CMSC 420 – 0201 – Fall 2019
Lecture 02

Basic Data Structures

Fun Challenge – Arrays for Busy People

You are given a large integer n, and are asked to implement an array data structure $A[1, \ldots, n]$ of some type T, with the following operations:

- `init`(v): all elements of A are defined to be v
- `get`(i): return the value of $A[i]$, where $1 \leq i \leq n$
- `set`(i, x): set $A[i] = x$, where $1 \leq i \leq n$

Here is the catch … All the above operations must run in time $O(1)$, irrespective of the value of n. (Thus, you cannot use a loop to initialize the array.)

Rules:

1. You may use additional arrays, but you cannot assume they are initialized
2. No fancy data structures other than arrays are allowed
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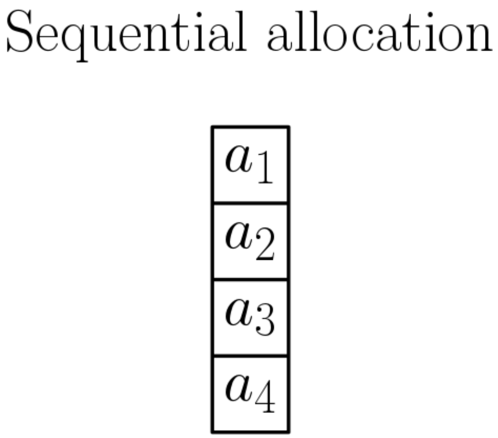
Linear Lists

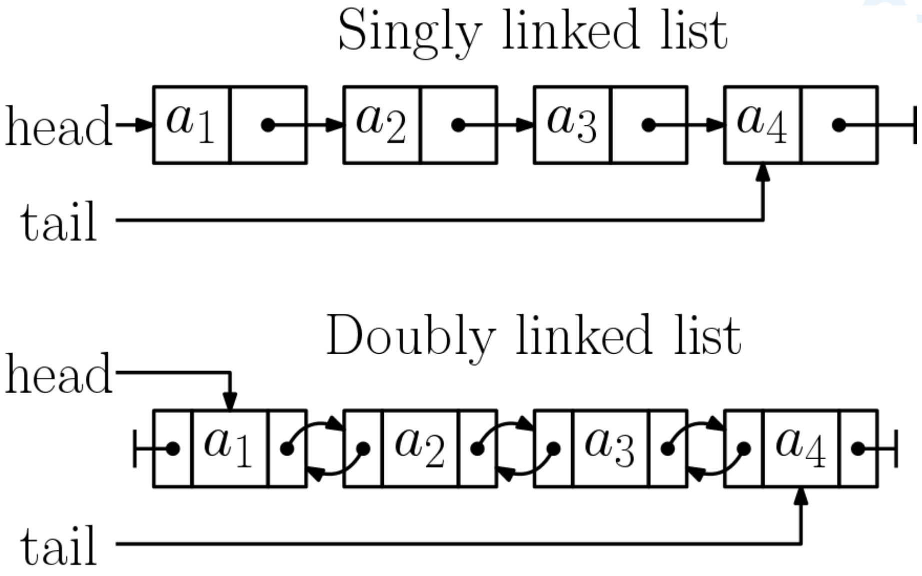
- Lists are among the most basic data types. Here is a simple interface.
  - `init()`: Initialize an empty list
  - `get(i)`: Returns element a_i
  - `set(i, x)`: Sets the ith element to x
  - `length()`: Returns the number of elements currently in the list
  - `insert(i, x)`: Insert element x just prior to element a_i (causing the index of all subsequent items to be increased by one)
  - `delete(i)`: Delete the ith element (causing the indices of all subsequent elements to be decreased by 1)

Linear Lists

Allocation Types

- Lists can be allocated in many ways. For example:
  - Sequential allocation - as an array
  - Singly linked - nodes, each referencing its successor
  - Doubly linked - nodes, each referencing its successor and predecessor





Linear Lists

Stacks, Queues, Deques

- There are a few very common types of lists:
  - Stacks - Supports insertion/removal from one end, called the top
    - push
    - pop
  - Queues - Supports insertion to tail end and removal from head end
    - enqueue
    - dequeue
  - Deque - Doubly-ended queue - Supports insertion/removal from either end
    - push-front, push-back
    - pop-front, pop-back
    - The name is a play on words, pronounced the same as "deck" of cards.

Linear Lists

Dynamic Storage Allocation

- When dealing with sequential allocation, what to do when we run out of space?
- Doubling:
  - Let n denote the current array size, and suppose we are asked to insert $(n + 1)$st item
  - Allocate a new array of size $2n$ (or generally any constant factor larger)
  - Copy the old contents to this array
  - Now, continue with the insertion
- Why double? Why not…
  - Less aggressive - Increase by a fixed number, say 100, more entries?
  - More aggressive - Increase by squaring the number of entries, say to n^2?

