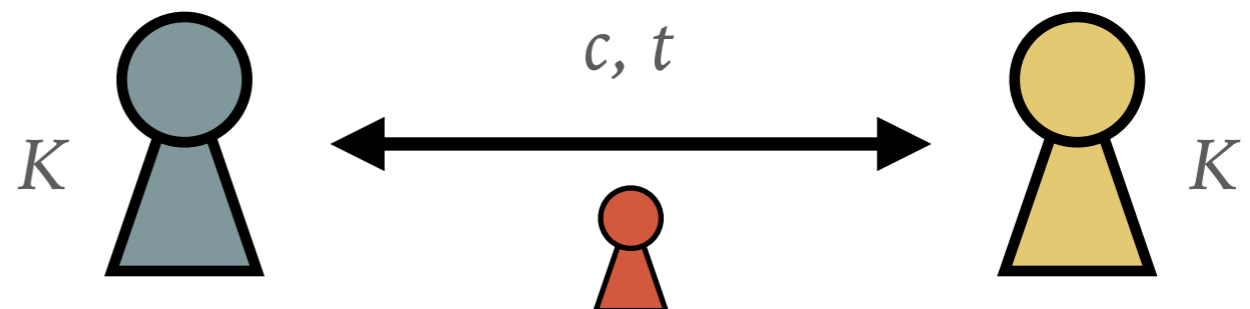
Fixed block size \Rightarrow use encryption "modes"

INTEGRITY

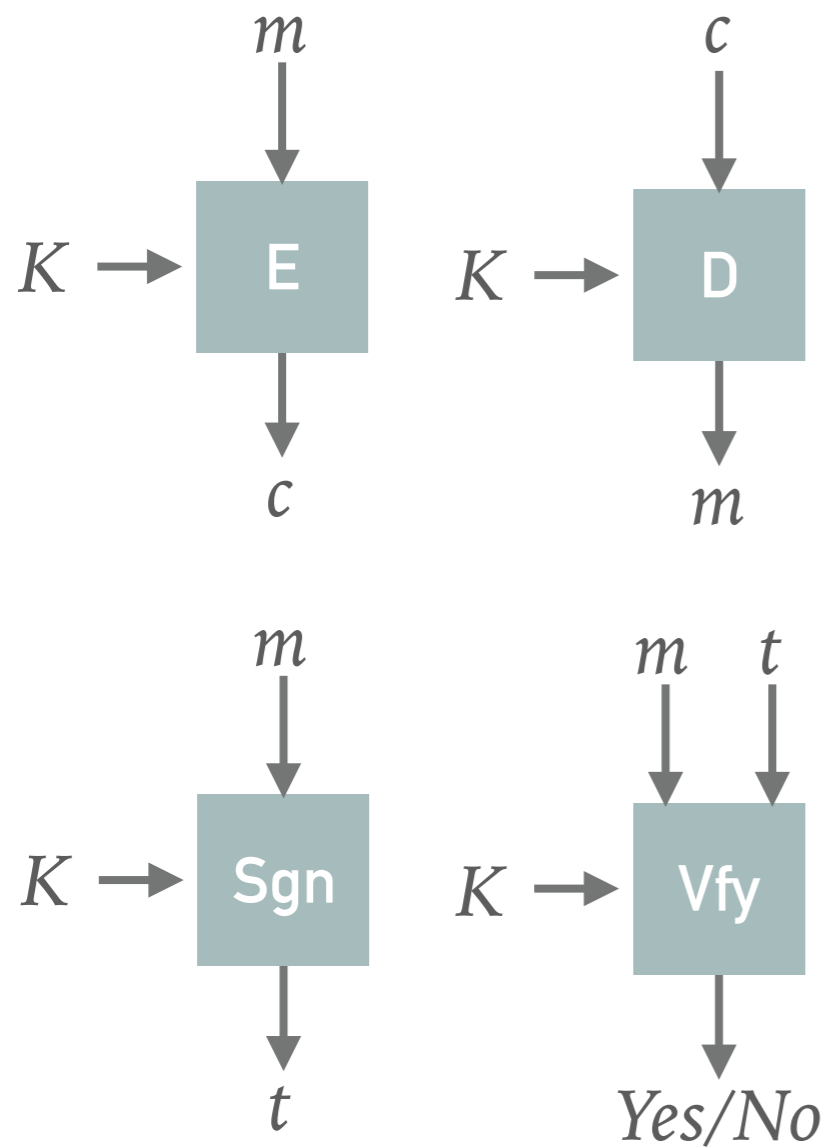
Message Authentication Codes (MACs)

