Motion planning: Configuration Space

CMSC425.01 Fall 2019

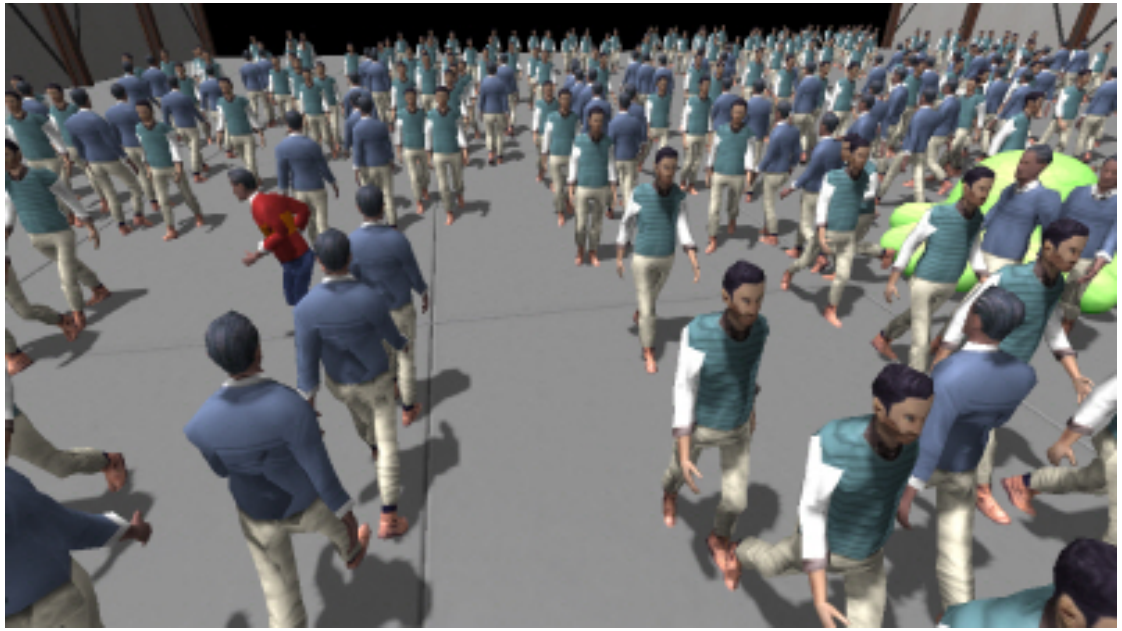
Administrivia

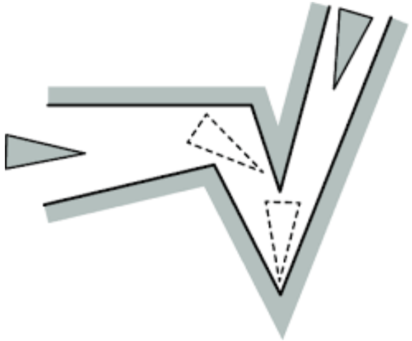
- Exam review

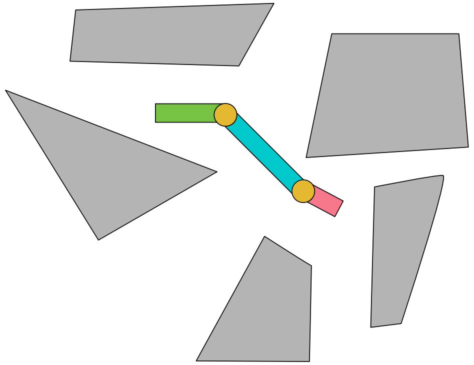
- Hw1 graded, Hw2 soon

Navigation problems

- Navigating from place to place
- Dense crowd navigation
- Coordinated team movement
- Pursuit
- Moving complex/articulated shape
  - Piano movers problem(rigid)
  - Skeleton (articulated)



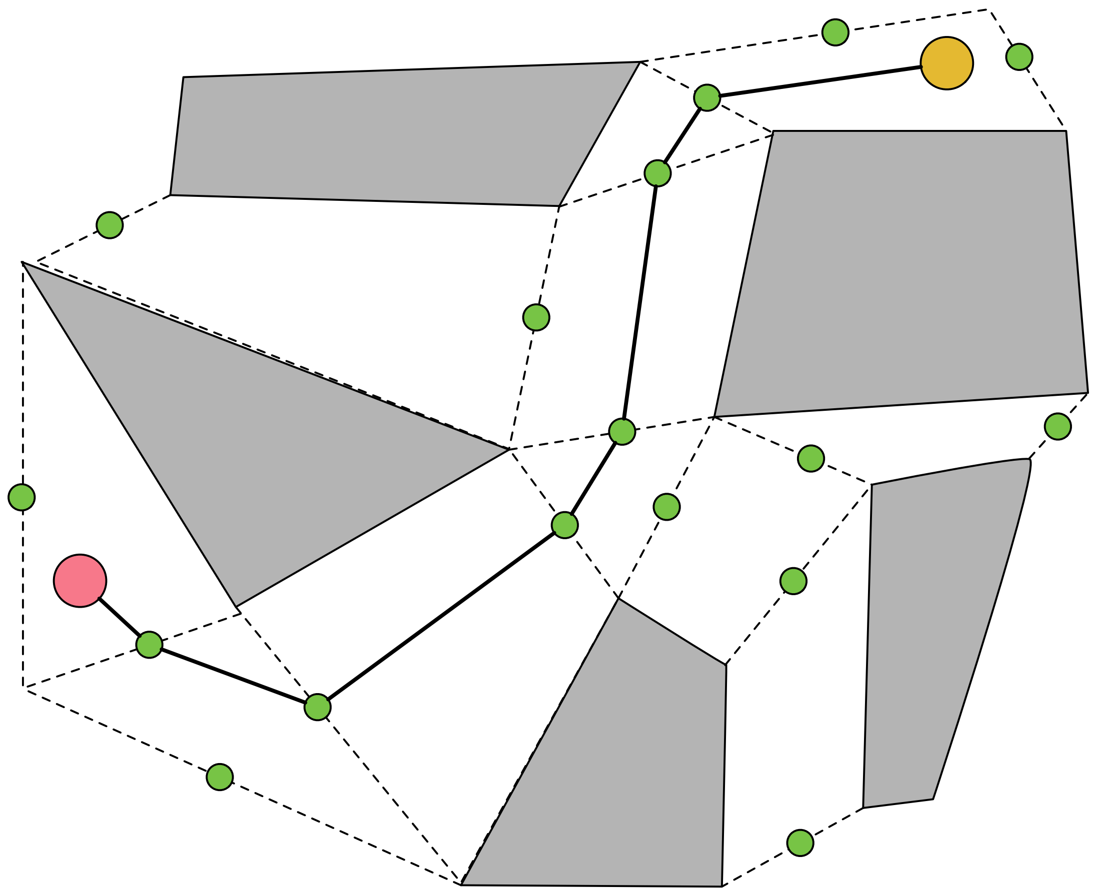




Navmesh

1. Mark navigable space
   - Use agent height/width/slope

2. Triangulate navigable area
   - Tile with triangles
3. Connect with graph
   - Connect in and out points
4. Search with algorithm
   - Dijkstra's or A*



Review: smoothing bounding

- Step 2: Simplify boundaries
  - Simplify polygon "map"

