Mining Your Ps and Qs: Detection of Widespread Weak Keys in Network Devices

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Abstract

RSA and DSA can fail catastrophically when used with malfunctioning random number generators, but the extent to which these problems arise in practice has never been comprehensively studied at Internet scale. We perform the largest ever network survey of TLS and SSH servers and present evidence that vulnerable keys are surprisingly widespread. We find that 0.75% of TLS certificates share keys due to insufficient entropy during key generation, and we suspect that another 1.70% come from the same faulty implementations and may be susceptible to compromise. Even more alarmingly, we are able to obtain RSA private keys for 0.50% of TLS hosts and 0.03% of SSH hosts, because their public keys shared nontrivial common factors due to entropy problems, and DSA private keys for 1.03% of SSH hosts, because of insufficient signature randomness. We cluster and investigate the vulnerable hosts, finding that the vast majority appear to be headless or embedded devices. In experiments with three software components commonly used by these devices, we are able to reproduce the vulnerabilities and identify specific software behaviors that induce them, including a boot-time entropy hole in the Linux random number generator. Finally, we suggest defenses and draw lessons for developers, users, and the security community.

1 Introduction and Roadmap

Randomness is essential for modern cryptography, where security often depends on keys being chosen uniformly at random. Researchers have long studied random number generation, from both practical and theoretical perspectives (e.g., [8, 13, 15, 17, 21, 23]), and a handful of major vulnerabilities (e.g., [5, 19]) have attracted considerable scrutiny to some of the most critical implementations. Given the importance of this problem and the effort and attention spent improving the state of the art, one might expect that today's widely used operating systems and server software generate random numbers securely. In this paper, we test that proposition empirically by examining the public keys in use on the Internet.

The first component of our study is the most comprehensive Internet-wide survey to date of two of the most important cryptographic protocols, TLS and SSH (Section 3.1). By scanning the public IPv4 address space, we collected 5.8 million unique TLS certificates from 12.8 million hosts and 6.2 million unique SSH host keys from 10.2 million hosts. This is 67% more TLS hosts than the latest released EFF SSL Observatory dataset [18]. Our techniques take less than 24 hours to scan the entire address space for listening hosts and less than 96 hours to retrieve keys from them. The results give us a macroscopic perspective of the universe of keys.

Next, we analyze this dataset to find evidence of several kinds of problems related to inadequate randomness. To our surprise, at least 5.57% of TLS hosts and 9.60% of SSH hosts use the same keys as other hosts in an apparently vulnerable manner (Section 4.1). In the case of TLS, at least 5.23% of hosts use manufacturer default keys that were never changed by the owner, and another 0.34% appear to have generated the same keys as one or more other hosts due to malfunctioning random number generators. Only a handful of the vulnerable TLS certificates are signed by browser-trusted certificate authorities.

Even more alarmingly, we are able to compute the private keys for 64,000 (0.50%) of the TLS hosts and 108,000 (1.06%) of the SSH hosts from our scan data alone by exploiting known weaknesses of RSA and DSA when used with insufficient randomness. In the case of RSA, distinct moduli that share exactly one prime factor will result in public keys that appear distinct but whose private keys are efficiently computable by calculating the greatest common divisor (GCD). We implemented an algorithm that can compute the GCDs of all pairs of 11 million distinct public RSA moduli in less than 2 hours (Section 3.3). Using the resulting factors, we are able to

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obtain the private keys for 0.50% of TLS hosts and 0.03% of SSH hosts (Section 4.2). In the case of DSA, if a DSA key is used to sign two different messages with the same ephemeral key, an attacker can efficiently compute the signer's long-term private key. We find that our SSH scan data contain numerous DSA signatures that used the same ephemeral keys during signing, allowing us to compute the private keys for 1.6% of SSH DSA hosts (Section 4.3).

To understand why these problem are occurring, we manually investigated hundreds of the vulnerable hosts, which were representative of the most commonly repeated keys as well as each of the private keys we obtained (Section 3.2). Nearly all served information identifying them as headless or embedded systems, including routers, server management cards, firewalls, and other network devices. Such devices typically generate keys automatically on first boot, and may have limited entropy sources compared to traditional PCs. Furthermore, when we examined clusters of hosts that shared a key or factor, in nearly all cases these appeared to be linked by a manufacturer or device model. These observations lead us to conclude that the problems are caused by specific defective implementations that generate keys without having collected sufficient entropy. We identified vulnerable devices and software from dozens of manufacturers, including some of the largest names in the technology industry, and worked to notify the responsible parties.

