CMSC 420 – 0201 – Fall 2019
Lecture 07

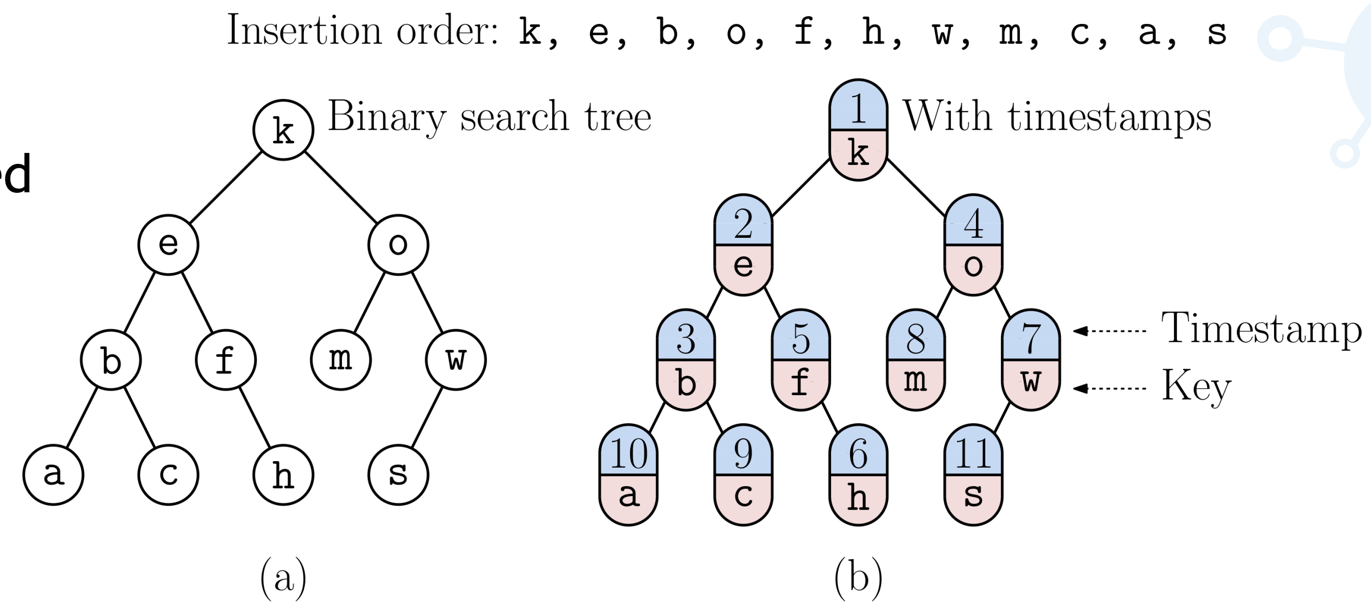
Treaps and Skip Lists

Randomized Search Structures

- Today we will discuss two randomized search structures:
  - Treaps
  - Skip lists
- We shall see that these structures are both very efficient and very simple
- Running times are measured in expectation over all random choices made
- Note that expected time does not depend on the distribution of keys or the order of operations