Linear Lists

Dynamic Storage Allocation

- Amortized cost: Given a sequence of m operations, the amortized cost of operations is the total cost of all the operations divided by the total number of operations, m.
- Theorem: When doubling reallocation is used, the amortized cost of stack, queue, and deque operations is O(1).
- Proof:
  - Charging argument - Each operation will perform a constant amount of work, and save a constant number of work tokens
  - When reallocation occurs, we will show that there are enough tokens to pay for the reallocation cost

Linear Lists

Dynamic Storage Allocation

- Theorem: When doubling reallocation is used, the amortized cost of stack operations is O(1).
- Proof:
  - Initialization - Assume a constant initial size O(1) initialization cost
  - Push - Do the operation, and deposit 4 work tokens in a bank account
  - Pop - Do the operation
  - Reallocation - Copy the current list of size n to a new array of size $2n$. We claim there are enough funds to pay for this. Why?
    - The last reallocation increased array size from $n/2$ to n
    - Since we overflowed, there must have been at least $n/2$ pushes since then
    - Bank account has at least $4(n/2) = 2n$. Thus, we have enough to pay for reallocation.

Multilists and Sparse Matrices

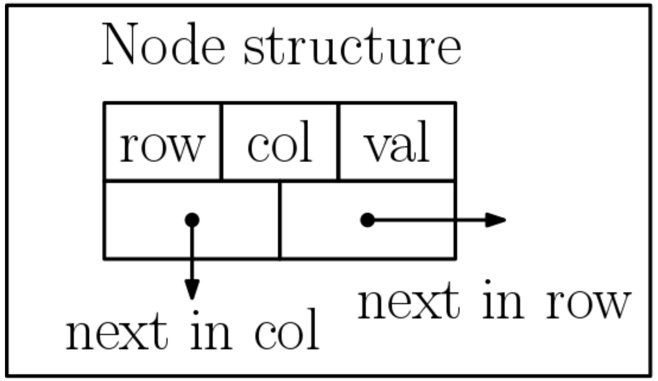
- Lists can be combined to perform more complex structures
- Example: Java’s ArrayList
- Better example: Sparse matrices
- Suppose you have a very large matrix, say $n \times m$, where n and m are in the tens of thousands.
- In many applications (particle dynamics in physics), almost all the matrix entries are zero

