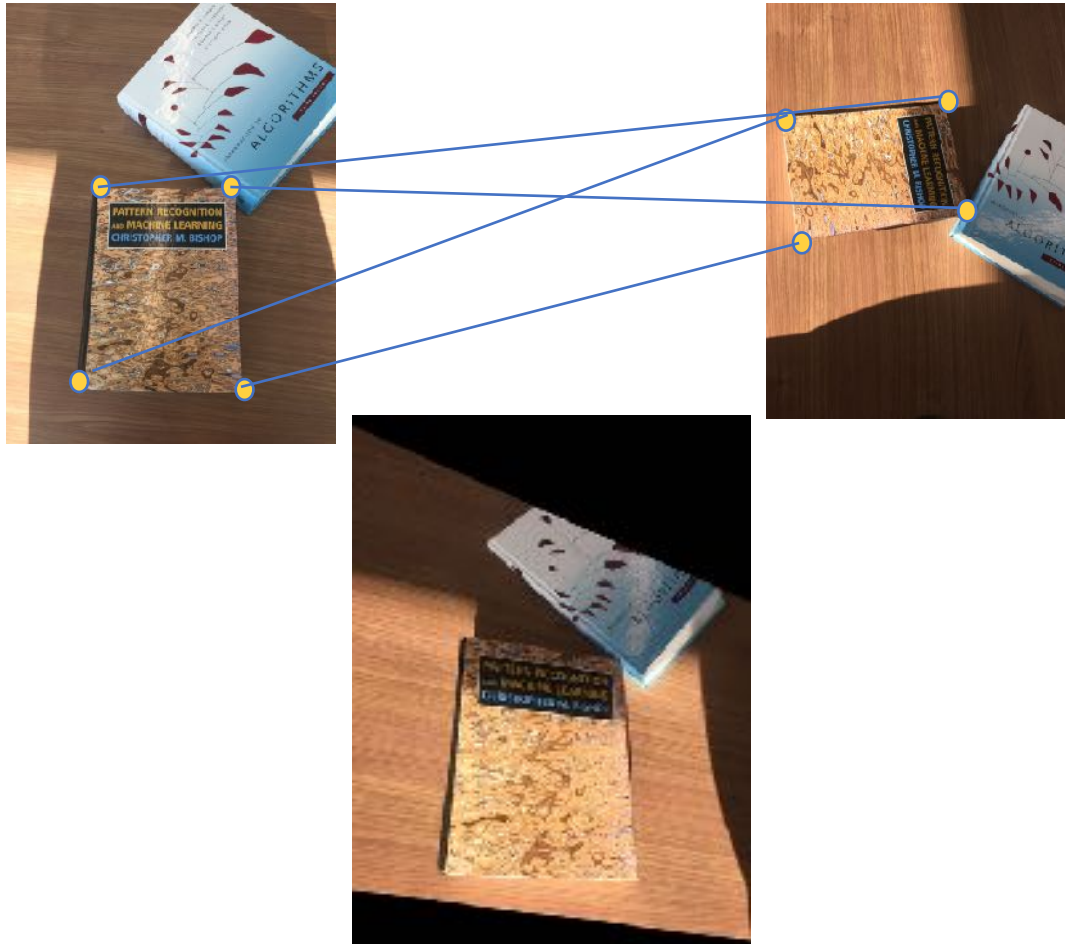


Homography

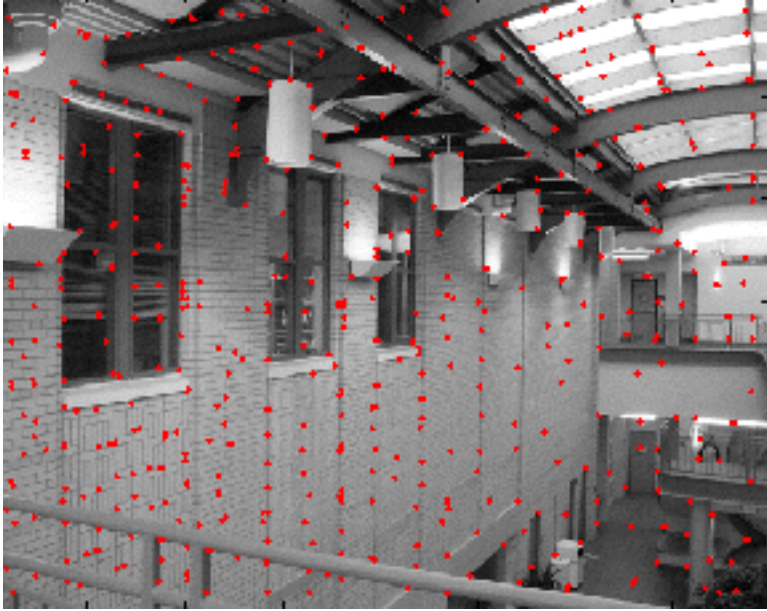