- Recursive refinement of straight line

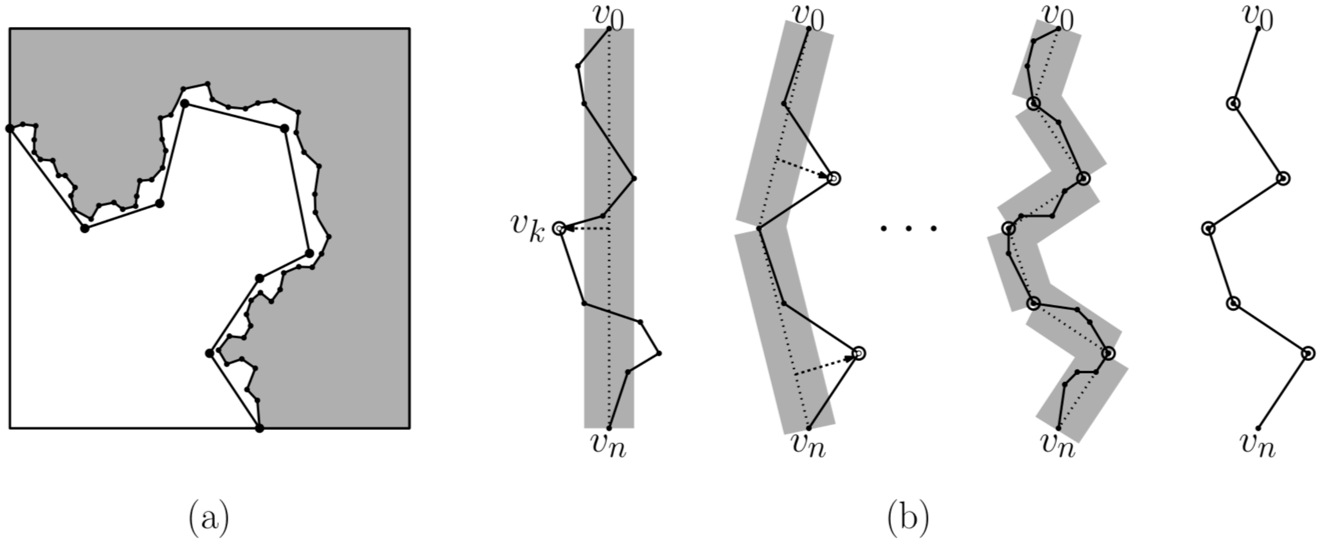
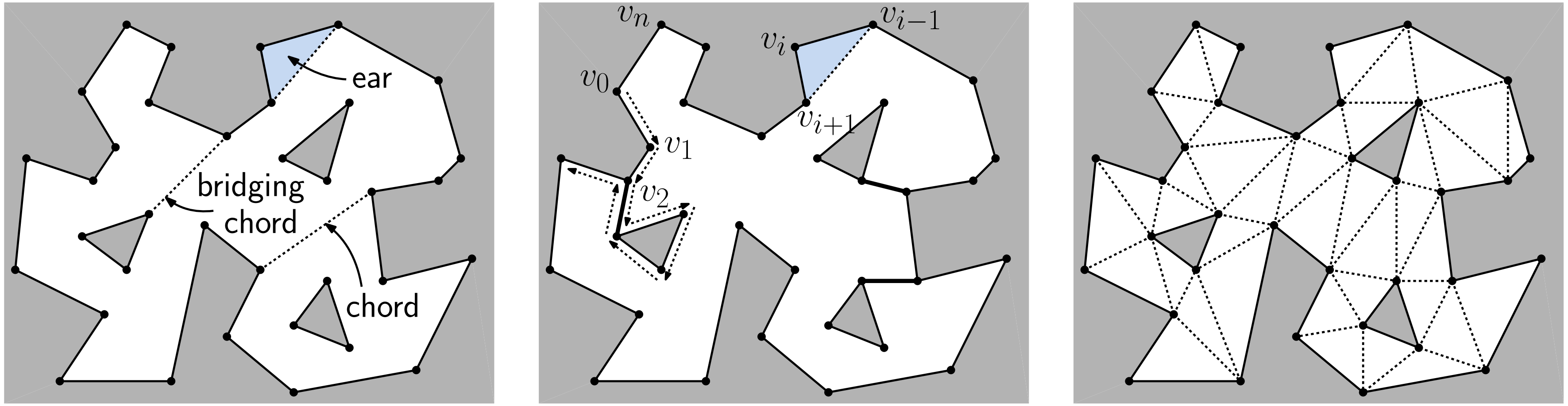


Fig. 3: The Ramer-Douglas-Peucker Algorithm.

Review of triangulation: how do efficiently?

- Step 3: Triangulate "map"
  - Cover with set of triangles

- Bridge holes
- Cut ears (!)



(a) (b) (c)

Beyond Navmesh

Navmesh: moving circle

1. Mark navigable space
   - Use agent height/width/slope
2. Triangulate navigable area
   - Tile with triangles
3. Connect with graph
   - Connect in and out points
4. Search with algorithm
   - Dijkstra's or A*

Generalizing: jointed polygon

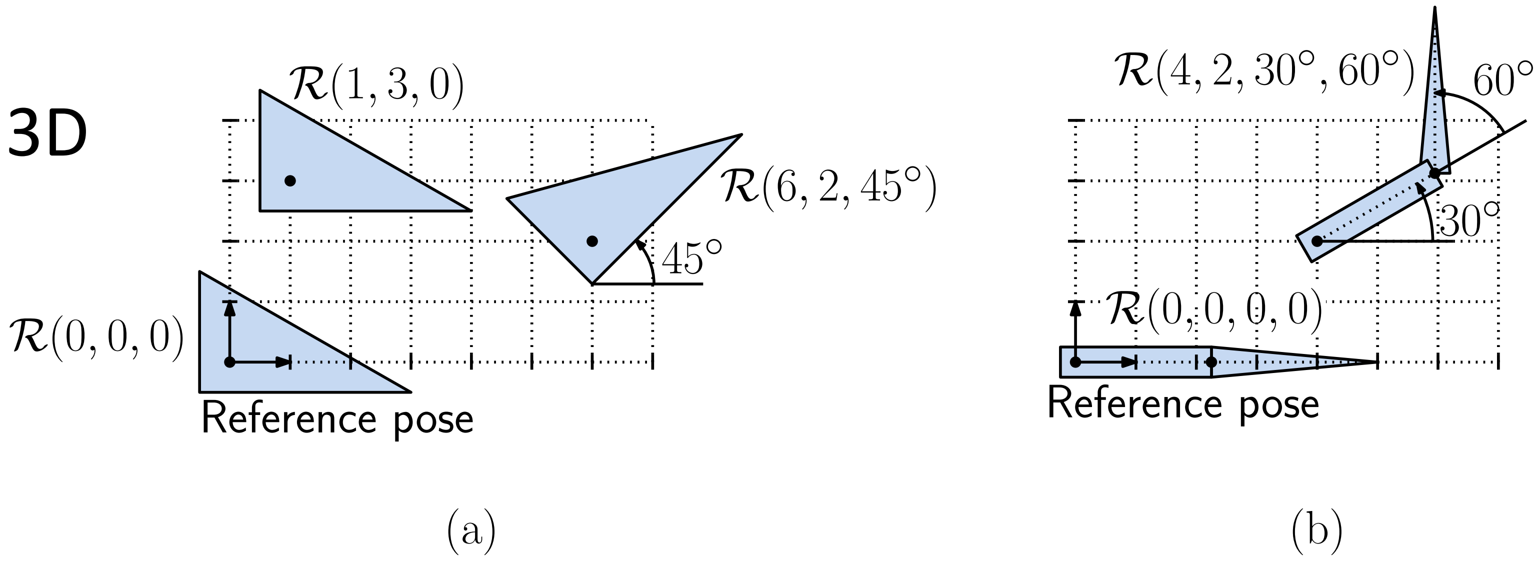
1. Define a navigable space
   - Jointed characters
   - Configuration space!
2. Find optimal paths in the space
3. Create a road network
4. Search the network