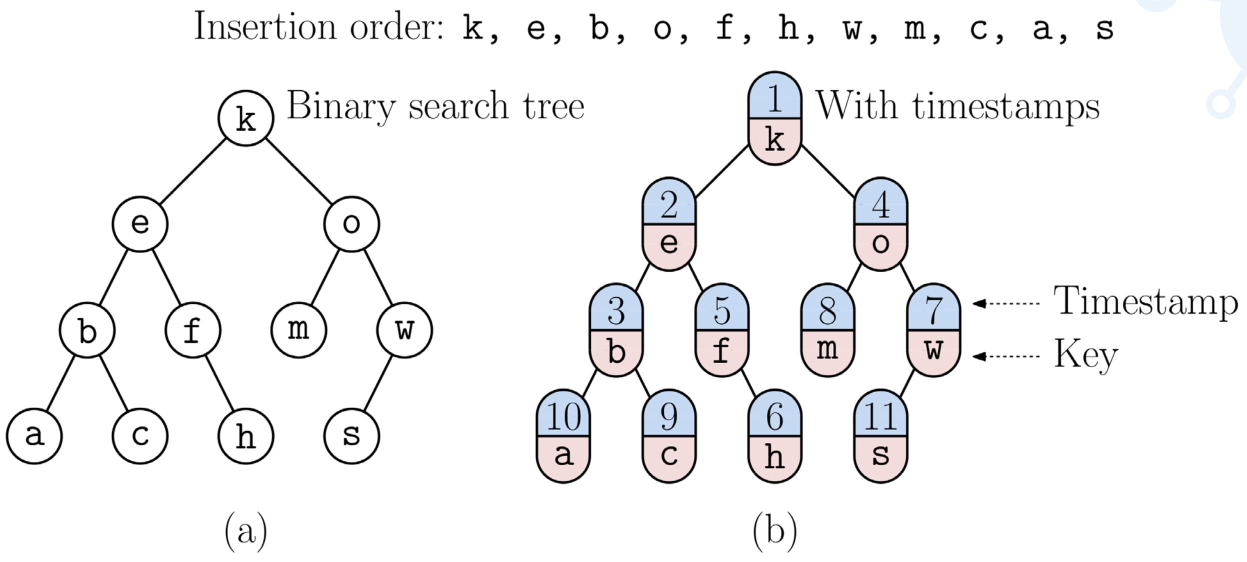
Treap

- Recall that if n keys are inserted into a (standard) binary search tree, the expected height is $O(\log n)$
- Treap - A binary tree that behaves "as if" keys were inserted in random order
- Intuition:
  - Label each item with its insertion time
  - These timestamps are ordered like a heap



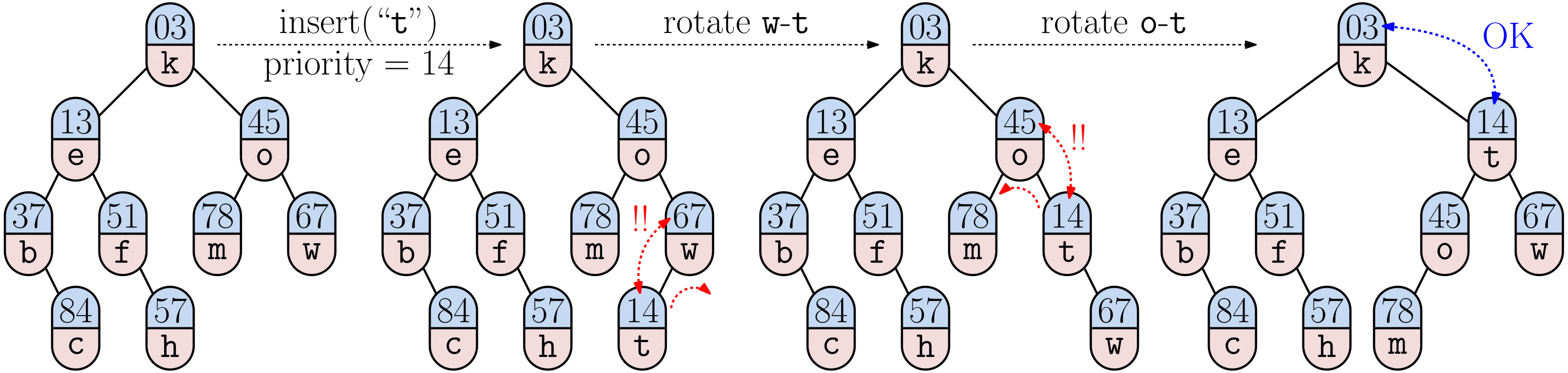
Treap

- A treap is a binary search tree, where every node p stores a priority `p.priority`
  - Priority values are chosen randomly when the key is inserted, and do not change
  - The tree is structured as if keys were inserted in priority order
- Theorem: Expected height is $O(\log n)$
- Ordering:
  - Keys - inorder
  - Priorities - heap order