Send (message, tag) pairs

Verify that they match



RECAP



CONFIDENTIALITY

Block ciphers

Deterministic \Rightarrow use IVs

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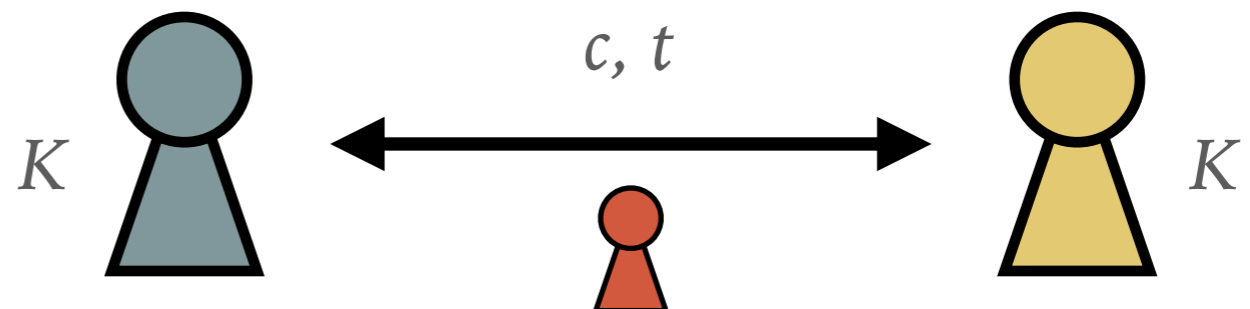
INTEGRITY

Message Authentication Codes (MACs)

Send (message, tag) pairs

Verify that they match

Today:
How do we establish K ?
How do we know with whom we are communicating?



BLACKBOX #4:
DIFFIE HELLMAN KEY ESTABLISHMENT

HIGH-LEVEL REVIEW OF MODULAR ARITHMETIC

$$x \bmod N$$

HIGH-LEVEL REVIEW OF MODULAR ARITHMETIC

$x \bmod N$

g is a **generator** of mod N if

$$\{1, 2, \dots, N-1\} = \{g^0 \bmod N, g^1 \bmod N, \dots, g^{N-2} \bmod N\}$$

HIGH-LEVEL REVIEW OF MODULAR ARITHMETIC

$$x \bmod N$$

g is a **generator** of mod N if

$$\{1, 2, \dots, N-1\} = \{g^0 \bmod N, g^1 \bmod N, \dots, g^{N-2} \bmod N\}$$

$$N=5, g=3$$

$$3^0 \bmod 5 = 1 \quad 3^1 \bmod 5 = 3 \quad 3^2 \bmod 5 = 4 \quad 3^3 \bmod 5 = 2$$

HIGH-LEVEL REVIEW OF MODULAR ARITHMETIC

$$x \bmod N$$

g is a **generator** of mod N if

$$\{1, 2, \dots, N-1\} = \{g^0 \bmod N, g^1 \bmod N, \dots, g^{N-2} \bmod N\}$$

$$N=5, g=3$$

$$3^0 \bmod 5 = 1 \quad 3^1 \bmod 5 = 3 \quad 3^2 \bmod 5 = 4 \quad 3^3 \bmod 5 = 2$$

Given x and g , it is efficient to compute

$$g^x \bmod N$$

HIGH-LEVEL REVIEW OF MODULAR ARITHMETIC

$$x \bmod N$$

g is a **generator** of mod N if

$$\{1, 2, \dots, N-1\} = \{g^0 \bmod N, g^1 \bmod N, \dots, g^{N-2} \bmod N\}$$

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Given x and g , it is efficient to compute

$$g^x \bmod N$$

Given g and g^x , it is efficient to compute x

(simply take $\log_g g^x$)

HIGH-LEVEL REVIEW OF MODULAR ARITHMETIC

$$x \bmod N$$

g is a **generator** of mod N if
 $\{1, 2, \dots, N-1\} = \{g^0 \bmod N, g^1 \bmod N, \dots, g^{N-2} \bmod N\}$

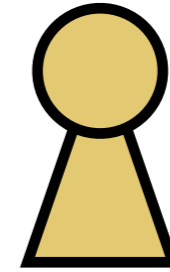
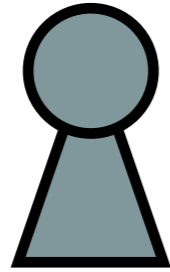
$$N=5, g=3$$
$$3^0 \bmod 5 = 1 \quad 3^1 \bmod 5 = 3 \quad 3^2 \bmod 5 = 4 \quad 3^3 \bmod 5 = 2$$

Given x and g , it is efficient to compute
 $g^x \bmod N$

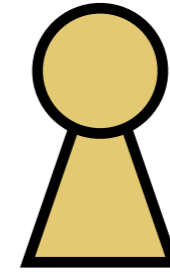
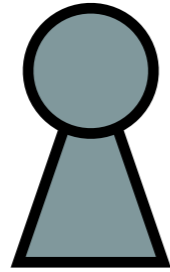
Given g and g^x , it is efficient to compute x
(simply take $\log_g g^x$)