In the final component of our study, we experimentally explore the root causes of these vulnerabilities by investigating several of the most common open-source software components from the population of vulnerable devices (Section 5). Based on the devices we identified, it is clear that no one implementation is solely responsible, but we are able to reproduce the vulnerabilities in plausible software configurations. Every software package we examined relies on `/dev/urandom` to generate cryptographic keys; however, we find that Linux's random number generator (RNG) can exhibit a boot-time entropy hole that causes `urandom` to produce deterministic output under conditions likely to occur in headless and embedded devices. In experiments with OpenSSL and Dropbear SSH, we show how repeated output from the system RNG can lead not only to repeated long-term keys but also to factorable RSA keys and repeated DSA ephemeral keys due to the behavior of application-specific entropy pools.

Given the diversity of the devices and software implementations involved, mitigating these problems will require action by many different parties. We draw lessons and recommendations for developers of operating systems, cryptographic libraries, and applications, and for device manufacturers, certificate authorities, end users, and the security and cryptography communities (Section 7). We have also created an online key-check service to allow users to test whether their keys are vulnerable.

It is natural to wonder whether these results should call into question the security of every RSA or DSA key. Based on our analysis, the margin of safety is slimmer than we might like, but we have no reason to doubt the security of most keys generated interactively by users on traditional PCs. While we took advantage of the details of specific cryptographic algorithms in this paper, we conclude that the blame for these vulnerabilities lies chiefly with the implementations. Ultimately, the results of our study should serve as a wake-up call that secure random number generation continues to be an unsolved problem in important areas of practice.

Online resources For an extended version of this paper, partial source code, and our online key-check service, visit https://factorable.net.

2 Background

In this section, we review the RSA and DSA public-key cryptosystems and discuss the known weaknesses of each that we used to compromise private keys. We then discuss how an adversary might exploit compromised keys to attack SSH and TLS in practice.

2.1 RSA review

An RSA [35] public key consists of two integers: an exponent e and a modulus N. The modulus N is the product of two randomly chosen prime numbers p and q. The private key is the decryption exponent

$$d = e^{-1} \bmod (p-1)(q-1).$$

Anyone who knows the factorization of N can efficiently compute the private key for any public key (e,N) using the preceding equation. When p and q are unknown, the most efficient known method to calculate the private key is to factor N into p and q and use the above equation to calculate d [9].

Factorable RSA keys No one has been publicly known to factor a well-generated 1024-bit RSA modulus; the largest known factored modulus is 768 bits, which was announced in December 2009 after a multi-year distributed-computing effort [28]. In contrast, the greatest common divisor (GCD) of two 1024-bit integers can be computed in microseconds. This asymmetry leads to a well-known vulnerability: if an attacker can find two distinct RSA moduli N_1 and N_2 that share a prime factor p but have different second prime factors q_1 and q_2, then the attacker can easily factor both moduli by computing their GCD, p, and dividing to find q_1 and q_2. The attacker can then compute both private keys as explained above.

2.2 DSA review

A DSA [32] public key consists of three so-called domain parameters (two prime moduli p and q and a generator g of the subgroup of order q mod p) and an integer $y = g^x \bmod p$, where x is the private key. The domain parameters may be shared among multiple public keys without compromising security. A DSA signature consists of a pair of integers (r,s): $r = g^k \bmod p \bmod q$ and $s = (k^{-1}(H(m)+xr)) \bmod q$, where k is a randomly chosen ephemeral private key and $H(m)$ is the hash of the message.

