

There are eight problems. Within reason, you should show your work.

Problem 1. Evaluate the following sums (by hand).

(a)

$$\sum_{i=1}^4 i(i+1)$$

(b)

$$\sum_{i=0}^4 2^i$$

Problem 2. Write

$$3 \sum_{i=1}^n (5i^2 - 4) - 2 \sum_{i=1}^n (3i^2 - 1)$$

as a single summation.

Problem 3. Use mathematical induction to show the following:

(a)

$$\sum_{i=1}^n i(i+1) = \frac{n(n+1)(n+2)}{3}$$

(b)

$$\sum_{i=0}^n 2^i = 2^{n+1} - 1$$

Problem 4. Assume that you guess that

$$\sum_{i=0}^n 2^i = a2^n + b$$

for constants  $a$  and  $b$ . Use constructive induction to verify the formula and derive  $a$  and  $b$ .

Problem 5.

(a) Assume  $b^x = a$ . What is  $x$  (in terms of  $a$  and  $b$ )?

(b) Using only part (a), show that  $\log_c(ab) = \log_c a + \log_c b$ .

(c) Show that  $a^{\log_b n} = n^{\log_b a}$ .

Problem 6. Differentiate the following functions:

- (a)  $\ln(x^2 + 5)$
- (b)  $\lg(x^2 + 5)$  [NOTE: In Computer Science we use  $\lg x$  to mean  $\log_2 x$ .]
- (c)  $\frac{1}{\ln(x^2+5)}$

Problem 7. Integrate the following functions:

- (a)  $\frac{1}{x}$
- (b)  $\frac{1}{7x+3}$
- (c)  $\ln x$  [HINT: Use integration by parts.]
- (d)  $x \ln x$  [HINT: Use integration by parts.]
- (e)  $x \lg x$

Problem 8. Consider the formula  $3n^4 + 7n^3 \log n + 2n^2$ .

- (a) What is the high order term?
- (b) What is the second order term?
- (c) Write the formula in  $\Theta$  notation (in simplest form).