Given g and $g^x \bmod N$ it is *infeasible* to compute x
Discrete log problem

DIFFIE-HELLMAN KEY EXCHANGE

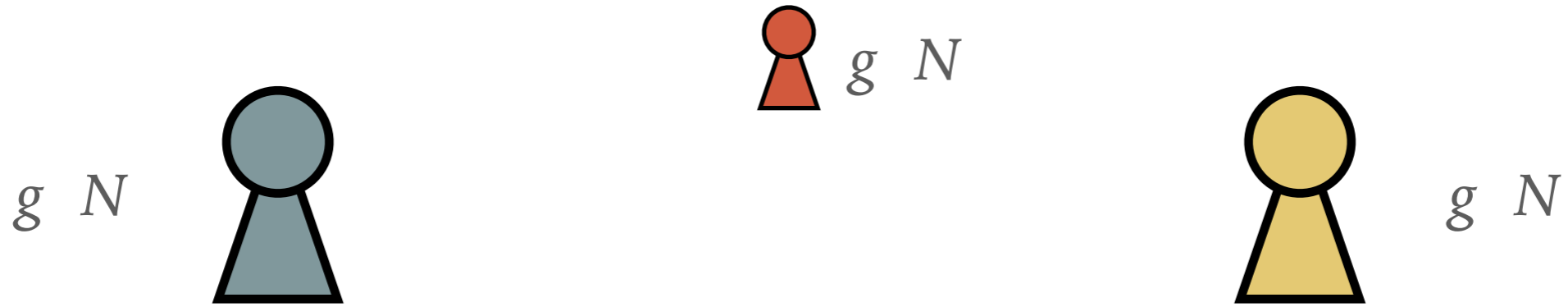


DIFFIE-HELLMAN KEY EXCHANGE



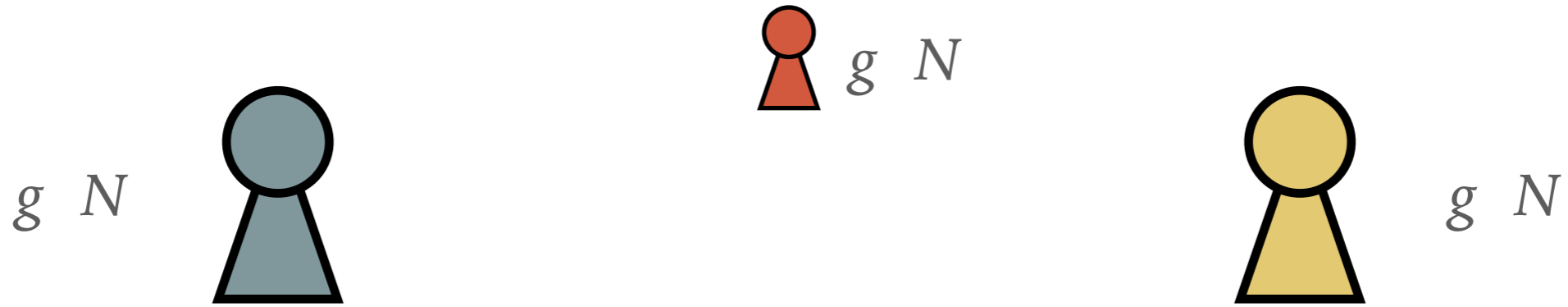
Public knowledge: g and N

DIFFIE-HELLMAN KEY EXCHANGE



Public knowledge: g and N

DIFFIE-HELLMAN KEY EXCHANGE

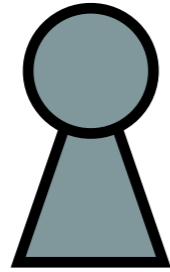


Public knowledge: g and N

Pick random a

DIFFIE-HELLMAN KEY EXCHANGE