Low-entropy DSA signatures DSA is known to fail catastrophically if the ephemeral key k used in the signing operation is generated with insufficient entropy [4]. (Elliptic curve DSA (ECDSA) is similarly vulnerable. [11]) If k is known for a signature (r,s), then the private key x can be computed from the signature and public key as follows:

$$x = r^{-1}(ks - H(m)) \bmod q.$$

If a DSA private key is used to sign two different messages with the same k, then an attacker can efficiently compute the value k from the public key and signatures and use the above equation to compute the private key x [29]. If two messages m_1 and m_2 were signed using the same ephemeral key k to obtain signatures (r_1,s_1) and (r_2,s_2), then this will be immediately clear as r_1 and r_2 will be equal. The ephemeral key k can be computed as:

$$k = (H(m_1) - H(m_2))(s_1 - s_2)^{-1} \bmod q.$$

2.3 Attack scenarios

The weak key vulnerabilities we describe in this paper can be exploited to compromise two of the most important cryptographic transport protocols used on the Internet, TLS and SSH, both of which commonly use RSA or DSA to authenticate servers to clients.

TLS In TLS [16], the server sends its public key in a TLS certificate during the protocol handshake. The key is used either to provide a signature on the handshake (when Diffie-Hellman key exchange is negotiated) or to encrypt session key material chosen by the client (when RSA-encrypted key exchange is negotiated).

If the key exchange is RSA encrypted, a passive eavesdropper with the server's private key can decrypt the message containing the session key material and use it to decrypt the entire session. If the session key is negotiated using Diffie-Hellman key exchange, then a passive attacker will be unable to compromise the session key from just a connection transcript. However, in both cases, an active attacker who can intercept and modify traffic between the client and server can man-in-the-middle the connection in order to decrypt or modify the traffic.

SSH In SSH, host keys allow a server to authenticate itself to a client by providing a signature during the protocol handshake. There are two major versions of the protocol. In SSH-1 [38], the client encrypts session key material using the server's public key. SSH-2 [39] uses a Diffie-Hellman key exchange to establish a session key. The user manually verifies the host key fingerprint the first time she connects to an SSH server. Most clients then store the key locally in a `known_hosts` file and automatically trust it for all subsequent connections.

As in TLS, a passive eavesdropper with a server's private key can decrypt an entire SSH-1 session. However, because SSH-2 uses Diffie-Hellman, it is vulnerable only to an active man-in-the-middle attack. In the SSH user authentication protocol, the user-supplied password is sent in plaintext over the encrypted channel. An attacker who knows a server's private key can use the above attacks to learn a user's password and escalate an attack to the system.

3 Methodology

In this section, we explain how we performed our Internet-wide survey of public keys, how we attributed vulnerable keys to devices, and how we efficiently factored poorly generated RSA keys.

3.1 Internet-wide scanning

We performed our data collection in three phases: discovering IP addresses accepting connections on TCP port 443 (HTTPS) or 22 (SSH); performing a TLS or SSH handshake and storing the presented certificate chain or host key; and parsing the collected certificates and host keys into a relational database. Table 1 summarizes the results.

Host discovery In the first phase, we scanned the public IPv4 address space to find hosts with port 443 or 22 open. We used the Nmap 5 network exploration tool [33]. We executed our first host discovery scan beginning on October 6, 2011 from 25 Amazon EC2 Micro instances spread across five EC2 regions (Virginia, California, Japan, Singapore, and Ireland). The scan ran at an average of 40,566 IPs/second and finished in 25 hours.

Certificate and host-key retrieval For TLS, we implemented a certificate fetcher in Python using the Twisted event-driven network framework. We fetched TLS certificates using an EC2 Large instance with five processes each maintaining 800 concurrent connections. We started fetching certificates on October 11, 2011.

To efficiently collect SSH host keys, we implemented a simple SSH client in C, which is able to process upwards of 1200 hosts/second by concurrently performing

	SSL Observatory (12/2010)	Our TLS scan (10/2011)	Our SSH scans (2-4/2012)
Hosts with open port 443 or 22	≈16,200,000	28,923,800	23,237,081
Completed protocol handshakes	7,704,837	12,828,613	10,216,363
Distinct RSA public keys	3,933,366	5,656,519	3,821,639
Distinct DSA public keys	1,906	6,241	2,789,662
Distinct TLS certificates	4,021,766	5,847,957	—
Trusted by major browsers	1,455,391	1,956,267	—

Table 1: **Internet-wide scan results** — We exhaustively scanned the public IPv4 address space for TLS and SSH servers listening on ports 443 and 22, respectively. Our results constitute the largest such network survey reported to date. For comparison, we also show statistics for the EFF SSL Observatory's most recent public dataset [18].