# Homography



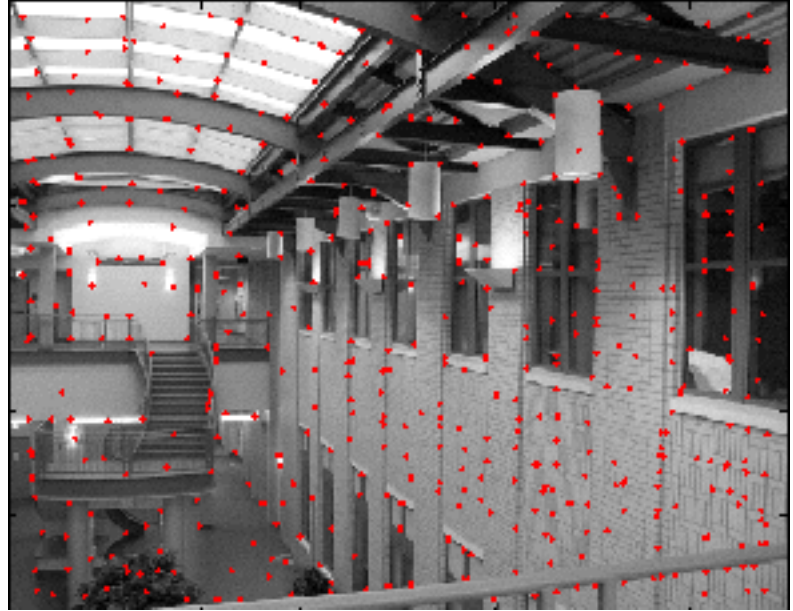
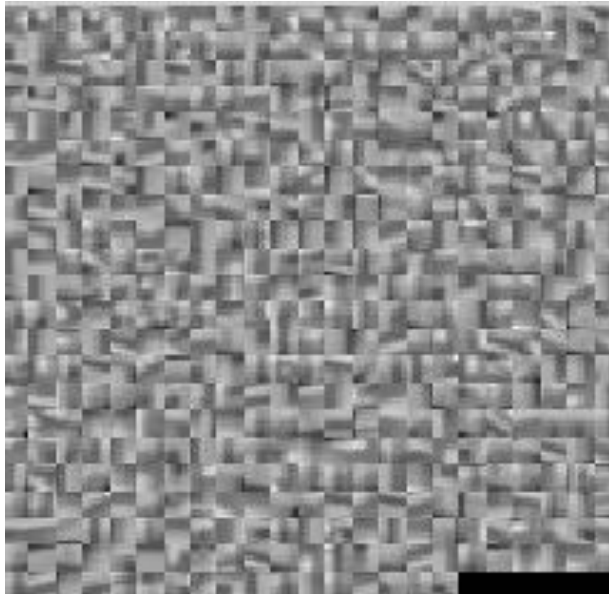
# Homography