Defining robot configuration *R*

- Multiple degrees of freedom

- 3DOF – translate/rotate $\mathcal{R}(x, y, \theta)$ (region covered by robot)
- 4DOF – translate/rotate/bend $\mathcal{R}(x, y, \theta, \phi)$

- 6DOF – rigid object in 3D
- Human – 244

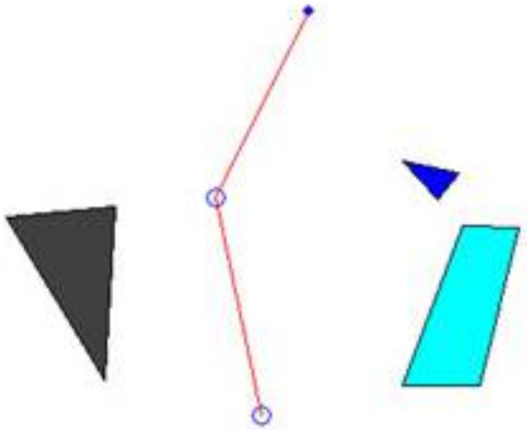


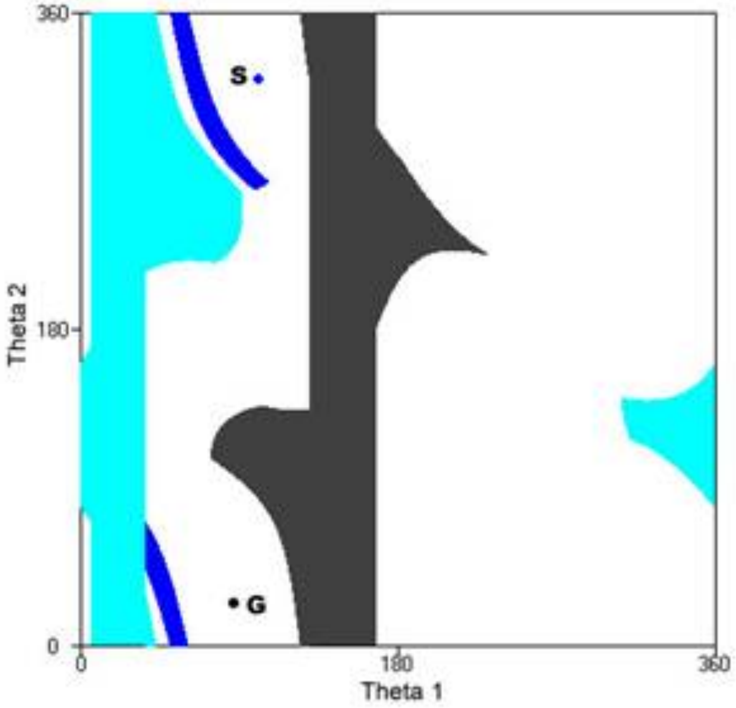
Defining workspace S

- Boundary of space + obstacles
- In same DOF space as robot

- Defines free and forbidden ranges of values of R

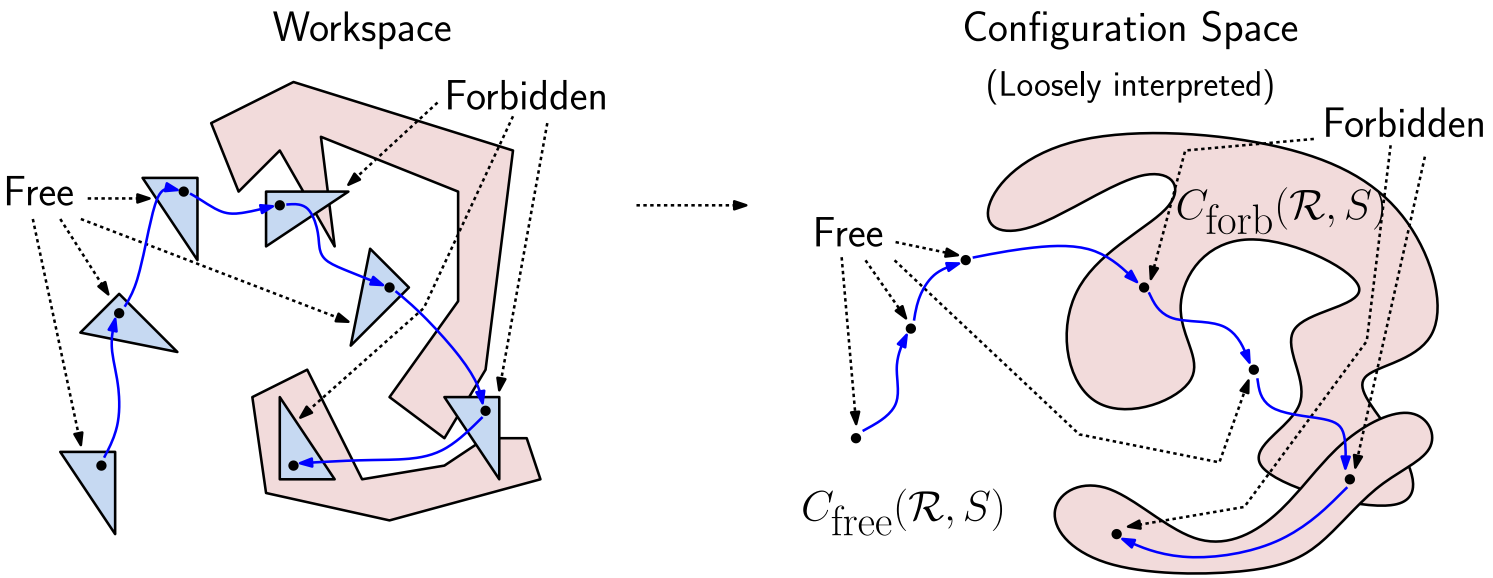
- $C_{free}(R, S)$
- $C_{forbidden}(R, S)$