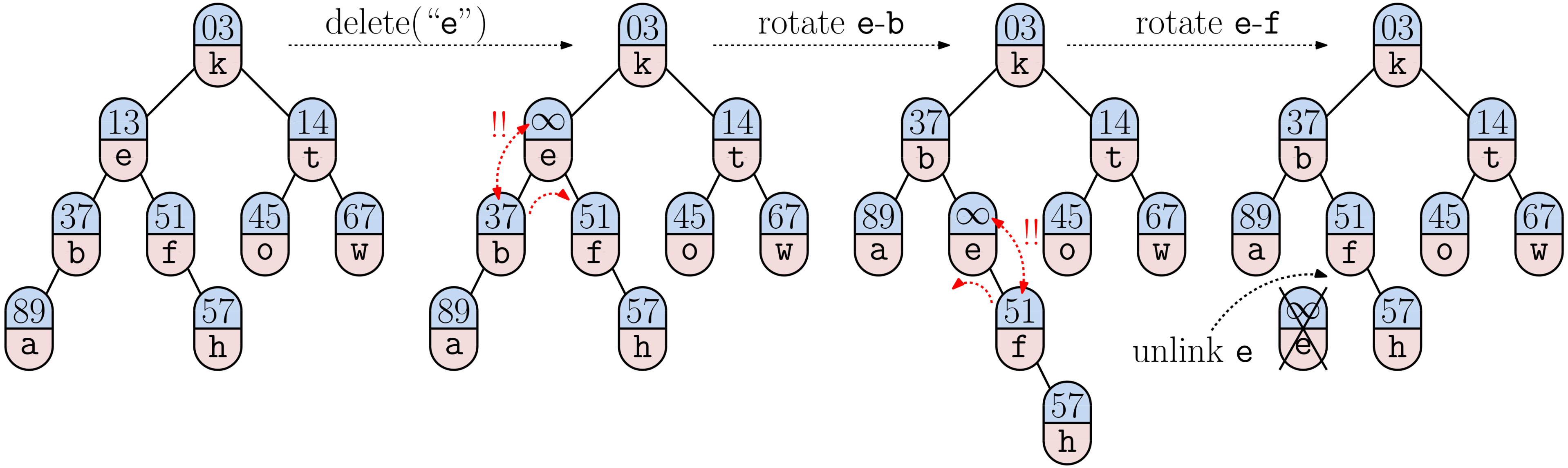
Treap Insertion

- Apply the standard insertion process – create node where we fall out of tree
- Assign a random priority value to the new node
- Apply rotations up the tree until it is in proper heap order
- Note: Inorder is maintained throughout