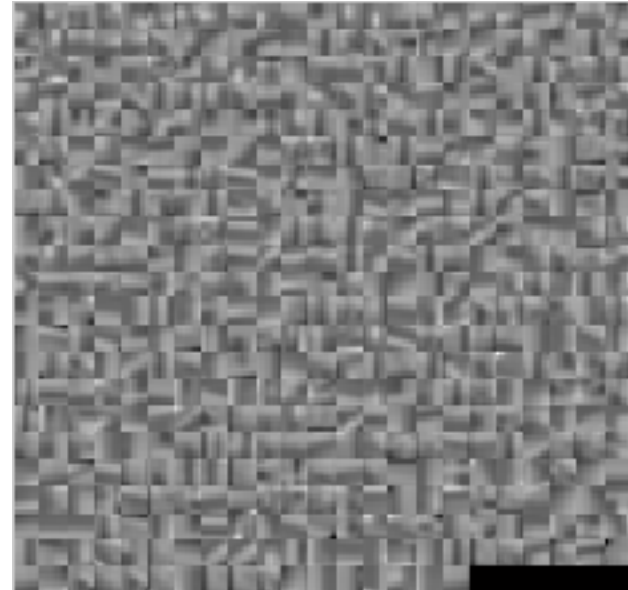
# Feature matching



descriptors for left image feature points



descriptors for right image feature points



# SIFT features

- Example



- (a) 233x189 image
- (b) 832 DOG extrema
- (c) 729 left after peak value threshold
- (d) 536 left after testing ratio of principle curvatures

# Strategies to match images robustly

(a) Working with individual features: For each feature point, find most similar point in other image (SIFT distance)

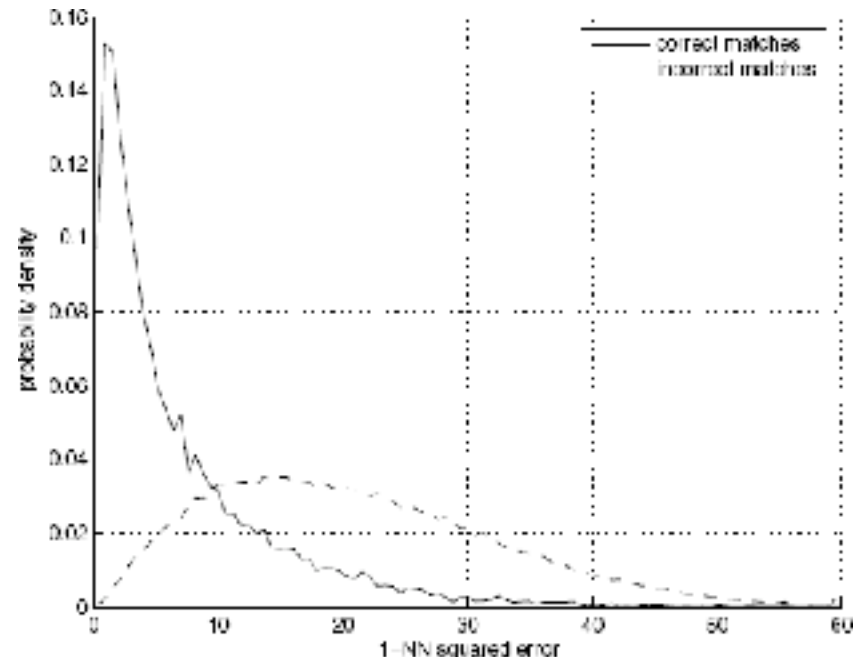
Reject ambiguous matches where there are too many similar points

(b) Working with all the features: Given some good feature matches, look for possible homographies relating the two images

Reject homographies that don't have many feature matches.