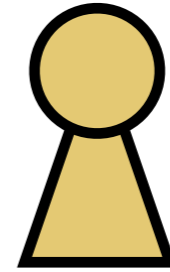
a g N



g N



g N

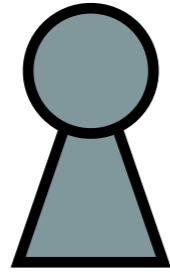


Public knowledge: g and N

Pick random a

DIFFIE-HELLMAN KEY EXCHANGE

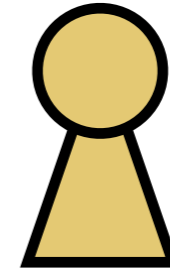
a g N



g N



g N



Public knowledge: g and N

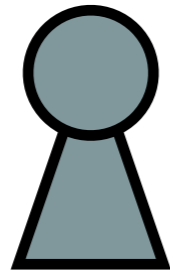
Pick random a

$g^a \bmod N$

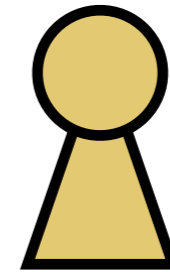


DIFFIE-HELLMAN KEY EXCHANGE

a g N



g N
 $g^a \pmod N$



g N
 $g^a \pmod N$

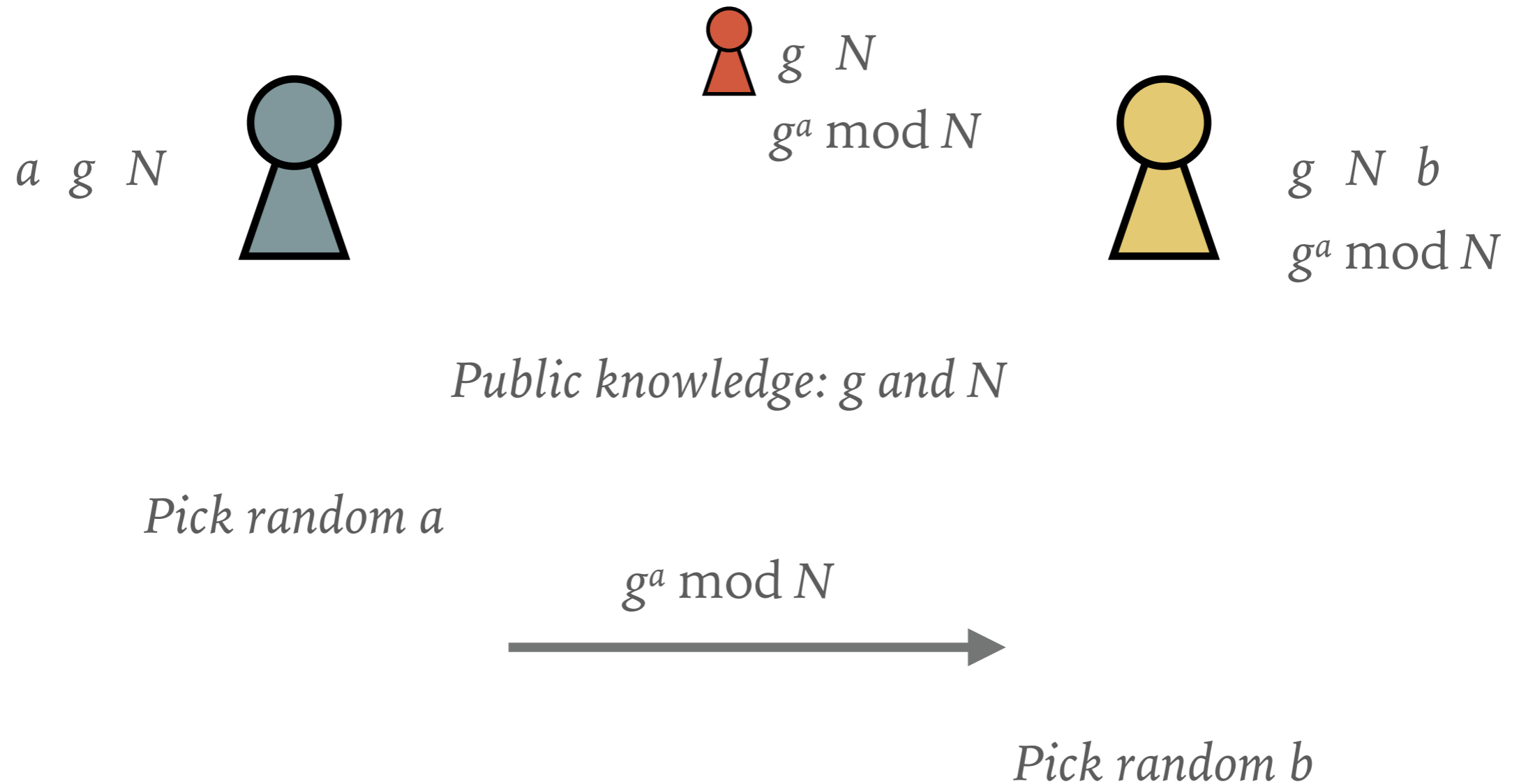
Public knowledge: g and N

Pick random a

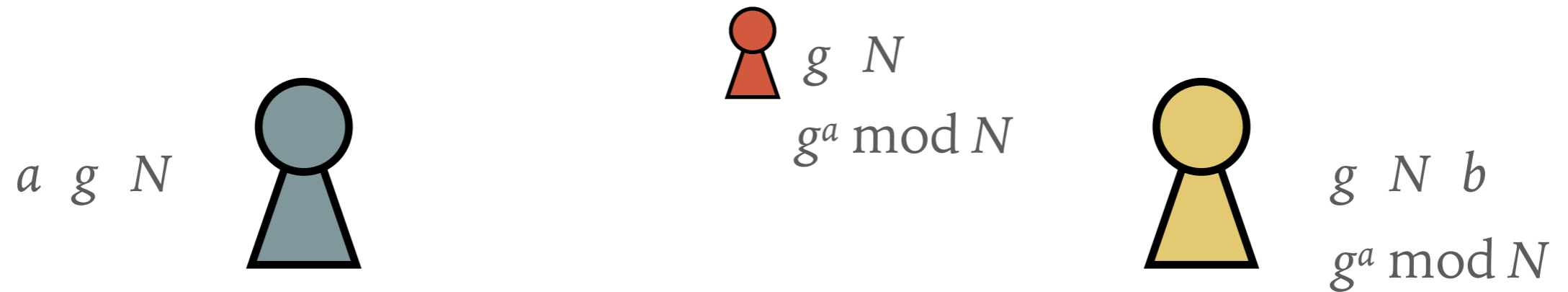
$g^a \pmod N$



DIFFIE-HELLMAN KEY EXCHANGE



DIFFIE-HELLMAN KEY EXCHANGE



Public knowledge: g and N

Pick random a

$g^a \bmod N$

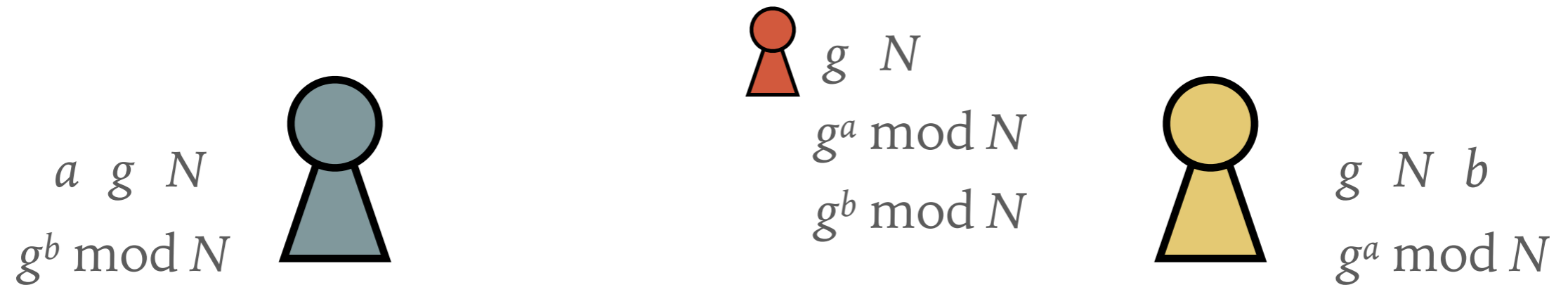


$g^b \bmod N$



Pick random b

DIFFIE-HELLMAN KEY EXCHANGE



Pick random a

$g^a \bmod N$

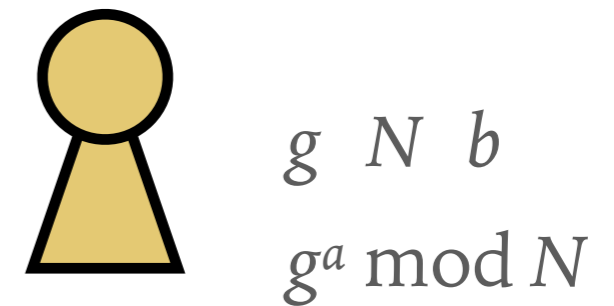
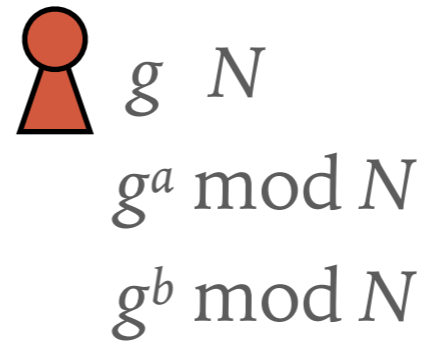