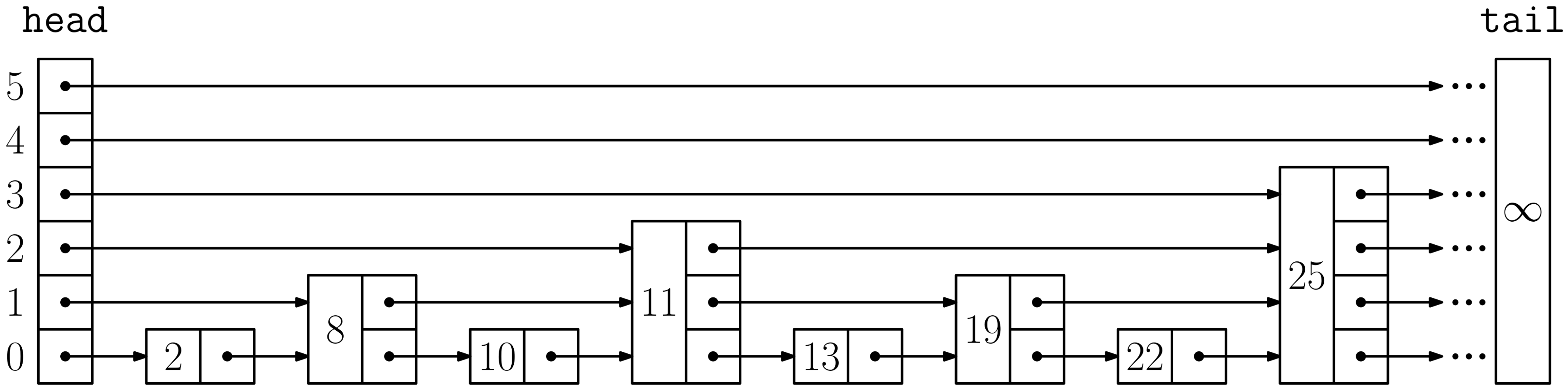
Treap Deletion

- Find the node to be deleted
- Set its priority value to ∞
- Rotate it down to the leaf level and unlink



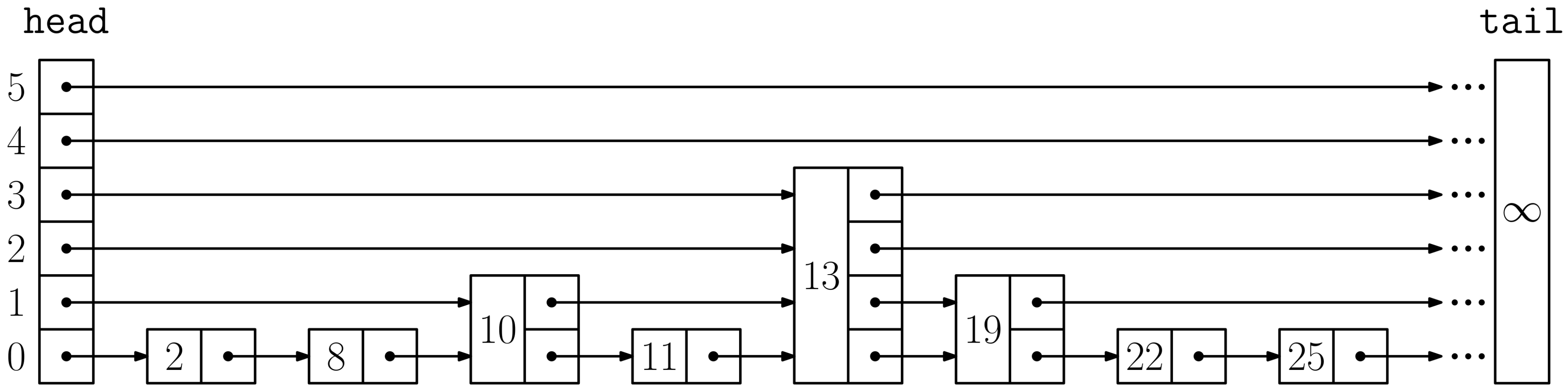
Skip List

- A "better" linked list
- Intuition: "Ideal" skip list
  - Store keys in a sorted link list (level 0)
  - Promote every other key from level $i - 1$ to level i
  - Number of levels is $O(\log n)$