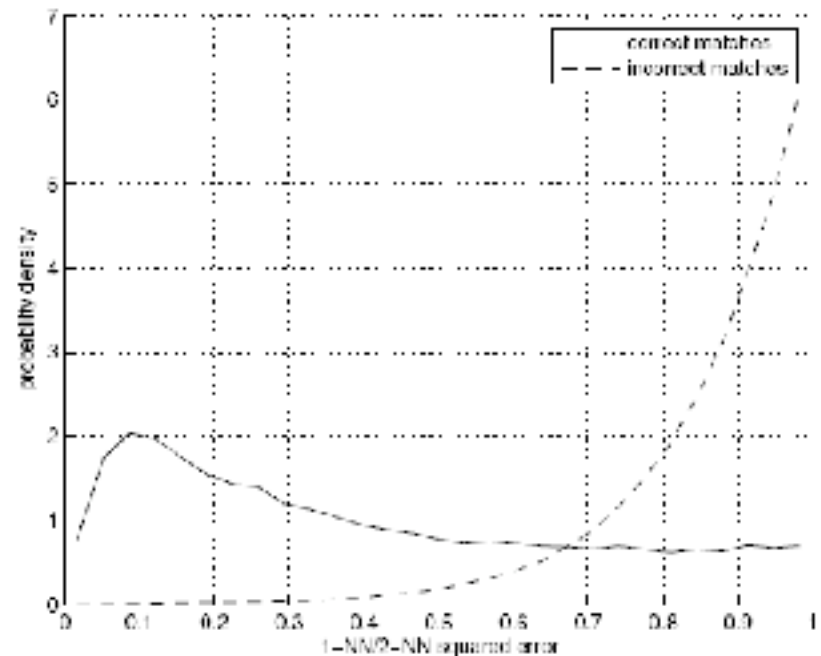
# (a) Feature-space outlier rejection

- Let's not match all features, but only these that have "similar enough" matches?
- How can we do it?
  - $\text{SSD}(\text{patch1}, \text{patch2}) < \text{threshold}$
  - How to set threshold?  
Not so easy.