$g^b \bmod N$



Pick random b

DIFFIE-HELLMAN KEY EXCHANGE



Public knowledge: g and N

Pick random a

$g^a \pmod N$



Pick random b

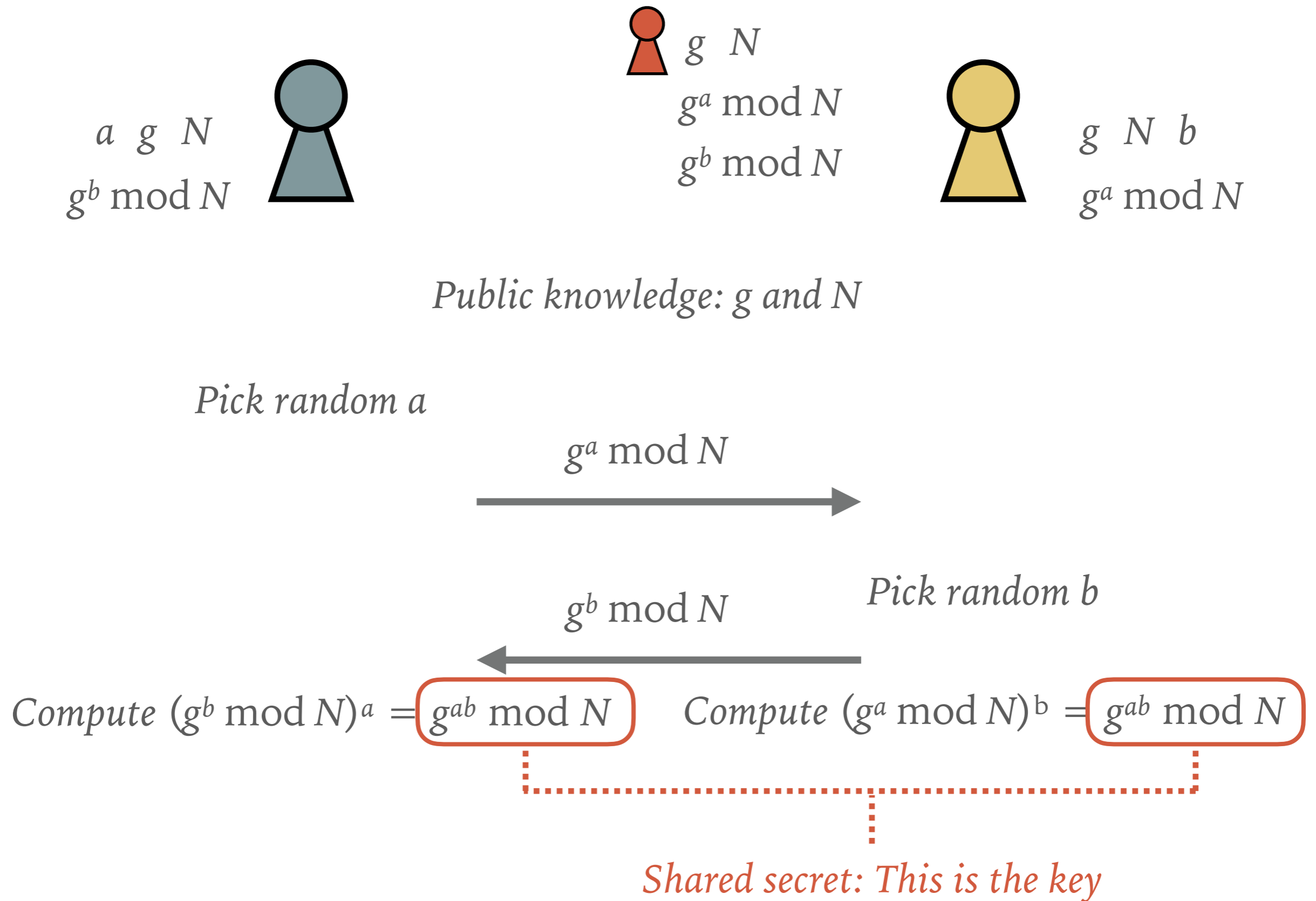
$g^b \pmod N$



Compute $(g^b \pmod N)^a = g^{ab} \pmod N$

Compute $(g^a \pmod N)^b = g^{ab} \pmod N$

DIFFIE-HELLMAN KEY EXCHANGE



DIFFIE-HELLMAN KEY EXCHANGE



g N

