CMSC425 Lecture notes 10/8
Skeletons, rigging and animation

These notes parallel the PowerPoint “Skeletons and Skins”, Day 12

Pages:

1-2) Observes that you can construct a rotation matrix if you know the angle, or if you know the vector into which you’re rotating the x-axis – that vector is the first column of the rotation matrix.

Much cheaper than doing the dot product and using sin/cos to compute rotation matrix.

3) Observes that rotation matrices are orthonormal, so each column is orthogonal (linearly independent) of the others so the dot product of pairs of different columns is 0, but each column is normalized so column i dot column is 1.

4) Does example from Mount notes on coordinate transforms.

5-6) Digression on projective geometry to explain homogenous coordinates. More on this later.

7) More on homogenous coordinates, points vs. vectors, and homogenous matrices.

8-9) Mount example on transforms for binding pose of arm, and rotation of joints.

Skeletons and Skinning CMSC 425
Oct. 8th

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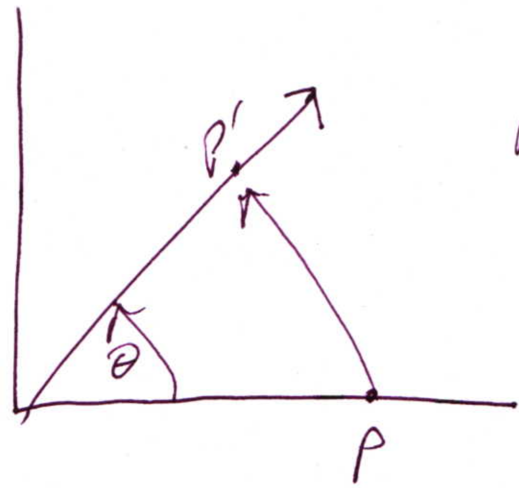
Alternative rotations



Rotate θ degrees

$$P' = M_R P$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$v = \hat{i} = \langle 1, 0 \rangle$

rotate to new axis.

$$\begin{bmatrix} .35 \\ .65 \end{bmatrix} = \begin{bmatrix} \underline{.35} & - \\ .65 & \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= [\, .35 \times 1 + 0,$$

$$\begin{bmatrix} -.65 \\ .35 \end{bmatrix} \begin{bmatrix} \cancel{.35} \\ \cancel{.65} \end{bmatrix} = \begin{array}{c} \begin{array}{cc} v & v^{\perp} \end{array} \\ \begin{bmatrix} .35 & \underline{-.65} \\ .65 & .35 \end{bmatrix} \end{array} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Rotate system into vectors