Motion planning in configuration space

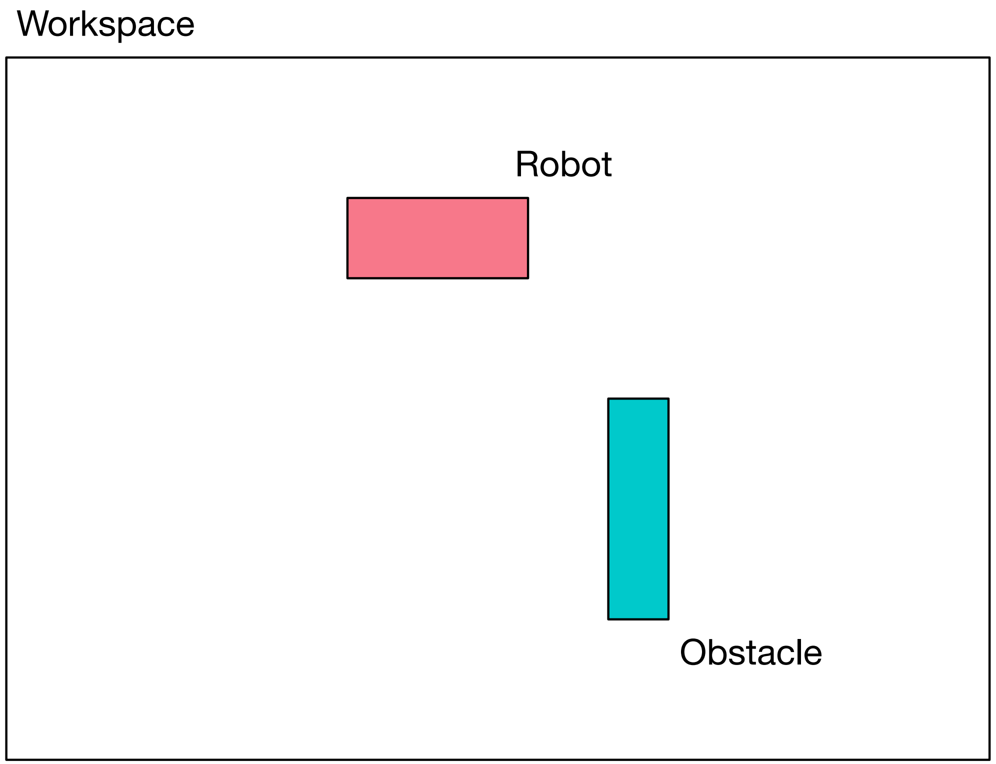
- Path from $s, t \in C_{free}(R, S)$
- if we have $\mathcal{R}(s) \rightarrow \mathcal{R}(t)$
- with all configurations in free space
- One path can be better than another based on length, maximum bend, etc



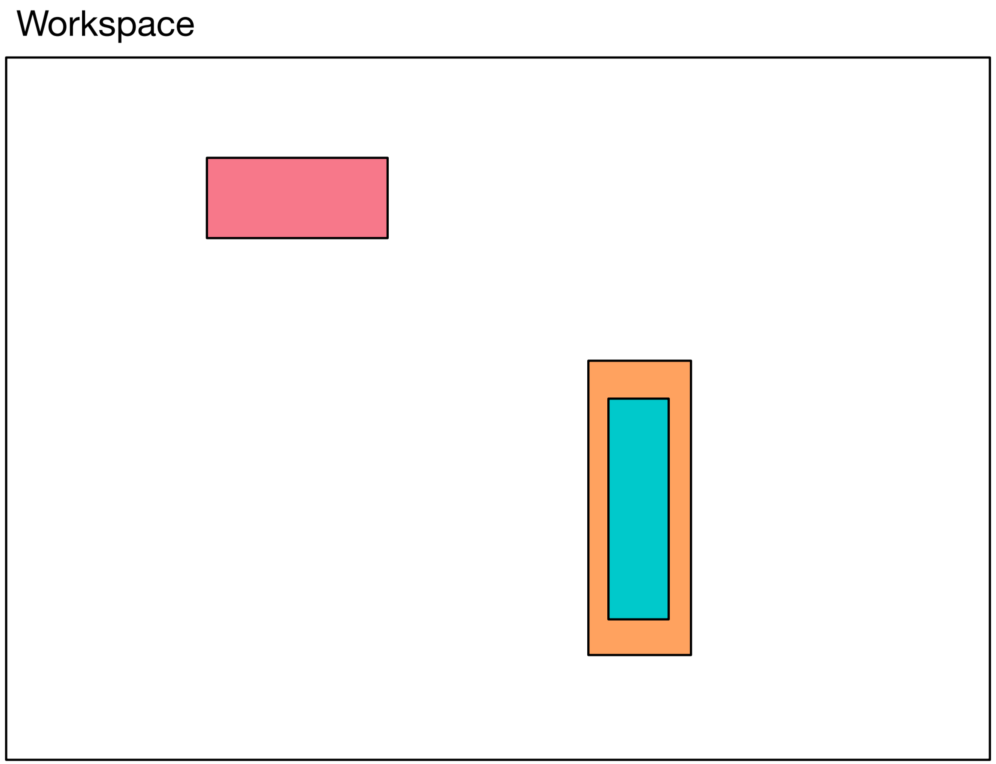
(a) (b)