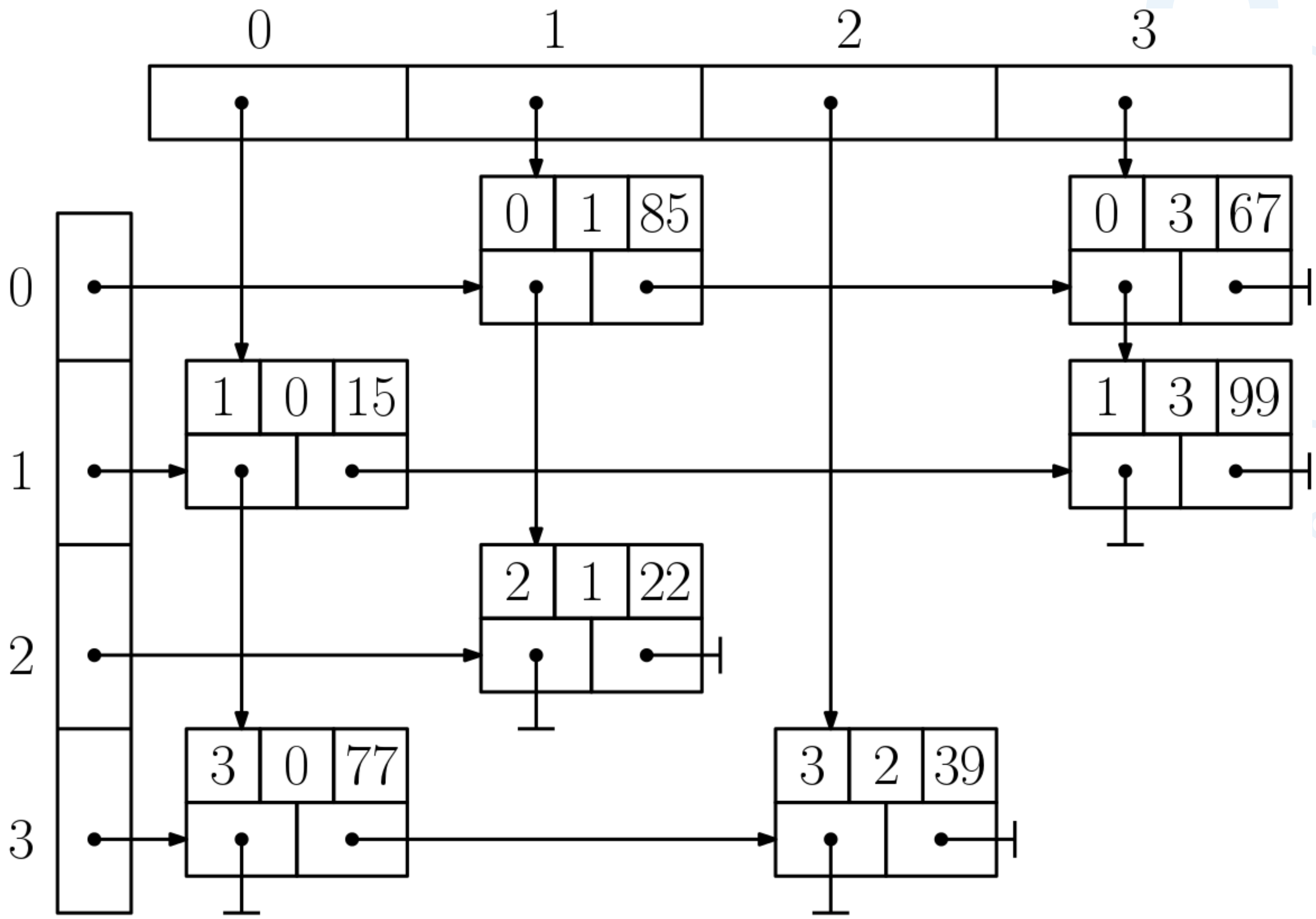
Multilist Representing a Sparse Matrix

Matrix contents

Column

Row	0	1	2	3
0	0	85	0	67
1	15	0	0	99
2	0	22	0	0
3	77	0	39	0





Fun Challenge – Arrays for Busy People

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Here is the catch … All the above operations must run in time $O(1)$, irrespective of the value of n. (Thus, you cannot use a loop to initialize the array.)

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Solution – Arrays for Busy People

We need to keep track of the entries of A that are defined, but how?

Idea 1: Maintain a parallel boolean array indicating which elements of A are defined: isDefined$[i]$ = true if $A[i]$ is defined.

$$\text{get}(i) := (\text{isDefined}[i] \;?\; A[i] : v)$$

Problem: We need to initialize this array, which will take $O(n)$ time. Too long!

Idea 2: Maintain a stack of indices of A that have been defined a value. (Can be initialized in constant time by setting $top = 0$.) We can "define" an entry $A[i]$ by pushing i on the stack.

Problem: Need to search the entire stack to test whether an entry is defined. Too long!

Solution – Arrays for Busy People

Here's the trick: Do both.

- Maintain a stack S containing the indices of defined elements.
- Maintain a parallel array $B[1, \ldots, n]$, where $B[i]$ indicates the element of the stack that witnesses that $A[i]$ is defined.
- Note that B may contain garbage, but the stack validates its "legitimate" entries.

	A		B		S	
1	cat	1	2	1	4	trusted
2	***	2	*	2	1	← top
3	***	3	*	3	*	
4	pig	4	1	4	*	
5	***	5	*	5	*	

set(5,"dog") →

	A		B		S	
1	cat	1	2	1	4	
2	***	2	*	2	1	trusted
3	***	3	*	3	5	← top
4	pig	4	1	4	*	
5	dog	5	3	5	*	

Solution – Arrays for Busy People

In addition to A, we maintain two arrays, $B[1, \ldots, n]$ and a stack S.

1. The command $\texttt{init}(v)$ saves the value of v and sets the stack empty ($top \leftarrow 0$).
2. When an entry $A[i]$ is first defined a value x, we push index i onto the stack, signaling that this entry has been initialized. We set $B[i] \leftarrow top$, which validates this entry. (Note that, $1 \leq B[i] \leq top$ and $S[B[i]] = i$.) Finally, we set $A[i] \leftarrow x$.
3. To test whether $A[i]$ is defined, test whether $1 \leq B[i] \leq top$ and $S[B[i]] = i$.
4. The command $\texttt{set}(i, x)$ applies Step 3 to test whether $A[i]$ is already defined. If not, we apply Step 2 to define it. If it was defined, we set $A[i] \leftarrow x$
5. The command get(x) applies Step 3 to test whether $A[i]$ is defined. If so, it returns $A[i]$. Otherwise it returns the default value v.

Do you believe it? You should be skeptical. Try it on a few examples to convince yourself.

Summary

- Basic data structures – Linear lists
- Stacks, queues, and deques
- Dynamic reallocation through doubling and amortized analysis
- Multilists
- (Fun problem – Not covered on the exams)