protocol handshakes using *libevent*. Initially, we ran the fetcher from an EC2 Large instance in a run that started on February 12, 2012. This run targeted only RSA-based host keys. In two later runs, we targeted DSA-based host keys, and rescanned those hosts that had offered DSA keys in the first SSH scan. For these, we also stored the authentication signature provided by the server; we varied the client string to ensure that each signature would be distinct. The first DSA run started on March 26, 2012 from a host at UCSD. The second run, from a host at the University of Michigan, started on April 1, 2012; it took 3 hours to complete.

TLS certificate processing For TLS, we performed a third processing stage in which we parsed the previously fetched certificate chains and generated a database from the X.509 fields. We implemented a certificate parser in Python and C primarily based on the M2Crypto SWIG interface to the OpenSSL library.

3.2 Identifying vulnerable device models

We attempted to determine what hardware and software generated or served the weak keys we identified using manual detective work. The most straightforward method was based on TLS certificate information—predominately the X.509 *subject* and *issuer* fields. In many cases, the certificate identified a specific manufacturer or device model. Other certificates contained less information; we attempted to identify these devices through Nmap host detection or by inspecting the public contents of HTTPS sites or other IP services hosted on the IP addresses.

When we could identify a pattern in vulnerable TLS certificates that appeared to belong to a device model or product line, we constructed regular expressions to find other similar devices in our scan results. Under the theory that the keys were vulnerable because of a problem with the design of the devices (where they were most likely generated), this allows us to estimate the total population of devices that might be potentially vulnerable, beyond those serving immediately compromised keys.

Identifying SSH devices was more problematic, as SSH keys do not include descriptive fields and the server identification string used in the protocol often indicated only a common build of a popular SSH server. We were able to classify many of the vulnerable SSH hosts using a combination of TCP/IP fingerprinting and examination of information served over HTTP and HTTPS.

The device names and manufacturers that we report here have been identified with moderate or high confidence given the available information. However, because we do not have physical access to the hosts, we cannot state with certainty that all our identifications are correct.

3.3 Efficiently computing all-pairs GCDs

We now describe how we efficiently computed the pairwise GCD of all distinct RSA moduli in our multimillion-key dataset. This allowed us to calculate RSA private keys for 66,540 vulnerable hosts that shared one of their RSA prime factors with another host in our survey.

The fastest known factoring method for general integers is the number field sieve, which has heuristic complexity $O(2^{n^{1/3}(\log n)^{2/3}})$ for n-bit numbers [30]. In contrast to factoring, the greatest common divisor (GCD) of two integers can be computed very efficiently using Euclid's algorithm. Using fast integer arithmetic, the complexity of GCD can be improved to $O(n(\lg n)^2 \lg\lg n)$ [7]. Computing the GCD of two 1024-bit RSA moduli using the GMP library [20] takes approximately 15 μs on a current mid-range computer.

The naïve way to compute the GCDs of every pair of integers in a large set would be to apply a GCD algorithm to each pair individually. There are 6×10^{13} distinct pairs of RSA moduli in our data; at 15 μs per pair, this calculation would take 30 years. We can do much better by using a more efficient algorithm.

To accomplish this, we implemented a quasilinear-time algorithm for factoring a collection of integers into coprimes, based on Bernstein [6]. The relevant steps, illustrated in Figure 1, are as follows:

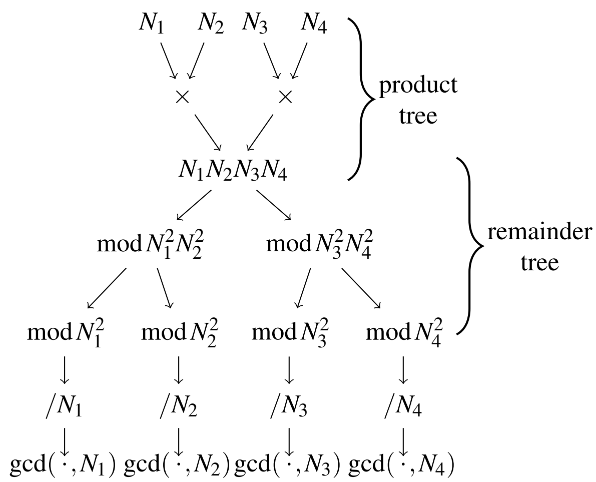


Figure 1: **Computing all-pairs GCDs efficiently** — We computed the GCD of every pair of RSA moduli in our dataset using an algorithm due to Bernstein [6].