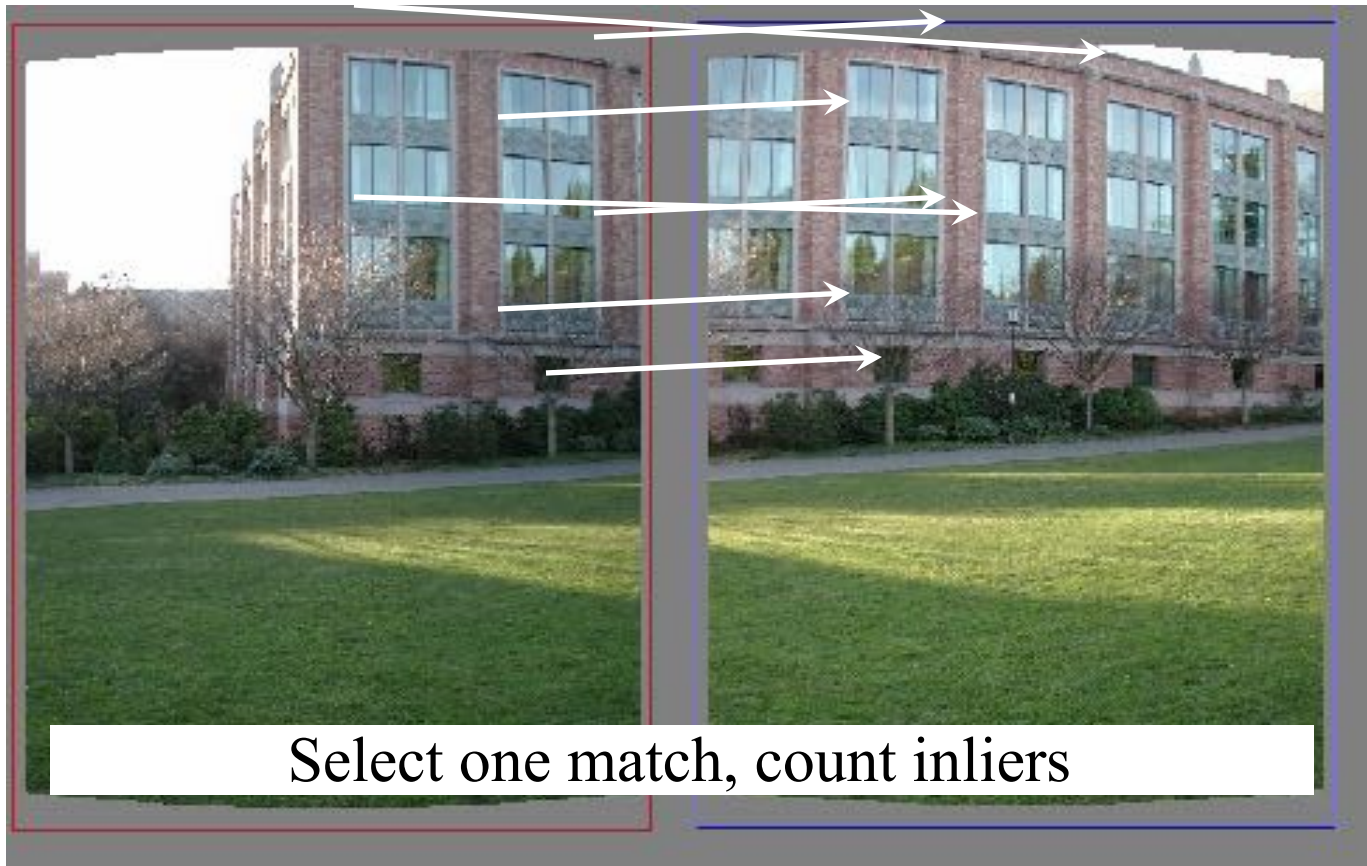
# Feature-space outlier rejection

- A better way [Lowe, 1999]:
  - 1-NN: SSD of the closest match
  - 2-NN: SSD of the second-closest match
  - Look at how much better 1-NN is than 2-NN, e.g.  $1\text{-NN}/2\text{-NN}$
  - That is, is our best match so much better than the rest?





# Random Sample Consensus



# RANSAC for estimating homography

RANSAC loop:

Select four feature pairs (at random)

Compute homography  $H$  (exact)

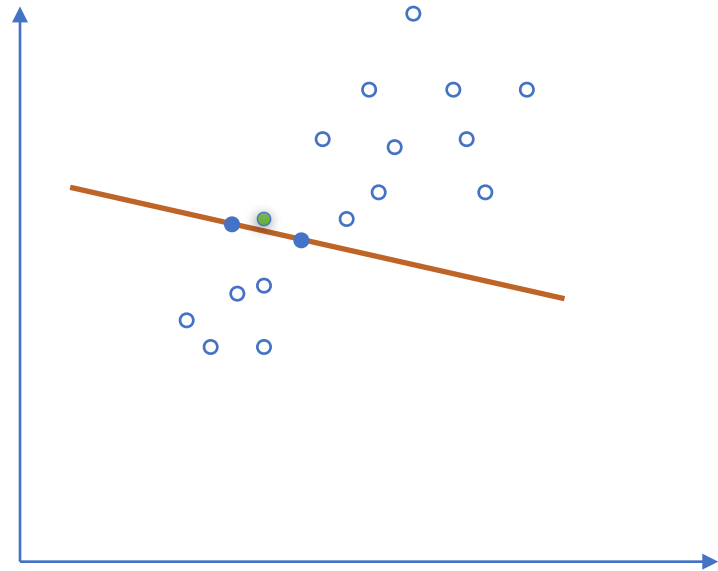
Compute inliers where  $\|p_i', H p_i\| < \epsilon$