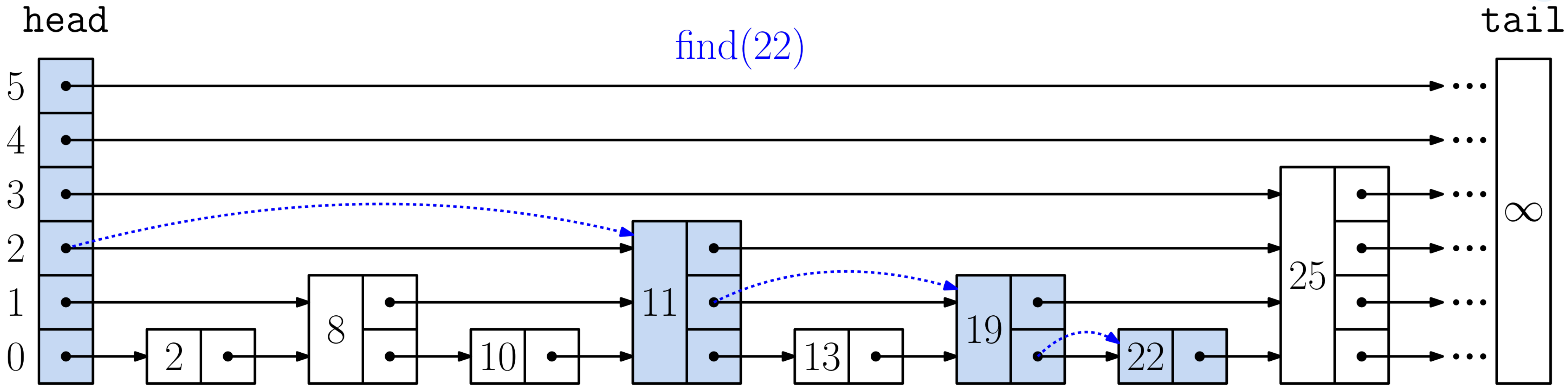
Skip List

- (Randomized) Skip List
  - Each node at level i tosses a coin
  - If the coin comes up heads (probability = ½) extend this node to level $i + 1$
  - Expected number of levels is $O(\log n)$



Skip List: Find(x)

- Start at the topmost level (`i = maxLevel`)
- Walk through level `i` until finding the rightmost node `p` such that `p.key <= x`
  - if `p.key == x`, found it!
  - else if `i > 0`, drop to next lower level (`i = i-1`)
  - else not found

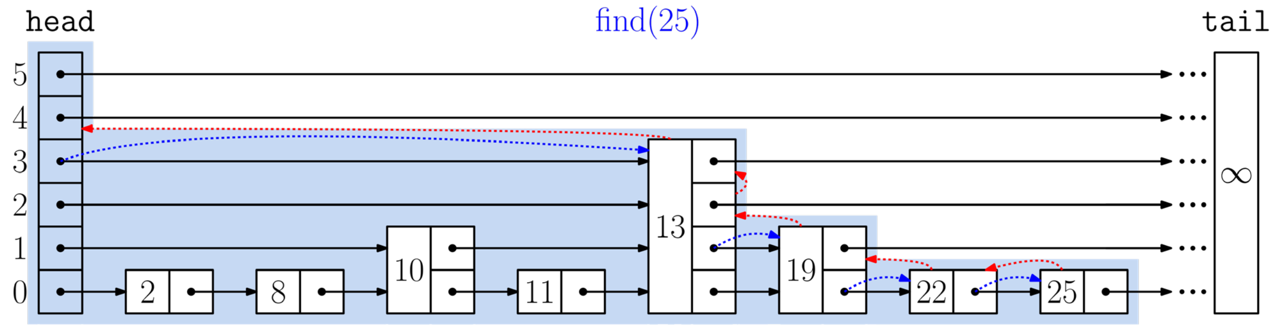


Skip List – Randomized Analysis

- Let $E(i)$ be expected number of nodes visited at level i and lower
- Backwards analysis: Walk backwards along the search path
- Suppose we are currently at level $i-1$
- If current node contributes to next higher level (with prob ½) search goes up a level (i), else we stay at current level ($i-1$). Thus:

$$E(i) = 1 + \tfrac{1}{2} E(i-1) + \tfrac{1}{2} E(i)$$

- Conclusion: $E(i) = 2 + E(i-1) = 2i$ (i.e., two nodes per level)
- Theorem: Expected search time is $O(\log n)$



