Problem 1. Find the upper and lower bounds for $\sum_{i=0}^{n}(i^4 + 3i^2)$ using the integral approximation method. Show your work.

Problem 2. Write a recurrence equation for a multiplication algorithm that squares any n-digit number by dividing the n-digit number into three parts, each comprised of $n/3$-digits. This way you are reducing the operation to multiplying six $n/3$-digit numbers. You mat assume n to be “nice”.

Solve the recurrence equation using the recursion tree approach to find the exact number of multiplications and additions to find the square of a number. You may represent an atomic multiplication between two, one-digit numbers, as μ and the atomic addition of two, one-digit numbers, as α.

Problem 3. Now that you have worked with divide and conquer algorithms and recurrences, we will try to combine it all together. In a divide and conquer algorithm, the problem is divided into smaller subproblems, each subproblem is solved *recursively*, and a *combine* algorithm is used to solve the original problem. Assume that there are a subproblems, each of size $1/b$ of the original problem, and that the algorithm used to combine the solutions of the subproblems runs in time cn^k, for some constants a, b, c, and k. For simplicity, we will assume, $n = b^m$, so that n/b is always an integer (b is an integer greater than 1). Answer the following:

(a) Write the generalized recurrence equation.

(b) Solve the recurrence equation using a recursion tree approach. Base case, $T(1) = c$.

(c) Once you obtain the solution to the recurrence equation in part(b), you will need to evaluate runtimes exactly for three cases:

  (a) $a > b^k$

  (b) $a = b^k$

  (c) $a < b^k$

In order to help you verify your exact runtime for these three cases, the asymptotic runtimes for the three cases are given as:

$$T(n) = \begin{cases} O(n^{lg_b^a}) & \text{if } a > b^k \\ O(n^k lg_b n) & \text{if } a = b^k \\ O(n^k) & \text{if } a < b^k \end{cases}$$

Show your work.

Congratulations! you have verified a very important theorem.