Keep largest set of inliers

Re-compute least-squares  $H$  estimate using all of the inliers

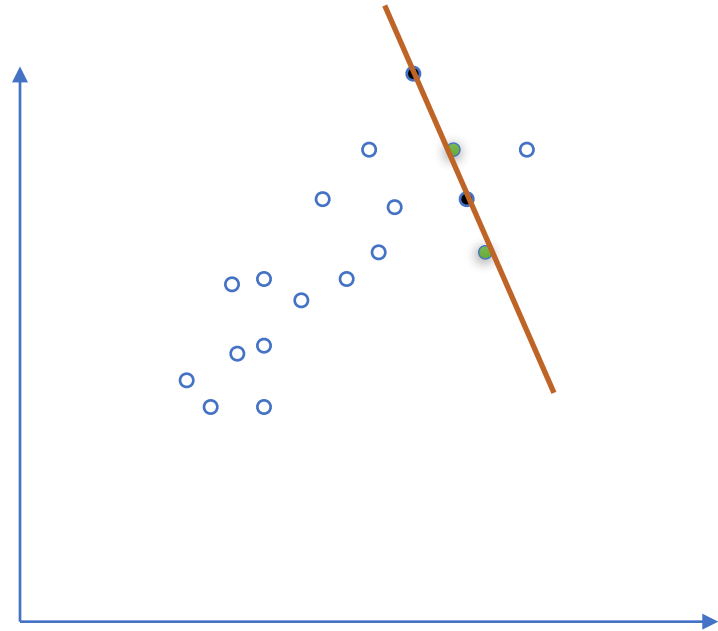
# Fit a line

- Pick 2 points
- Fit line
- Count inliers (3 inliers)



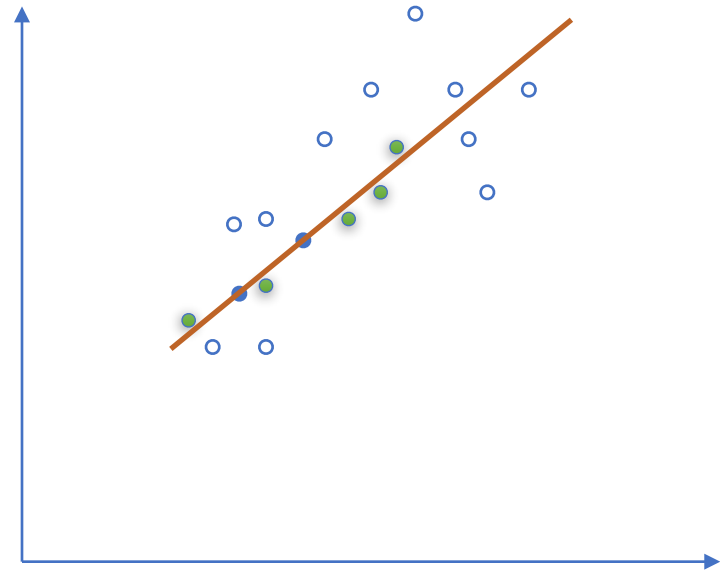
# Fit a line

- Pick 2 points
- Fit line
- Count inliers (4 inliers)



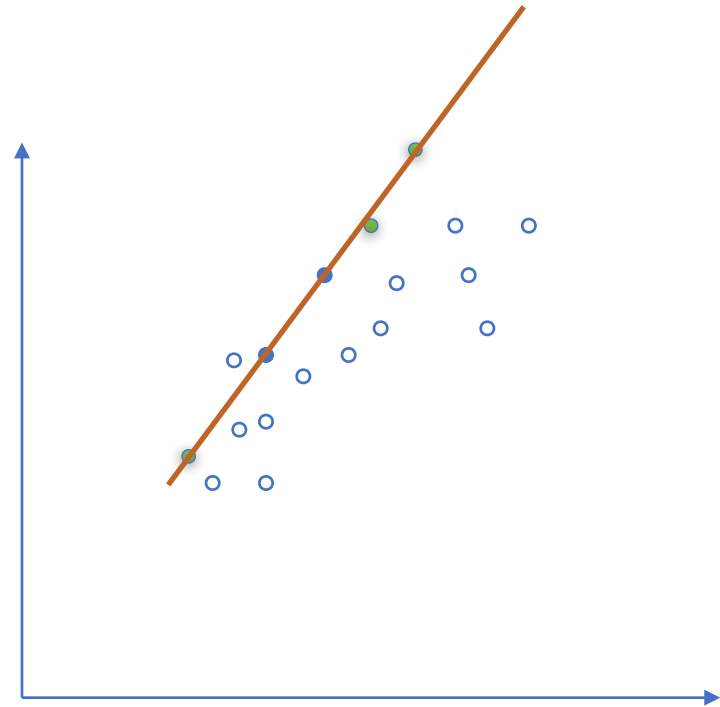
# Fit a line

- Pick 2 points
- Fit line
- Count inliers (7 inliers)



