MATH299M/CMSC389W
Spring 2019 – Dan Zou, Devan Tamot, Vlad Dobrin
Model H3: Constructive/Destructive Interference with Sine Waves
Assigned: September 13th, 2019
Due: September 23th, 2019 11:59PM

By now you should be comfortable with Table, Map and Manipulate, and getting used to Plot; these are the tools you need for this assignment. Also for this assignment I want you all to start focusing on *aesthetics*, cleaning up your assignments before submitting them. Use text cells, add headings, label your Plots, remove extraneous code, etc. – I want you all to practice getting your models presentation-ready (okay, maybe not that sophisticated yet, but we're moving in that direction) finished products so that you have cool polished things to show people and to organize/display your work!

Okay, let's jump in. Sound waves are modeled by sine waves, let's remind ourselves how sine waves work:

$$a \cdot Sin(bx + c)$$

a is *amplitude*, that is, the height of the wave at its crest is proportional to it.
b is the *frequency*, so how many times the wave does a full period each interval of 2π, this makes sense because the scalar on the x speeds up (or slows down if b < 1) how fast the sine wave runs through the domain, thus scaling the frequency.
c is the *phase shift*, which moves the sine wave along the x-axis. There's never a point to using it outside of a range of $[0, 2\pi)$ since the sine wave is periodic anyway (okay, the range it matters is actually $[0, 2\pi/a)$ because $2\pi/a$ is the wavelength). Recall that cosine waves are just sine waves phase shifted by $\pi/2$, so if you were wondering why we only use sine waves, we really are using both if you consider phase shift.

Sound waves in the real world are a *superposition* of sine waves of several frequencies, that is, a linear combination (weighted sum) of sine waves of different frequencies. The idea of *Fourier Analysis* is to construct functions out of infinite weighted sums of sine waves, similar to how a Taylor Approximation works. A superposition of three sine waves would look like this:

$$a_1 Sin\,(b_1 t) + a_2 Sin\,(b_2 t) + a_3 Sin(b_3 t)$$

and you can add phase shifts if you would like. Here t is time, which is suggestive of how things work physics-wise.

If you make the frequency negative then this correlates to a wave moving the opposite direction, since its crests would be moving to the left as t increases… okay, but what do we mean by that exactly? Isn't the dependent axis t, and graphs of course are static once you stick them on some axes? When you plot Sin(t) vs. t, and you restrict the t-axis to, say, -5 to 5, you're looking at a continuum of snapshots of the wave's value over a range of time. If you instead plotted Sin(t+s) vs. t and then animated it as s increases, that window of time you're observing will start sliding. This is a subtle detail, but important to think about what you're actually animating. Whenever you see a graphic of a wave waving, what are you actually looking at? This is part of the answer. Another interpretation is that this is a wave on an infinitely long string and waves with negative amplitude are ones that are traveling the other direction down the string.

Constructive/destructive interference is when a negative wave and positive wave hit each other and cancel out or stack – wow so many subtleties, okay, by "negative" or "positive" wave we more precisely we mean "left moving" and "right moving" wave, and whether it's constructive or destructive refers to whether the waves are the same sign at the point we're looking at or opposite signs (in amplitude), which means we're really just talking about the behavior at one point.

Problem: Make a model that animates a few sine waves in superposition. The compound waves get pretty complicated pretty quickly. You could visualize interference, maybe visualize overtones (little, higher frequency waves running along bigger waves). Don't forget to format things to make it look nice – PlotLabel, colors, etc.

Got Physics? : You could visualize $e^{-i\omega t}$, a solution to differential equations you've seen if you've done any physics. It's generally complex, but Euler's formula conveniently separates it into a sum of real and imaginary periodic pieces. The Mathematica functions Re[] and Im[] take the real and imaginary parts of numbers – do you think you can use these tools to get an idea of what this differential equation solution actually means? Why does the solution to the overdamped harmonic oscillator look periodic, even though the graph doesn't look periodic? (http://hyperphysics.phy-astr.gsu.edu/hbase/oscda.html)

Standing waves would also be very cool to see - they're waves with fixed points. To tackle this we're going to need to look at things a bit differently, let's start using this as out general form for sine waves:

$$ASin\left(\frac{2\pi x}{\lambda} \pm \omega t\right)$$

This represents a wave on a string (or more abstractly), meaning that at time t and position x on the string, that is the amplitude of the string that you will see. λ is the wavelength, A is the amplitude scalar and ω is the angular frequency; $\omega = 2\pi\lambda$ always. If you take two of these with equal frequency (and thus equal magnitude) and equal amplitude, but opposite sign in the argument, you get two waves moving the opposite direction on the string that consistently cancel out at certain point(s); these are fixed points, and those superpositions of waves are standing waves. All solutions to the wave equation with boundary are built up as linear combinations of the standing wave solutions – the wave equation is the coolest thing ever and makes intimate use of calculus, linear algebra and differential equations.