Building configuration space

Robot and obstacle

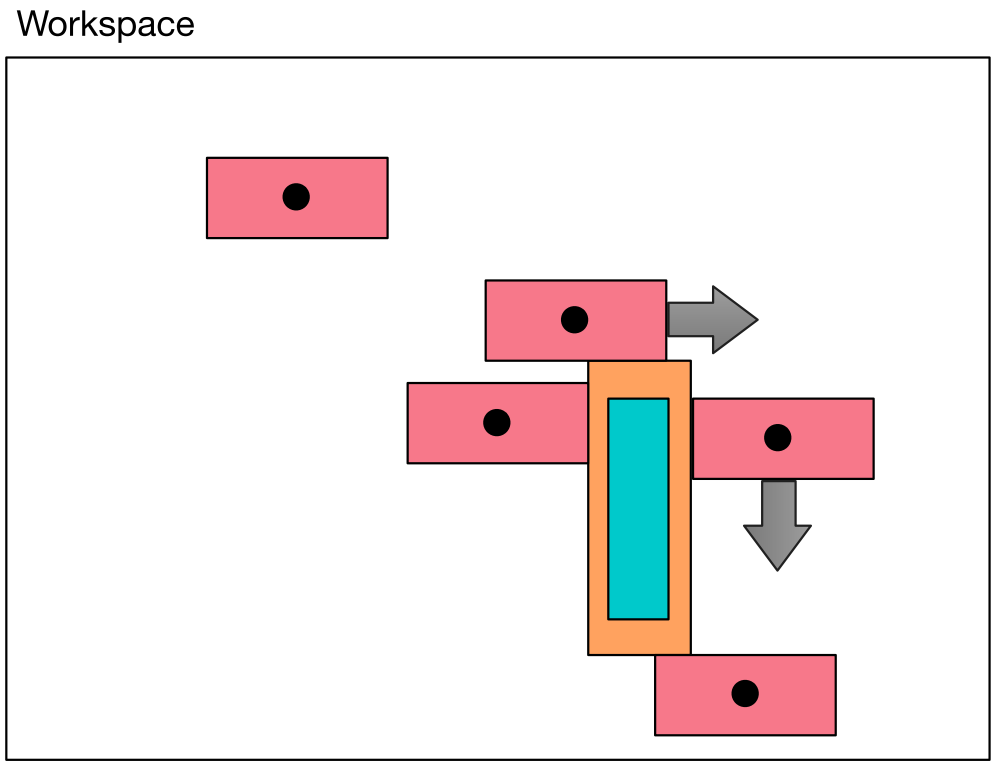


Step 1: Establish buffer distance

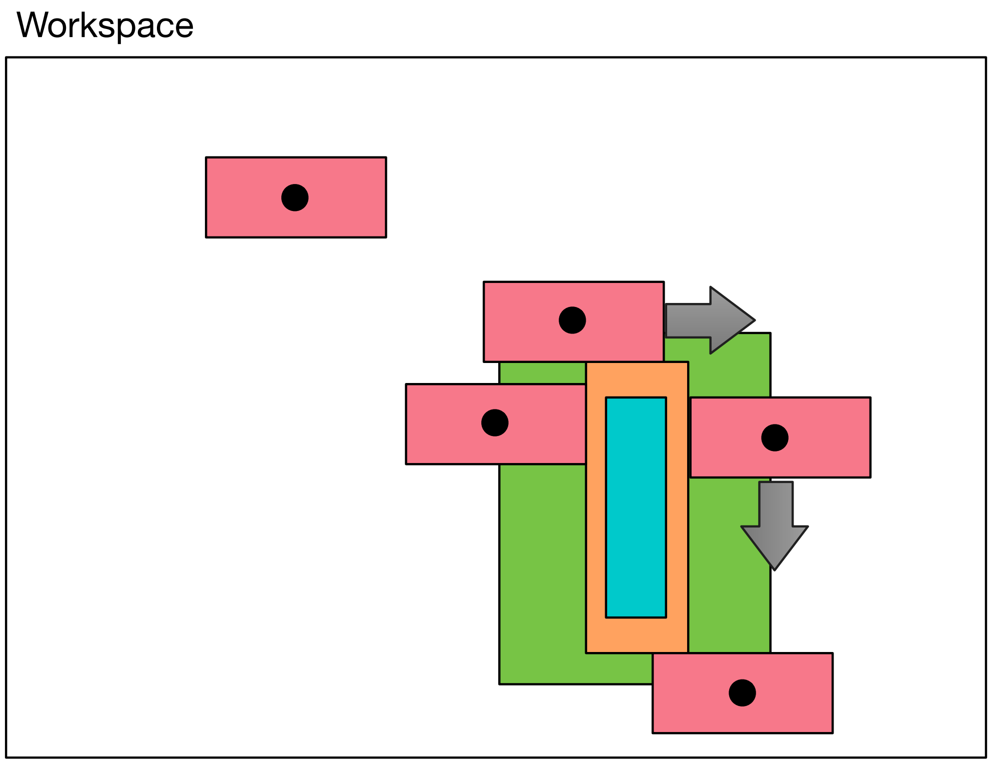


Building configuration space

Step 2: Move shape around obstacle

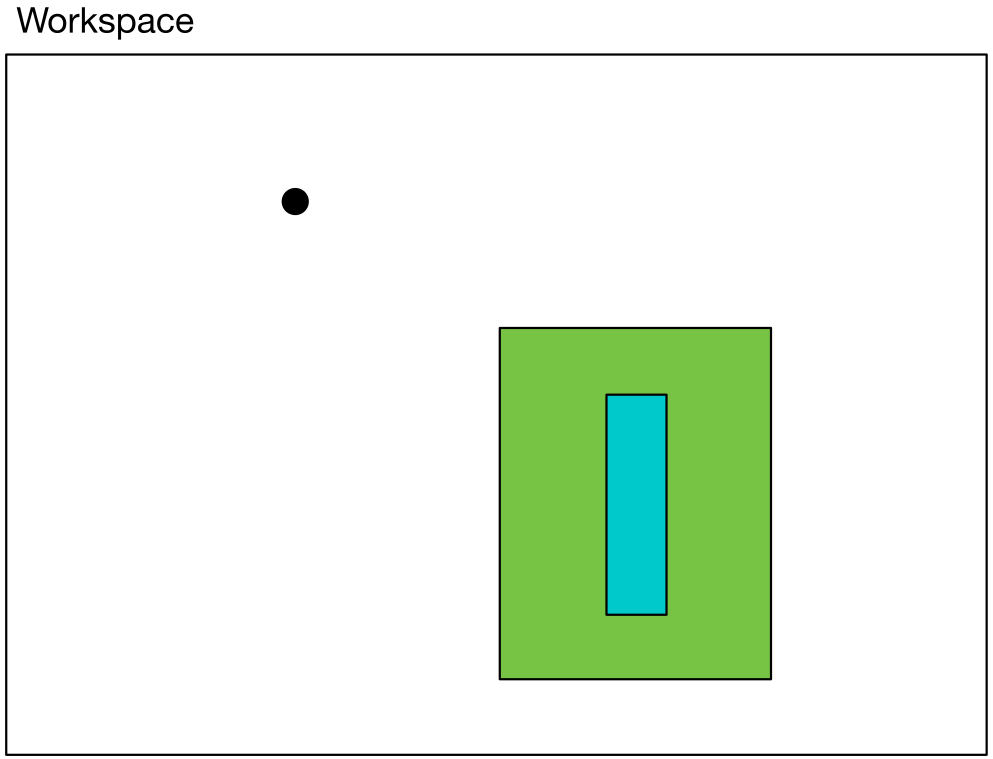


Step 3: Create extended obstacle (green) by midpoint



Building configuration space

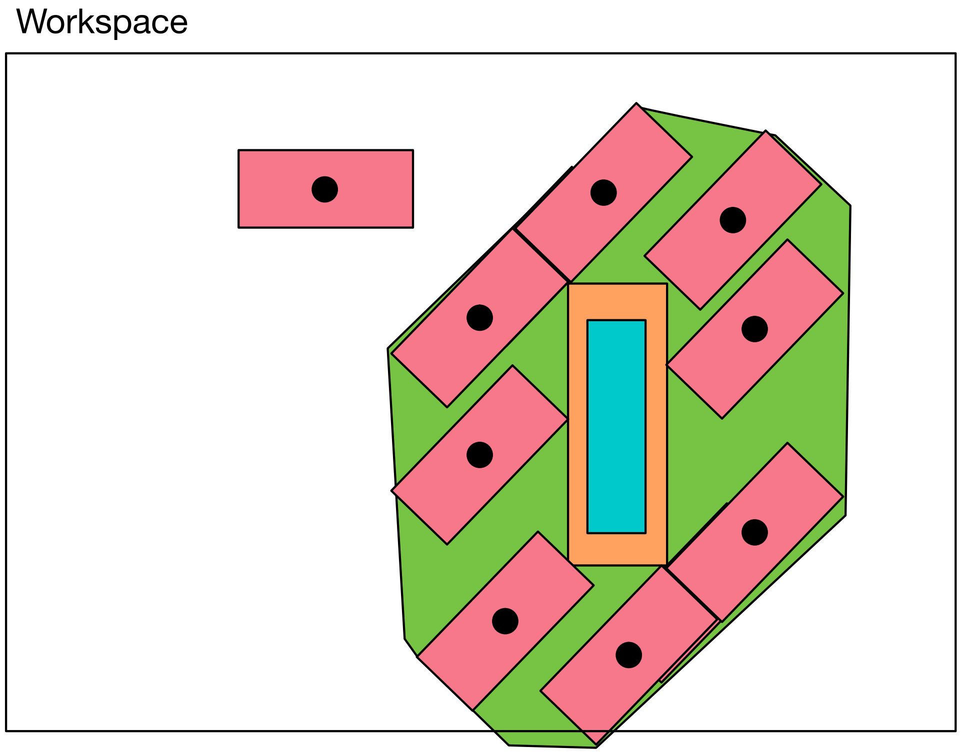
Step 4: Reduce robot to point



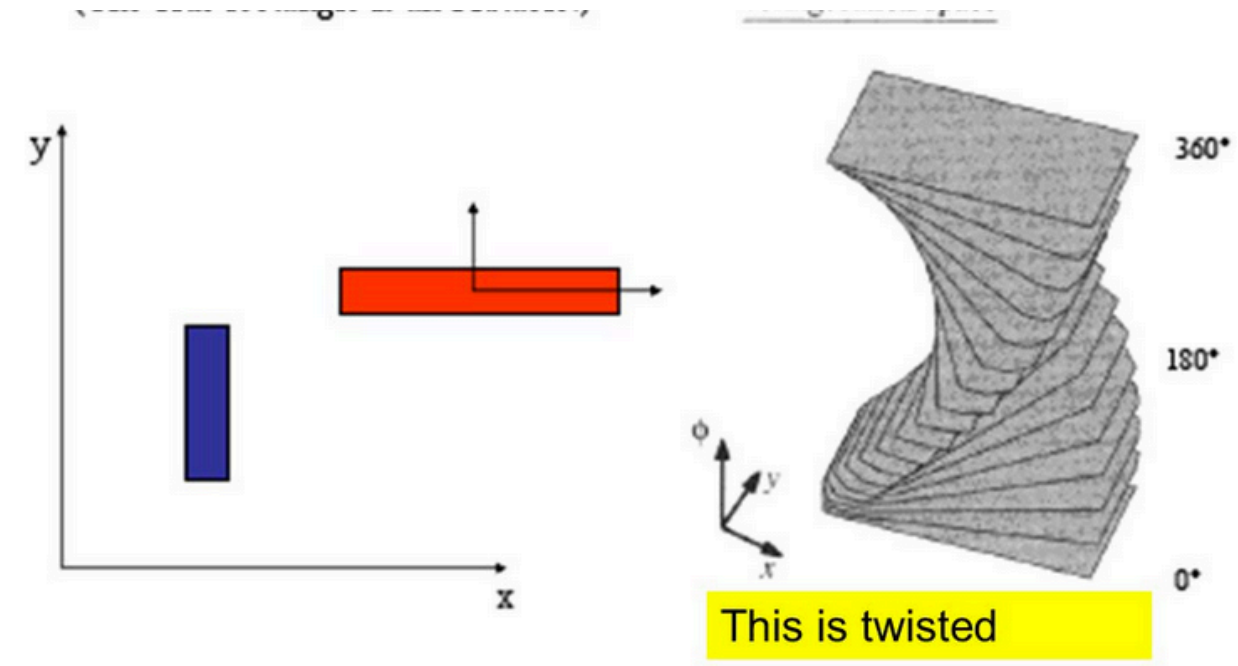
- Robot becomes point
- Obstacle become C-obstacle
- Path finding reduces to finding a path for a single point around extended obstacles

In higher dimensions

Step 6: Rotate and repeat (0-180 degrees)



Creates solid in 3D space



- $\mathcal{R} = (x, y, \theta)$