Algorithm 1 Quasilinear GCD finding

Input: $N_1, \ldots, N_m$ RSA moduli

Compute $P = \prod N_i$ using a product tree.

Compute $z_i = (P \bmod N_i^2)$ for all i using a remainder tree.

Output: $\gcd(N_i, z_i/N_i)$ for all i.

A product tree computes the product of m numbers by constructing a binary tree of products. A remainder tree computes the remainder of an integer modulo many integers by successively computing remainders for each node in their product tree. For further discussion, see [7].

The final output of the algorithm is the GCD of each modulus with the product of all the other moduli. We are interested in the moduli for which this GCD is not 1. However, if a modulus shares both of its prime factors with two other distinct moduli, then the GCD will be the modulus itself rather than one of its prime factors. This occurred in a handful of instances in our dataset; we factored these moduli using the naïve quadratic algorithm for pairwise GCDs.

We implemented the algorithm in C using the GMP library for the arithmetic operations and ran it on the 11,170,883 distinct RSA moduli from our TLS and SSH datasets and the EFF SSL Observatory [18] dataset.

The entire computation finished in 5.5 hours using a single core on a machine with a 3.30 GHz Intel Core i5 processor and 32 GB of RAM. The remainder tree took approximately ten times as long to process as the product tree. Parallelized across sixteen cores on an EC2 Cluster Compute Eight Extra Large Instance with 60.5 GB of RAM and using EBS-backed storage for scratch data, the same computation took 1.3 hours at a cost of about $5.

4 Vulnerabilities

We analyzed the data from our TLS and SSH scans and identified several patterns of vulnerability that would have been difficult to detect without a macroscopic view of the Internet. This section discusses the details of these problems, as summarized in Table 2.

4.1 Repeated keys

We found that 7,770,232 of the TLS hosts (61%) and 6,642,222 of the SSH hosts (65%) served the same key as another host in our scans. To understand why, we clustered certificates and host keys that shared the same public key and manually inspected representatives of the largest clusters. In all but a few cases, the TLS certificate subjects, SSH version strings, or WHOIS information were identical within a cluster, or pointed to a single manufacturer or organization. This sometimes suggested an explanation for the shared keys.

Not all of the repeated keys were due to vulnerabilities. For instance, many of the most commonly repeated keys appeared in shared hosting situations. Six of the ten most common DSA host keys and three of the ten most common RSA host keys were served by large hosting providers (see Figure 2). Another frequent reason for repeated keys was distinct TLS certificates all belonging to the same organization. For example, TLS hosts at google.com, appspot.com, and doubleclick.net all served distinct certificates with the same public key. We excluded these cases and attributed remaining clusters of shared keys to several classes of problems.

Default keys A common reason for hosts to share the same key that we *do* consider a vulnerability is

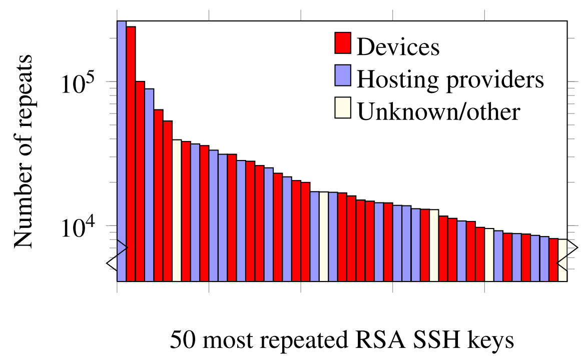


Figure 2: **Commonly repeated SSH keys** — We investigated the 50 most repeated SSH host keys for both RSA and DSA. Nearly all of the repeats appeared to be due either to hosting providers using a single key on many IP addresses or to devices that used a default key or generated keys using insufficient entropy. Note log scale.