$V, V^{\perp}$

$$\Rightarrow M = \begin{bmatrix} V^T & (V^{\perp})^T \end{bmatrix}$$

$V, V^{\perp}$ orthonormal

$$|V| = |V^{\perp}| = 1$$

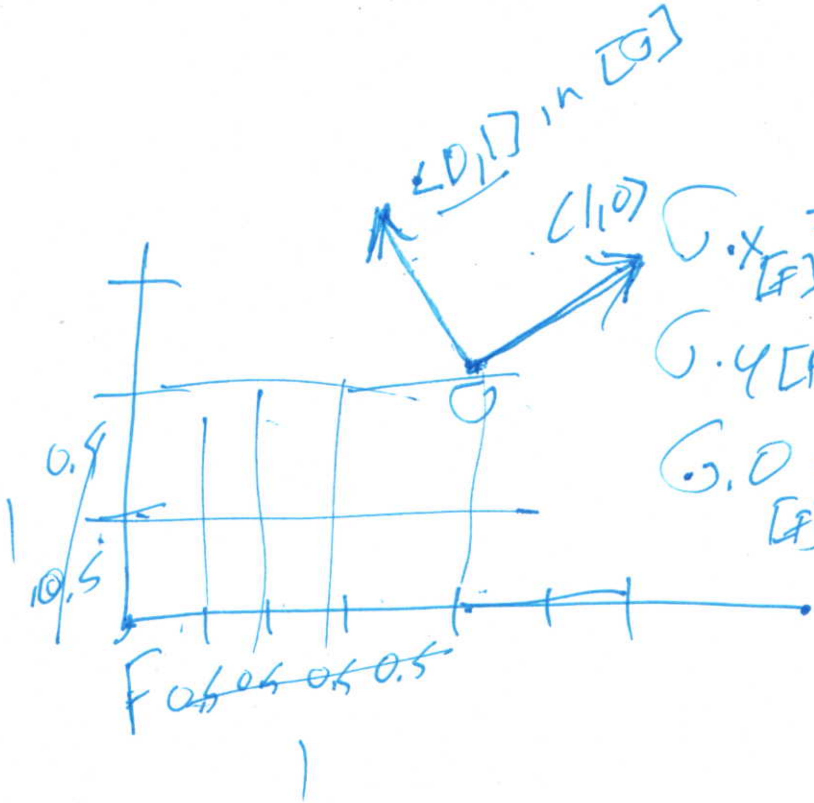
$$V \cdot V^{\perp} = 0$$

$$M = \begin{bmatrix} V^T & V^{\perp T} \end{bmatrix} = \begin{bmatrix} V_x & V^{\perp}_x \\ V_y & V^{\perp}_y \end{bmatrix}$$

$$|M| = 1$$

$$M^{INV} = M^T$$

$$M^{INV} = \begin{bmatrix} V \\ V^{\perp} \end{bmatrix} = \begin{bmatrix} V_x & V_y \\ V^{\perp}_x & V^{\perp}_y \end{bmatrix}$$



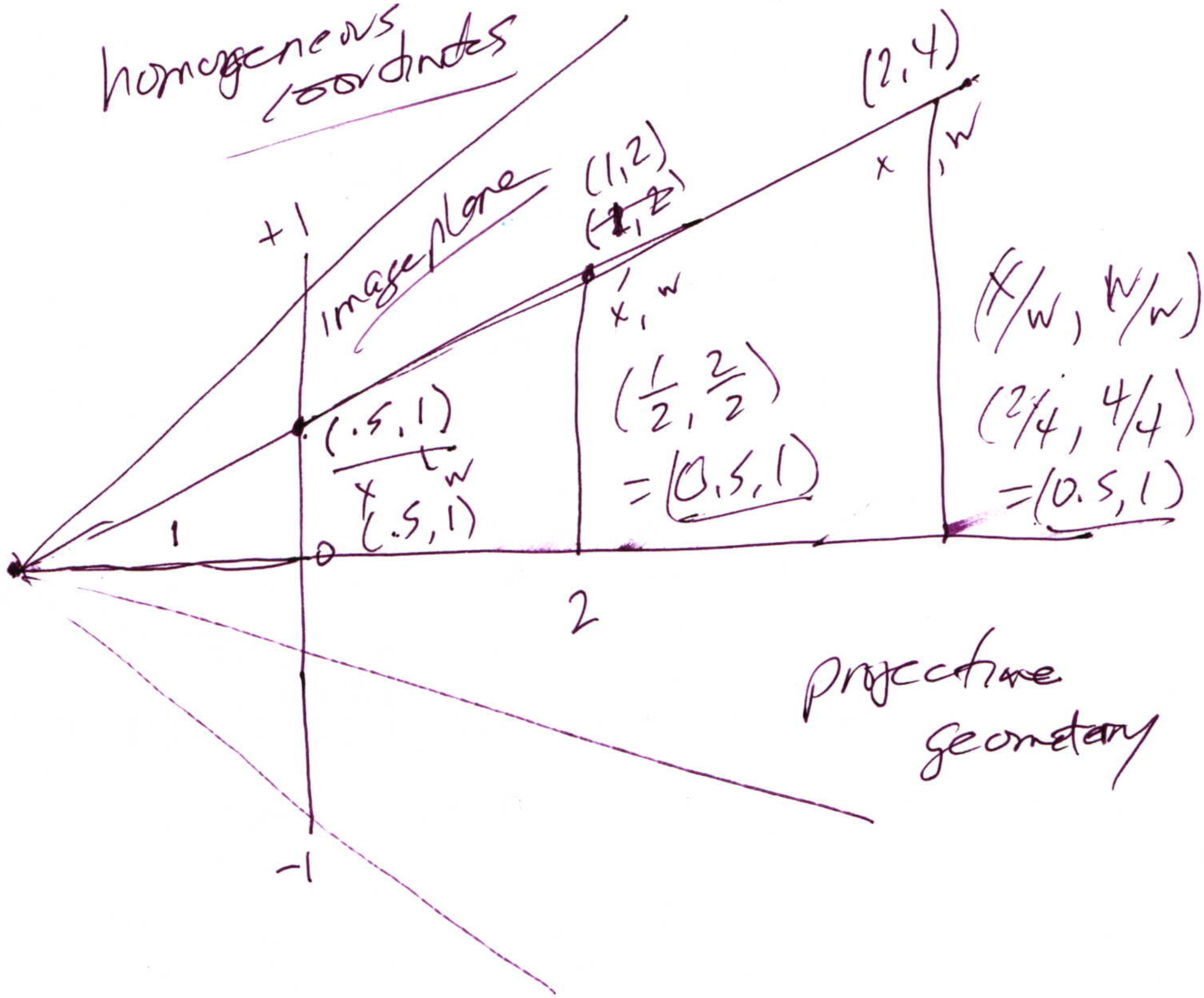
$$G.x_{[F]} = \langle 2,1,0\rangle \quad \text{vectors}$$
$$G.y_{[F]} = \langle -1,2,0\rangle$$
$$G.O_{[F]} = (4,2,1) \text{ point}$$