- 3D space
- (Howie Choset CMU)

Creating in higher dimensional space

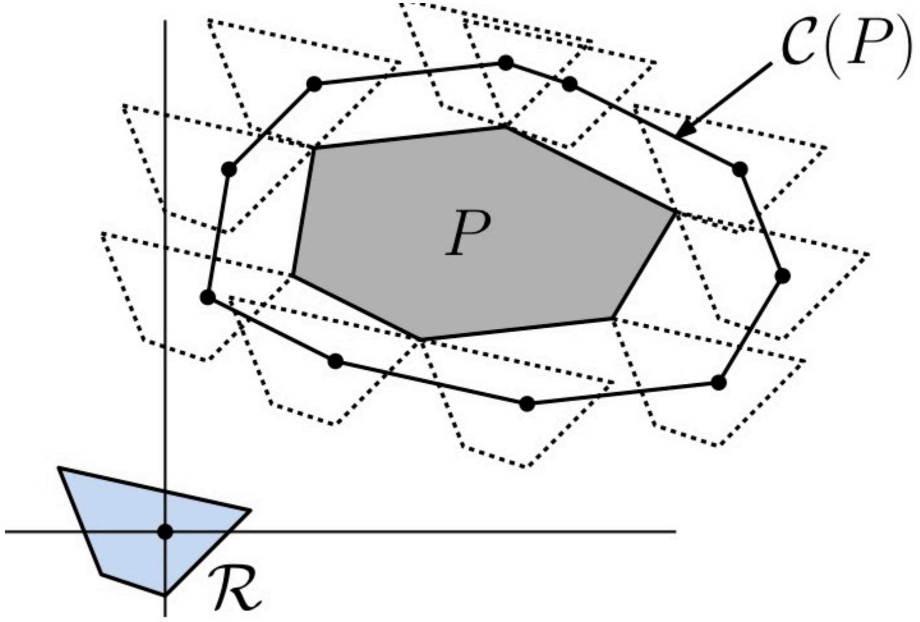
- Expensive!
  - 5DOF or 7DOF arm?
  - Robot base with arm? 10DOF

- Sample space, fit surface, approximate

(a) (b) (c) (d) (e)

Formalizing: Minkowski sums

- Motivation
- $\mathcal{R}(p)$ is region of $\mathcal{R}$ translated to p
- P is an obstacle region
- $C(p) = \{p : \mathcal{R}(p) \cap P \neq \emptyset\}$



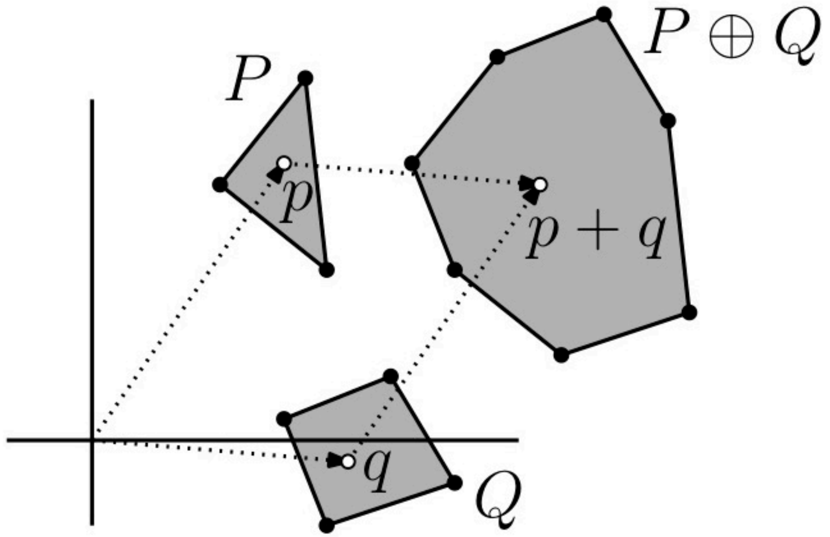
(a)

Definitions

- Minkowski sum
- $P \oplus Q = \{p + q : p \in P,\ q \in Q\}$

- Negated region
- $-P = \{-p : p \in P\}$

- Sum with point
- $P \oplus p = P \oplus \{p\}$



(b)

Claim: $C(P) = P \oplus (-\mathcal{R})$

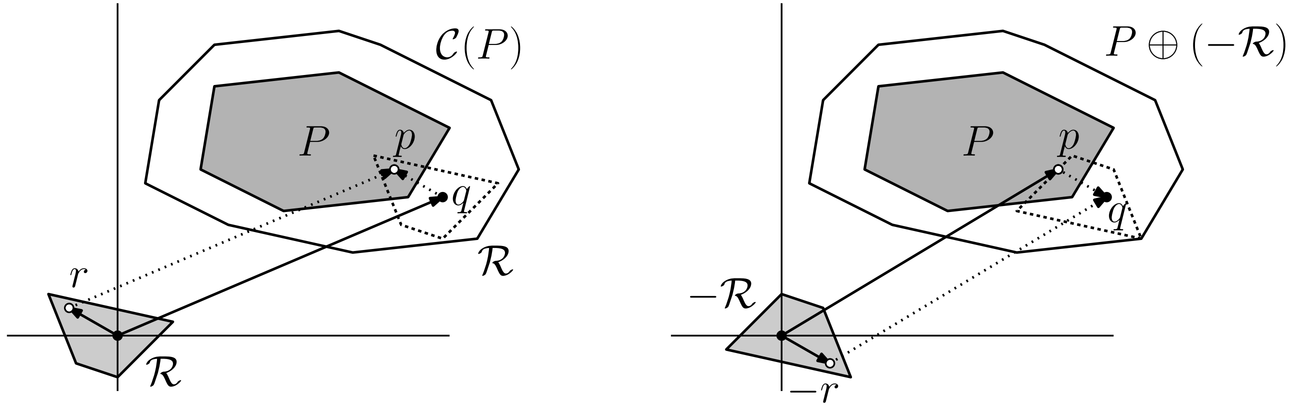
- "Proof":