	Our TLS Scan		Our SSH Scans	
Number of live hosts	12,828,613	(100.00%)	10,216,363	(100.00%)
. . . using repeated keys	7,770,232	(60.50%)	6,642,222	(65.00%)
. . . using vulnerable repeated keys	714,243	(5.57%)	981,166	(9.60%)
. . . using default certificates or default keys	670,391	(5.23%)		
. . . using low-entropy repeated keys	43,852	(0.34%)		
. . . using RSA keys we could factor	64,081	(0.50%)	2,459	(0.03%)
. . . using DSA keys we could compromise			105,728	(1.03%)
. . . using Debian weak keys	4,147	(0.03%)	53,141	(0.52%)
. . . using 512-bit RSA keys	123,038	(0.96%)	8,459	(0.08%)
. . . identified as a vulnerable device model	985,031	(7.68%)	1,070,522	(10.48%)
. . . model using low-entropy repeated keys	314,640	(2.45%)		

Table 2: **Summary of vulnerabilities** — We analyzed our TLS and SSH scan results to measure the population of hosts exhibiting several entropy-related vulnerabilities. These include use of repeated keys, use of RSA keys that were factorable due to repeated primes, and use of DSA keys that were compromised by repeated signature randomness. Under the theory that vulnerable repeated keys were generated by embedded or headless devices with defective designs, we also report the number of hosts that we identified as these device models. Many of these hosts may be at risk even though we did not specifically observe repeats of their keys.

manufacturer-default keys. These are preconfigured in the firmware of many devices, such that every device of a given model shares the same key pair unless the user changes it. The private keys to these devices may be accessible through reverse engineering, and published databases of default keys such as littleblackbox [24] contain private keys for thousands of firmware releases.

At least 670,391 (5.23%) of the TLS hosts appeared to serve manufacturer-default certificates or keys. We classified a certificate as a manufacturer default if nearly all the devices of a given model used identical certificates, or if the certificate was labeled as a default certificate.

The most common default certificate that we could ascribe to a particular device belonged to a model of consumer router. Our scan uncovered 90,779 instances of this device model sharing a single certificate. We also found a variety of enterprise products serving default keys, including server management devices, network storage devices, routers, remote access devices, and VoIP devices.

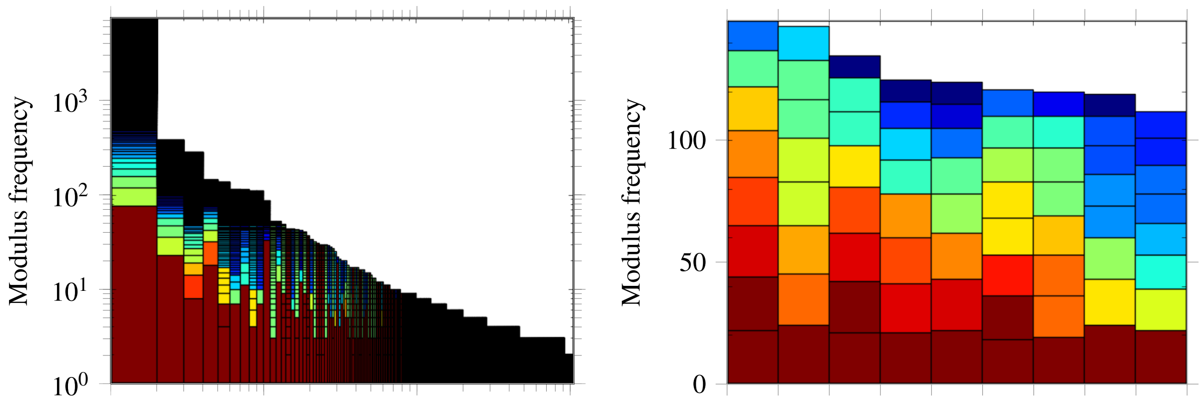
For most of the repeated SSH keys, the lack of uniquely identifying host information prevents us from distinguishing default keys from keys generated with insufficient entropy, so we address these together in the next section.