$G.x_{[F]} \Rightarrow$ x basis vector in f coordinate system

$$T_{[F\leftarrow G]} = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

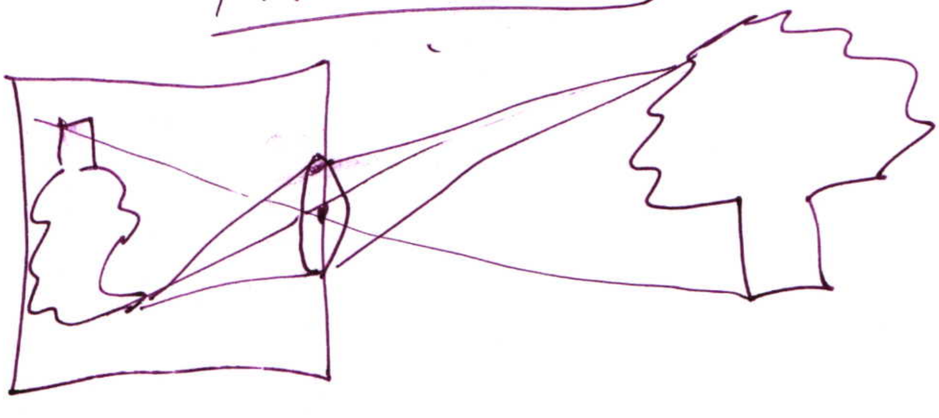
$$T_{[F\leftarrow G]} \cdot P_{(0,1)} = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