If robot R intersects obstacle P when at location q (R(q) in P)

Then we have for r in R that p = q+r

Then we can deduce q = p – r

The points q are those that compose C(P)

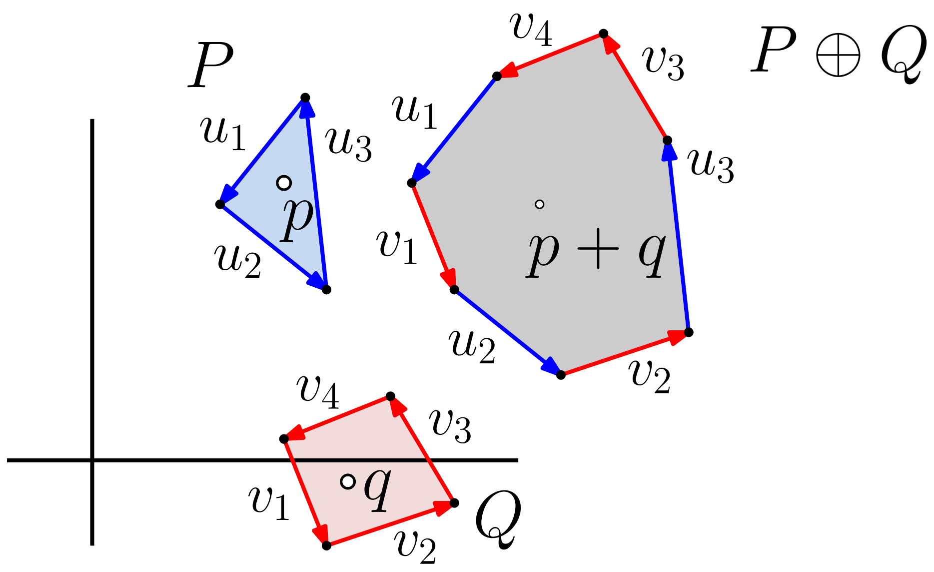


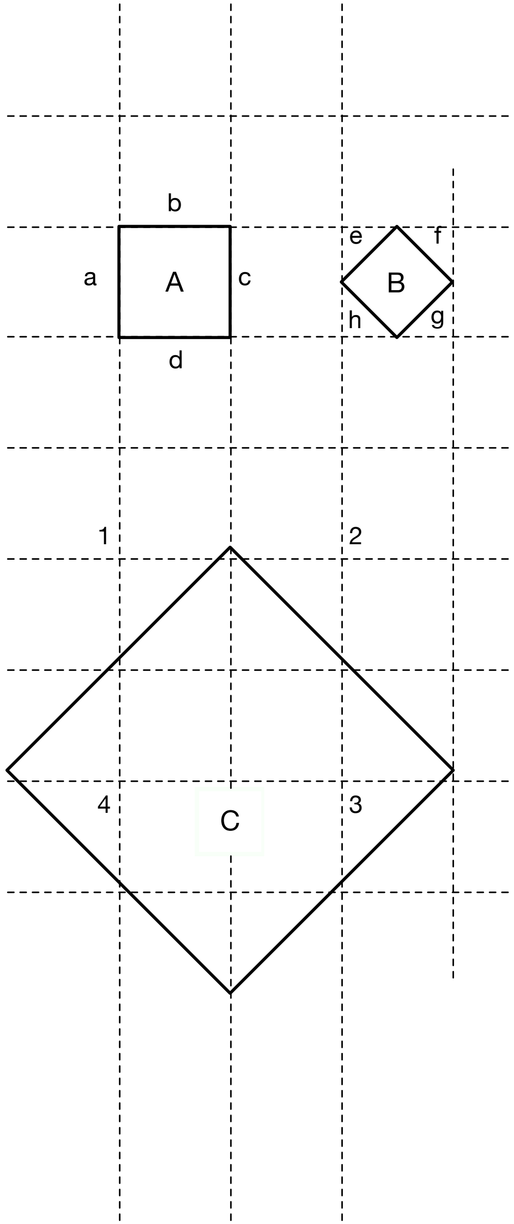
(a) (b)

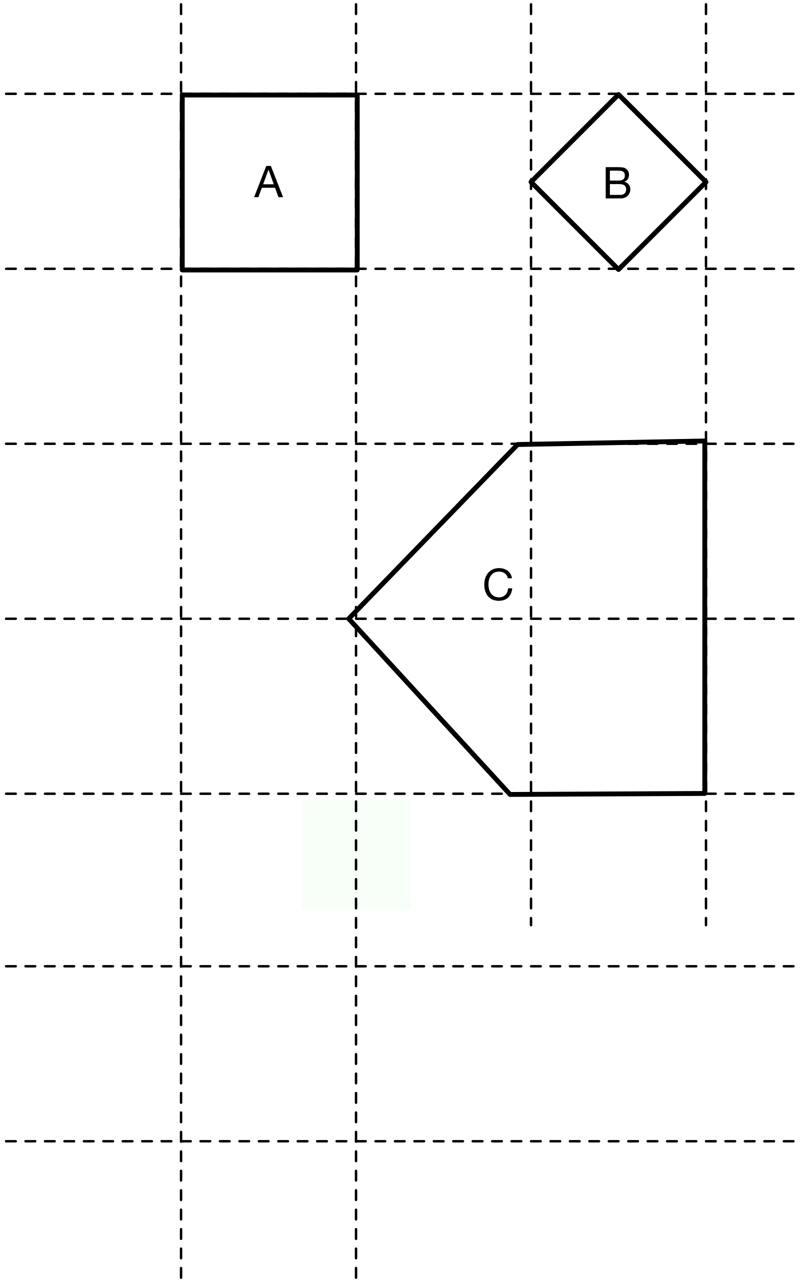
Algorithm: Computing Minkowski sum

- Input: two polygons
- Output: polygon of M-sum

- Algorithm:
  - Take each edge in CCW direction
  - Sort by angle
  - Combine




b
a
A
c
e
f
B
h
g
d
1
2
4
C
3


A
B
C

Finding paths in polygonal configuration space

- Version 1: Navmesh
- Others?