Skip List Insertion

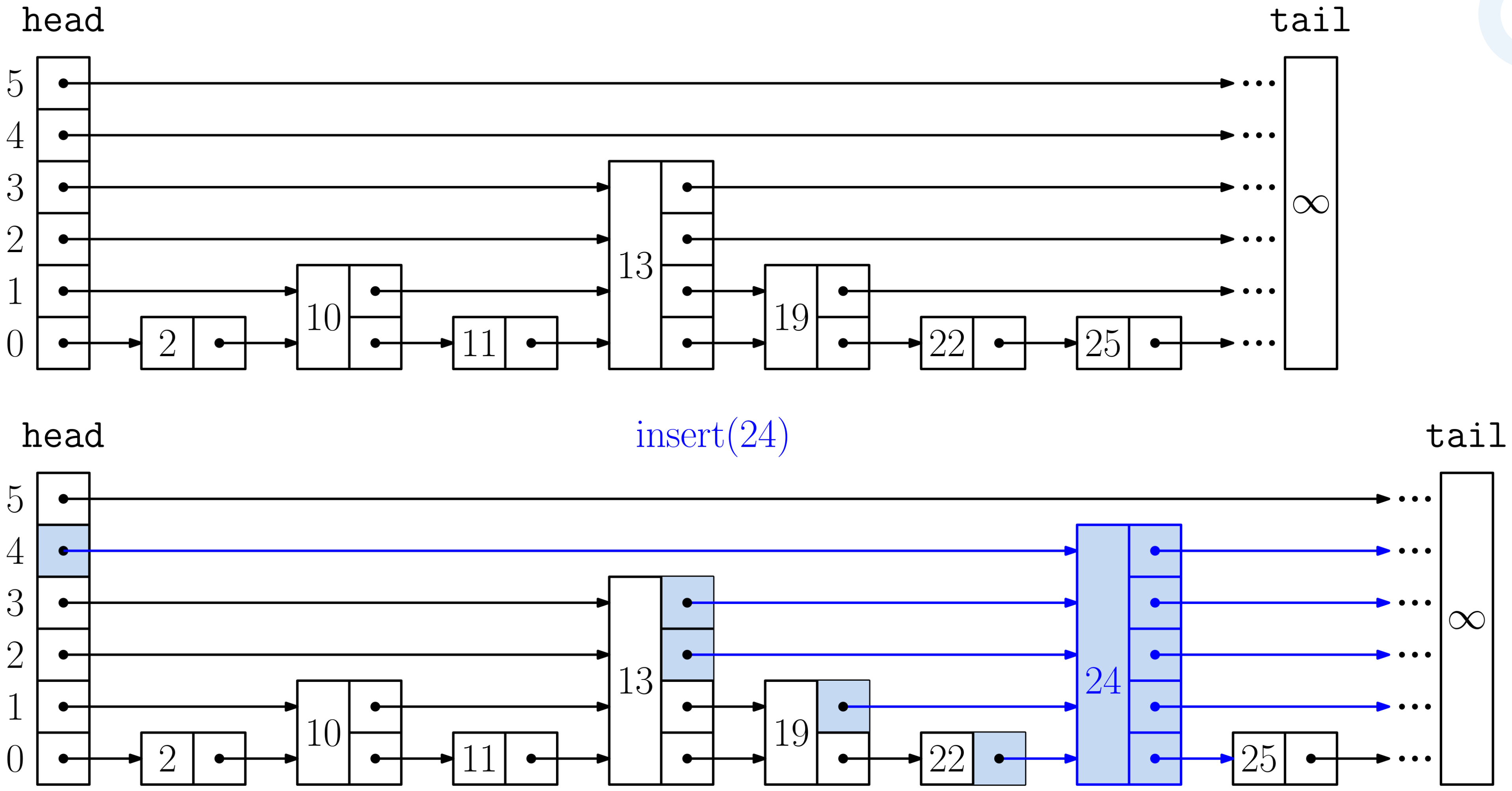
insert(x, v)

- Walk through structure, as if doing find(x)
- Keep track of the last node visited before dropping down a level
- When we find where to insert x at level 0, apply coin flipping to determine level

```
k = 0
while (k <= maxLevel && Math.random() % 2 == 0) k++;
generate node of level k, with key x and value v
```

- Link this node into levels 0 through k

Skip List - Insertion



Summary

- Randomized search structures
- Treaps
- Skip lists