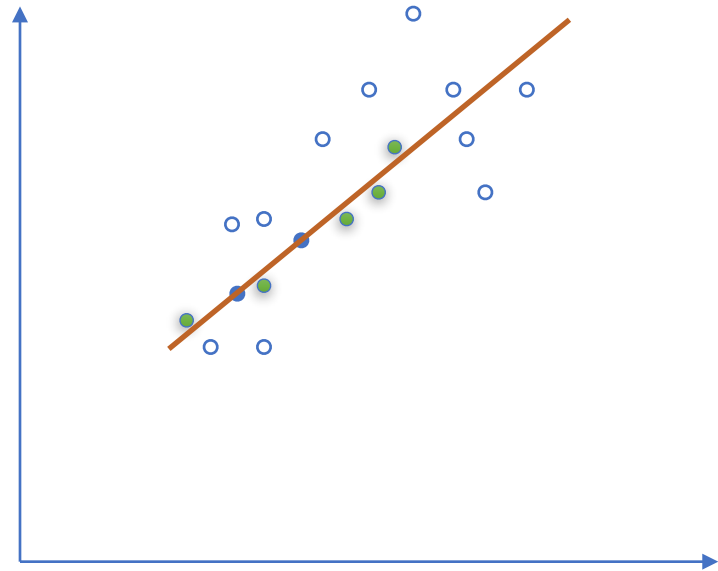
# Fit a line

- Pick 2 points
- Fit line
- Count inliers (5 inliers)

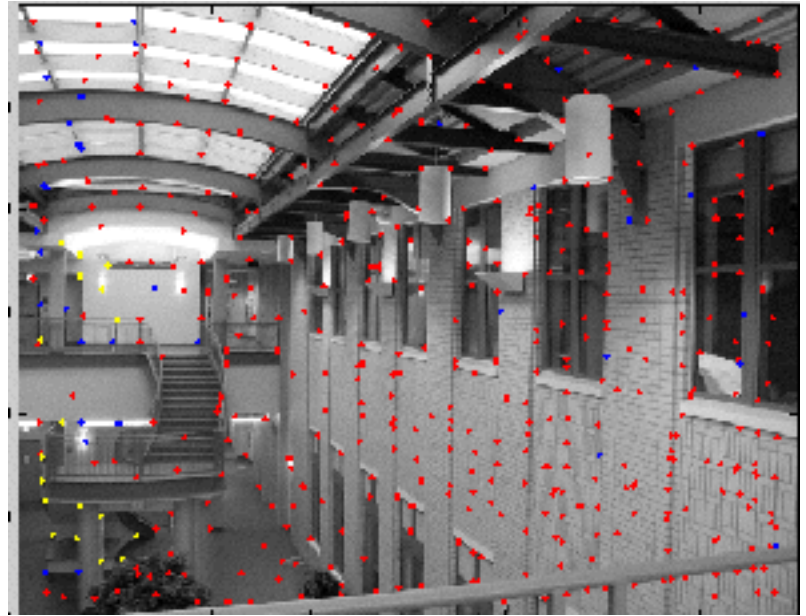
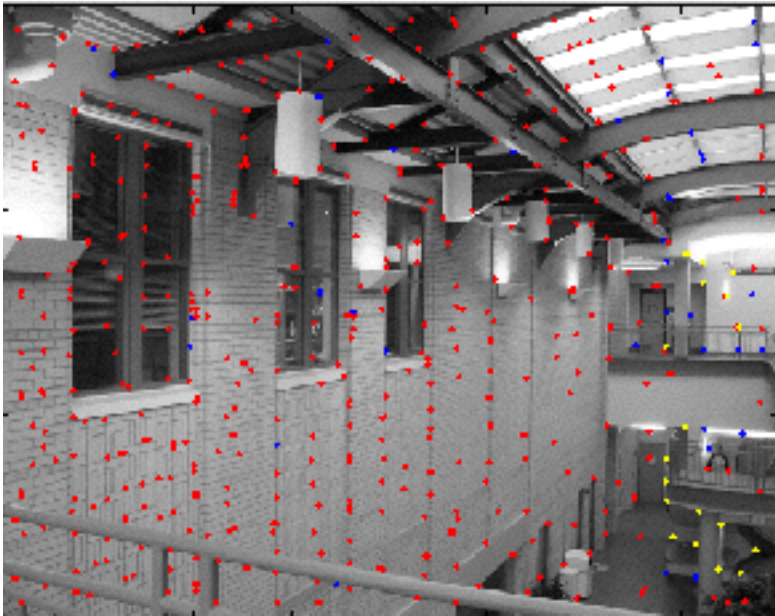