Finding paths in polygonal configuration space

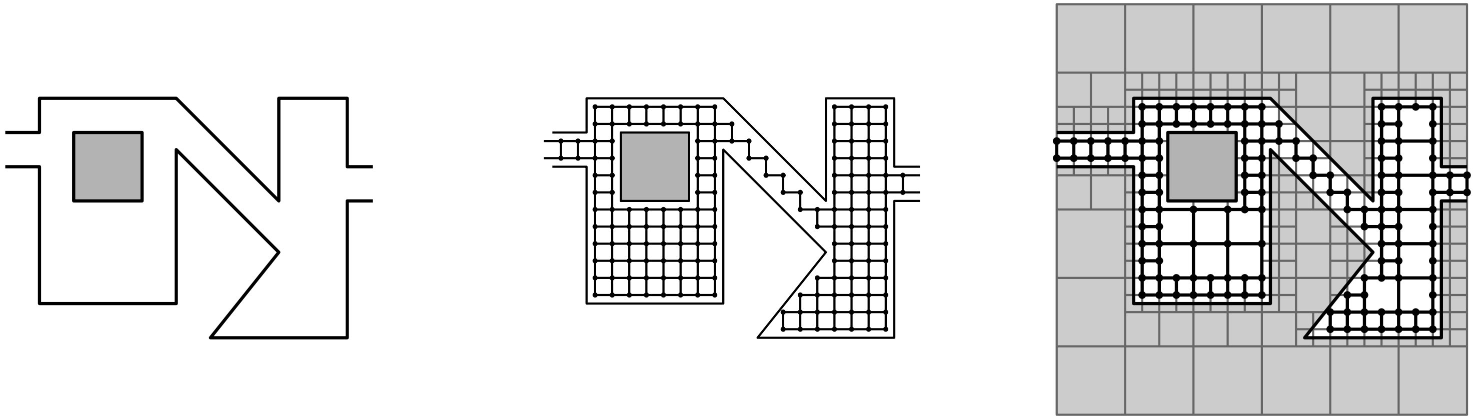
- Version 1: Navmesh
- Others?

- Version 2: Game designer draws …

Finding paths in polygonal configuration space

- Version 1: Navmesh
- Others?

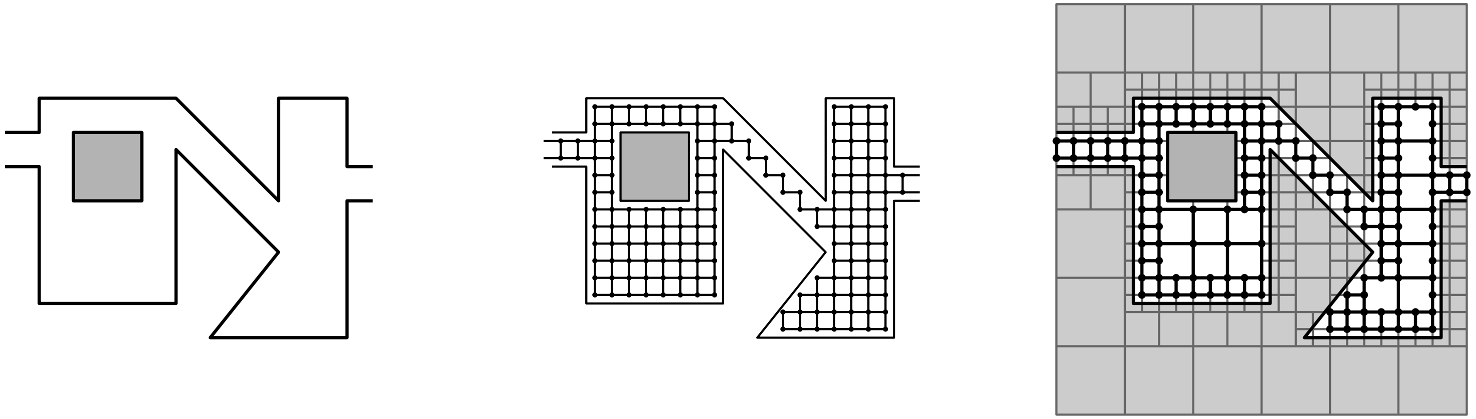
- Version 3: Grid


(a) (b) (c)

Finding paths in polygonal configuration space

- Version 1: Navmesh
- Others?

- Version 4: Multiresolution grid

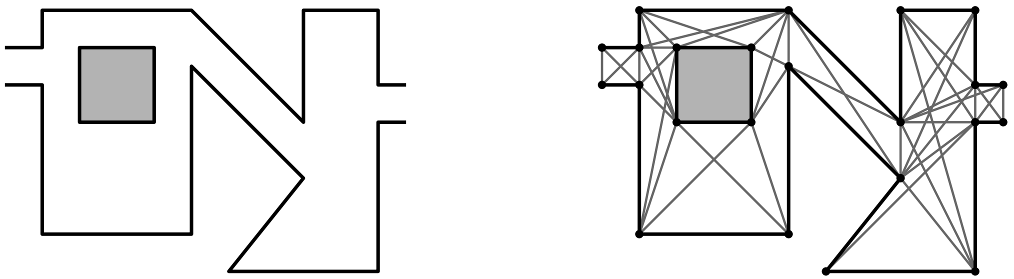


(a) (b) (c)

Finding paths in polygonal configuration space

- Version 1: Navmesh
- Others?

- Version 5: Visibility graph



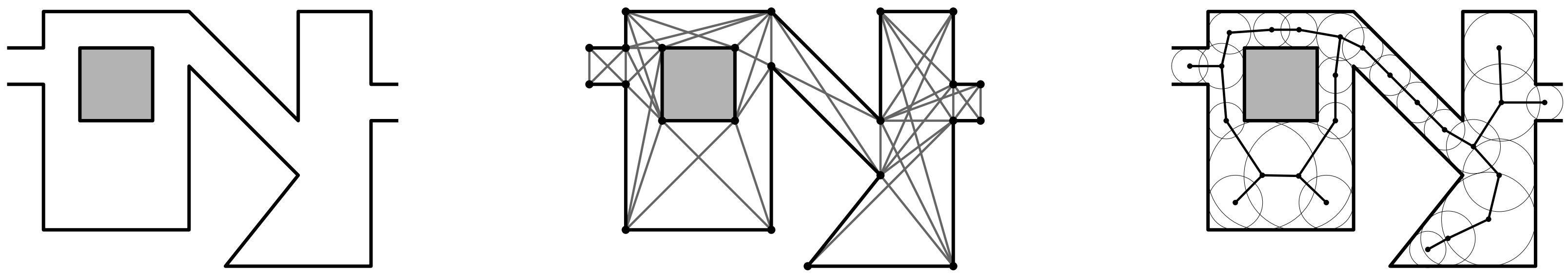
(a)

(b)

Finding paths in polygonal configuration space

- Version 1: Navmesh
- Others?

- Version 6: Medial axis (c)



(a) (b) (c)