$g^a \bmod N$

$g^b \bmod N$

$g^{ab} \bmod N$

$$g^a \bmod N * g^b \bmod N = g^{a+b} \bmod N$$

DIFFIE-HELLMAN KEY EXCHANGE



g N

$g^a \bmod N$

$g^b \bmod N$

$g^{ab} \bmod N$

Given g and $g^x \bmod N$ it is *infeasible* to compute x
Discrete log problem

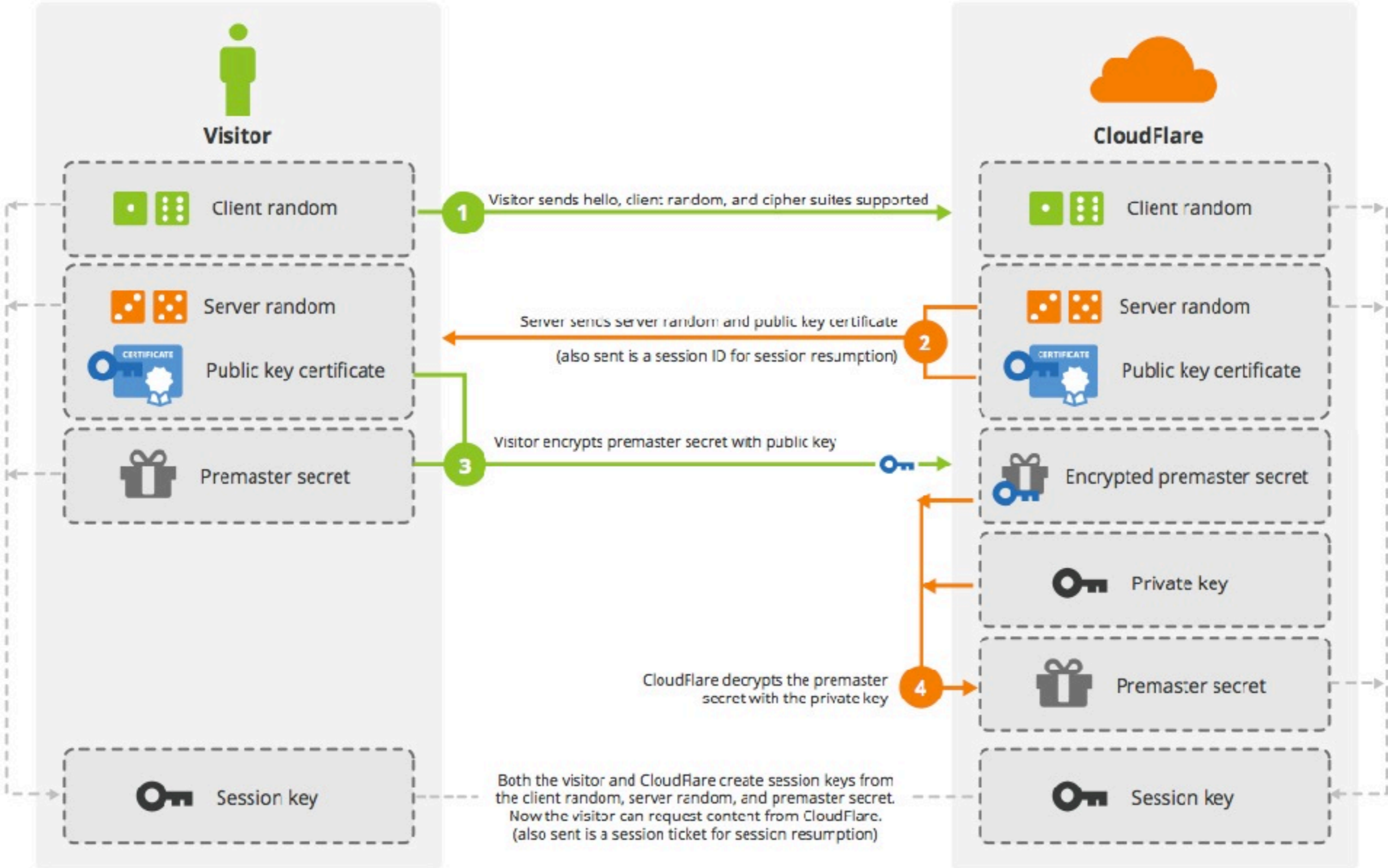
$$g^a \bmod N * g^b \bmod N = g^{a+b} \bmod N$$