# Fit a line

- Use biggest set of inliers
- Least squares fit



# RANSAC



red:  
rejected by 2nd nearest  
neighbor criterion  
blue:  
Ransac outliers  
yellow:  
inliers





# RANSAC parameters

Decision Threshold:  $|X' - HX| < \epsilon$

Number of trials:  $N$

Size of the inlier set:  $M$

# Robustness

- Proportion of inliers in our pairs is  $g$  (for “g o o d”)
- Our model needs  $P$  pairs
  - $P=4$  for homography
- Probability that we pick  $P$  inliers?  
 $g^P$
- Probability that after  $N$  RANSAC iterations we have not picked a set of inliers?
  - $(1 - g^P)^N$

# Robustness: example

- Proportion of inliers  $G=0.5$
- Probability that we pick  $P=4$  inliers?
  - $0.5^4 = 0.0625$  (6 % *chance*)
- Probability that we have not picked a set of inliers?

$N = 100$  iterations

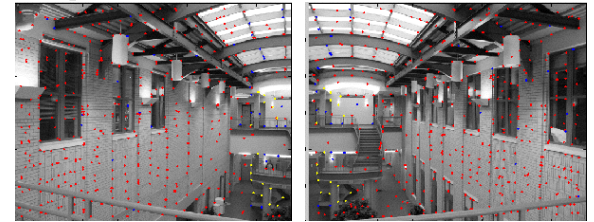
$$(1 - 0.5^4)^{100} = 0.00157 \text{ (1 chance in 600)}$$

$N = 1000$  iterations

$$(1 - 0.5^4)^{1000} = 0.00157 \text{ (1 chance in } 10^{28}\text{)}$$

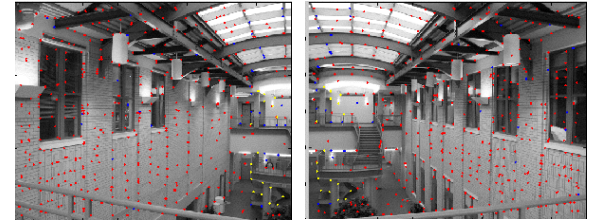
1 chance in 1e28

# Robustness: example



- Proportion of inliers  $G=0.3$
- Probability that we pick  $P=4$  inliers?
  - $0.3^4=0.0081$  (0.8% chance)
- Probability that we have not picked a set of inliers?
  - $N=100$  iterations:  
 $(1-0.3^4)^{100}=0.44$  (1 chance in 2)
  - $N=1000$  iterations:  
1 chance in 3400

# Robustness: example



- Proportion of inliers  $G=0.1$
- Probability that we pick  $P=4$  inliers?
  - $0.1^4=0.0001$  (0.01% chances, 1 in 10,000)
- Probability that we have not picked a set of inliers?
  - $N=100$  iterations:  $(1-0.1^4)^{100}=0.99$
  - $N=1000$  iterations: 90%
  - $N=10,000$ : 36%
  - $N=100,000$ : 1 in 22,000

# Robustness: conclusions

- Effect of number of parameters of model/  
number of necessary pairs
  - Bad exponential
- Effect of percentage of inliers
  - Base of the exponential
- Effect of number of iterations
  - Good exponential

# RANSAC recap

- For fitting a model with low number  $P$  of parameters (8 for homographies)
- Loop
  - Select  $P$  random data points
  - Fit model
  - Count inliers  
(other data points well fit by this model)
- Keep model with largest number of inliers

# RANSAC for Homography





# RANSAC for Homography



# RANSAC for Homography