homogeneous coordinates

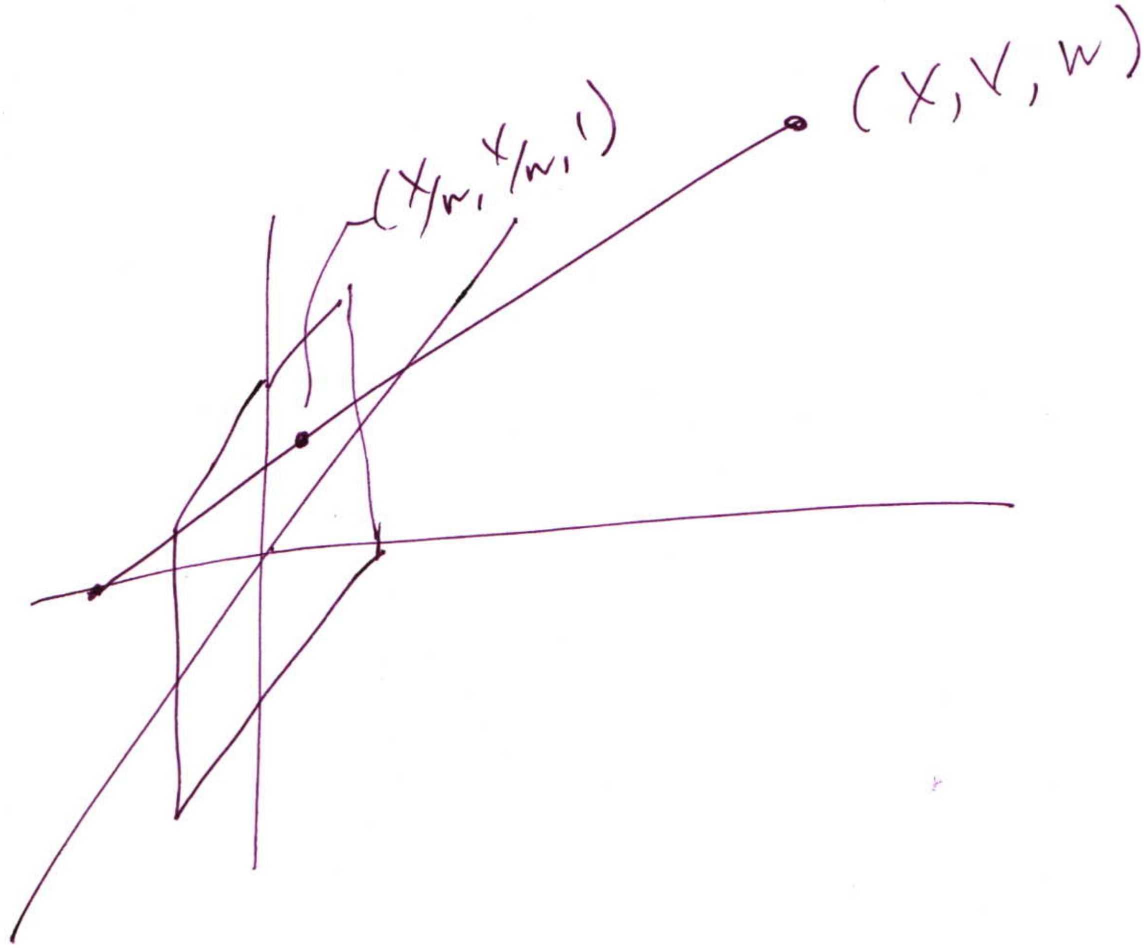


projective geometry

Pinhole camera



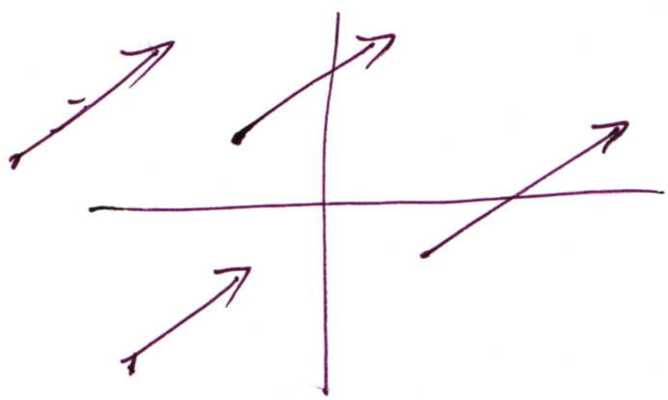
2D projection



point

$$\begin{bmatrix} x+\Delta x \\ y+\Delta y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$w=1$



vector

$$\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$w=0$

$$= \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

vector = pt − pt

$= (x, y, 1)$

$- (w, z, 1)$

$= (x-w, y-z, 0)$

$V_{[k]} = (2,0,1)$

$V_{[j]} = (6,0,1)$

$T_{[j \leftarrow k]}$ takes k into j

$V_{[i]} = (3,6,1)$

$$T_{[j \leftarrow k]} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{[j \leftarrow k]} V_{[k]}^T = (6,0,1)$$

$$T_{[i \leftarrow j]} = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

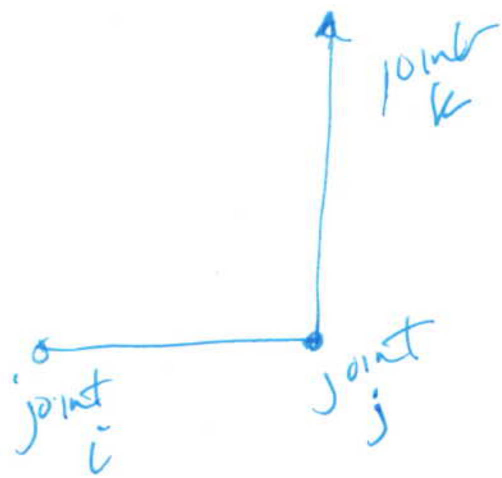
Translate

Rotate

$$\begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}$$

$$T_{[j \leftarrow k]} = \begin{bmatrix} \phi & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{[i \leftarrow j]} = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$M = \underline{M_{R(45)} * T_{[i \leftarrow j]} * M_{R(30)} * T_{[j \leftarrow k]} * P_{[k]}}$$