Repeated keys due to low entropy Another common reason that hosts share the same key appears to be entropy problems during key generation. In these instances, when we investigated a key cluster, we would typically see thousands of hosts across many address ranges, and, when we checked the keys corresponding to other instances of the same model of device, we would see a long-tailed distribution in their frequencies. Intuitively, this type of distribution suggests that the key generation process may be using insufficient entropy, with distinct keys due to relatively rare events. For TLS, our investigations began with the keys that occurred in at least 100 distinct self-signed certificates. For SSH, we started from the 50 most commonly repeated keys for each of RSA (appearing on more than 8000 hosts) and DSA (more than 4000 hosts).

With this process, we identified 43,852 TLS hosts (0.34%) that served repeated keys due apparently to low entropy during key generation. 27,545 certificates (98%) containing these repeated keys were self-signed; all 577 CA-signed certificates identified Iomega StorCenter network storage devices. For most SSH hosts we were unable to distinguish between default keys and keys repeated due to entropy problems, but 981,166 of the SSH hosts (9.60%) served keys repeated for one of these reasons.

We used the techniques described in Section 3.2 to identify apparently vulnerable devices from 27 manufacturers. These include enterprise-grade routers from Cisco and Juniper; server management cards from Dell, Hewlett-Packard, and IBM; virtual-private-network (VPN) devices; building security systems; network attached storage devices; and several kinds of consumer routers and VoIP products.

Duplicated keys are a red flag that calls the security of the device's key generation process into question, and all keys generated with the same model device should be considered suspect. For 14 of the TLS devices generating repeated keys, we were able to infer a fingerprint for the device model and estimate the total population of the device in our scan. The prevalence of duplicated keys varied greatly for different device models, from as low as 0.2%



(a) Primes generated by Juniper SSG 20 firewall (b) Primes generated by IBM Remote Supervisor Adapter

Figure 3: **Visualizing RSA common factors** — Different implementations displayed different patterns of vulnerable keys. These plots depict the distribution of vulnerable keys divisible by common factors generated by two different device models. Each column represents a collection of RSA moduli divisible by a single prime factor p. The color and internal rectangles show, for each p, the frequencies of each distinct prime factor q. The Juniper device (*left*; note log-log scale) follows a long-tailed distribution that is typical of many of the devices we identified. In contrast, the IBM remote access device (*right*) was unique among those we observed in that it generates RSA moduli roughly uniformly distributed among nine distinct prime factors.

in the case of one router to 98% for one thin client. The total population of these identified, potentially vulnerable TLS devices was 314,640 hosts, which represents 2.45% of the TLS hosts in our scan.

In the above analyses, we excluded repeated keys that were due to the infamous Debian weak-key vulnerability [5, 37], which we report separately in Table 2.

4.2 Factorable RSA keys

A second way that entropy problems might manifest themselves during key generation is if an RSA modulus shares one of its prime factors p or q with another key. As explained in Section 2.1, finding such a pair immediately allows an adversary to efficiently factor both moduli and obtain their private keys. In order to find such keys, we computed the GCD of all pairs of distinct RSA moduli by applying the algorithm described in Section 3.3.

The 11,170,883 distinct RSA moduli yielded 2,314 distinct prime divisors, which divided 16,717 distinct public keys. This allowed us to obtain private keys for 23,576 (0.40%) of the TLS certificates in our scan data, which were served on 64,081 (0.50%) of the TLS hosts, and 1,013 (0.02%) of the RSA SSH host keys, which were served on 2,459 (0.027%) of the RSA SSH hosts.

The vast majority of the vulnerable keys appeared to be system-generated certificates and SSH host keys used by routers, firewalls, remote administration cards, and other types of headless or embedded network devices. Only two of the factorable TLS certificates had been signed by a browser trusted authority and both have expired. Some devices generated factorable keys both for TLS and SSH, and a handful of devices shared common factors across SSH and TLS keys.

We classified these factorable keys in a similar manner to the repeated keys. We found that, in all but a small number of cases, the TLS certificates and SSH host keys divisible by a common factor all belonged to a particular manufacturer, which we were able to identify in most cases using the techniques described in Section 3.2.

We identified devices from 41 manufacturers in this way, which constituted 99% of the hosts that generated RSA keys we could factor. The devices range from 100% (576 of 576 devices) vulnerable to 0.01% vulnerable (2 out of 10,932). As with repeated keys, we would not expect to see well-generated cofactorable keys; any device model observed generating factorable keys should be treated as potentially vulnerable.