BLACKBOX #5:
PUBLIC KEY CRYPTOGRAPHY

**PUTTING IT ALL TOGETHER:
PUBLIC KEY INFRASTRUCTURE (PKI)**

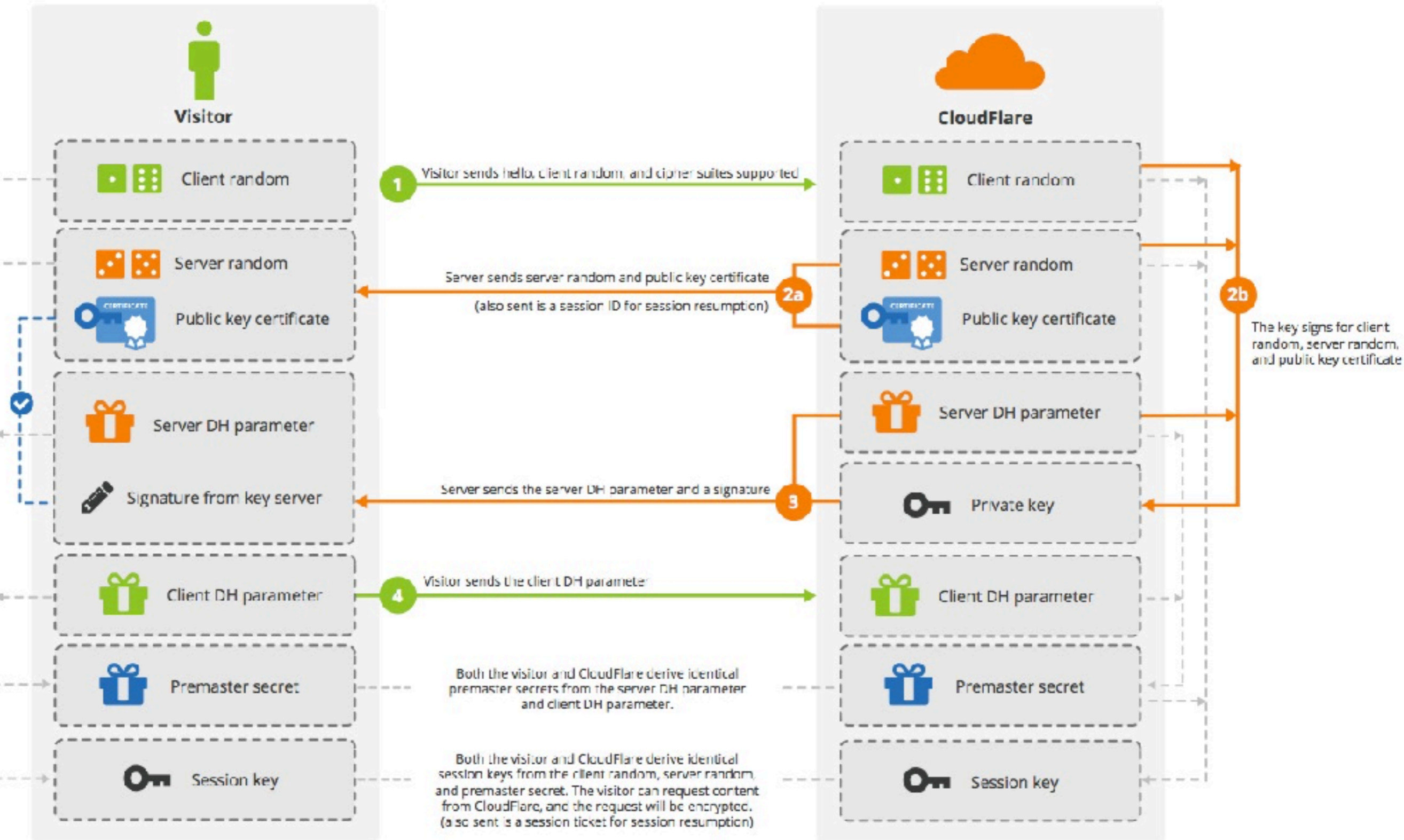
SSL Handshake (RSA) Without Keyless SSL

Handshake



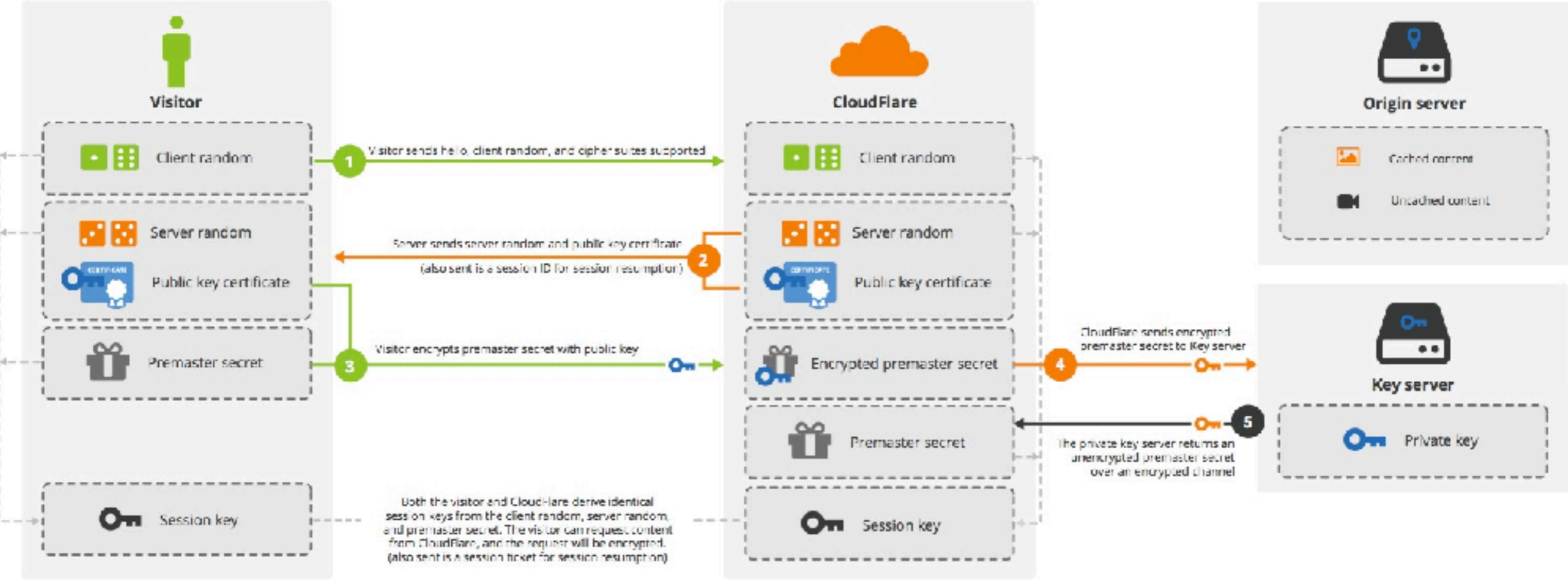
SSL Handshake (Diffie-Hellman) Without Keyless SSL

Handshake



CloudFlare Keyless SSL (RSA)

Handshake



CloudFlare Keyless SSL (Diffie-Hellman)

Handshake