The majority of the devices serving factorable keys were Juniper firewalls. We identified 46,993 of these devices in our dataset, and we factored the keys for 12,688 (27%). Of these keys, 7,510 (59%) share a single common divisor. The distribution of common factors among its moduli is shown in Figure 3a.

The most remarkable devices were IBM Remote Server Administration cards and BladeCenter devices, which displayed a distribution of factors unlike any of the other vulnerable devices we found. There were only 9 distinct prime factors that had been used to generate the keys for 576 devices. Each device's key was the product of two

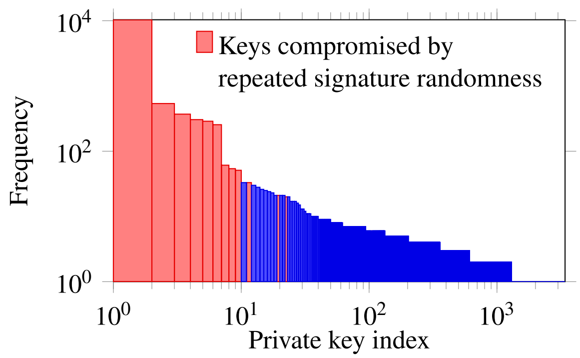


Figure 4: **Vulnerable DSA keys for one SSH device** — We identified 18,684 SSH DSA keys that appeared to have been generated by Gigaset DSL routers, of which 16,575 were repeated at least once. Shown in red in this log-log plot are 12,378 keys further compromised due to repeated DSA signature values.

different primes from this list. The 36 possible moduli that could be generated with this process were roughly uniformly distributed, as shown in Figure 3b. In addition, this was the only device we observed to generate RSA moduli where *both* prime factors were shared with other keys.

4.3 DSA signature weaknesses

The third class of entropy-related vulnerability that we searched for was repeated ephemeral keys in DSA signatures. As explained in Section 2.2, if a DSA key is used to sign two different messages using the same ephemeral key, then the long-term private key is immediately computable from the public key and the signatures. The presence of this vulnerability indicates entropy problems at later phases of operation, after initial key generation. We searched for signatures from identical keys containing repeated r values. Then we used the method in Section 2.2 to compute the corresponding private keys.

Our combined SSH DSA scans collected 9,114,925 signatures (in most cases, two from each SSH host serving a DSA-based key) of which 4,365 (0.05%) contained the same r value as at least one other signature. 4,094 of these signatures (94%) used the same r value and the same public key. This allowed us to compute the 281 distinct private keys used to generate these signatures. These compromised keys were served by 105,728 (1.6%) of the SSH DSA hosts in our combined scans.

We clustered the vulnerable signatures by r values and manufacturer. 2,026 (46%) of the colliding r values appeared to have been generated by Gigaset SX762 consumer-grade DSL routers and revealed private keys for 17,349 (66%) of the 26,374 hosts we identified as this device model (see Figure 4). Another 934 signature collisions appeared to be from ADTran Total Access business-grade phone/network routers and revealed private keys for 62,807 (73%) out of 86,301 such hosts. Several vulnerable device models, including the IBM RSA II remote administration cards and Juniper NetScreen, also generated factorable RSA keys.

While we collected multiple signatures from some SSH hosts, 3,917 (89.7%) out of 4,365 of the collisions were from *different* hosts that had generated the same long-term key and also used the same ephemeral key during the key exchange protocol. This problem compounds the danger of the repeated key vulnerability: a single signature collision between any pair of hosts sharing the same key at any point during runtime reveals the private key for every host using that key. In our dataset we observed tens of thousands of hosts using the same public key. While a single host may never repeat an ephemeral key, two hosts sharing a private key due to poor entropy can put each others' keys at risk.

We note that any estimation of vulnerability based on our data is an extreme lower bound, as we gathered at most two signatures from each host in our scans. It is likely that many more private keys would be revealed if we collected additional signatures.

5 Experimental Investigation

Based on the results the previous section, we conjectured that the problems we observed were an implementational phenomenon. To more deeply understand the causes, we augmented our data analysis with experimental investigation of specific implementations. While there are many independently vulnerable implementations, we chose to examine three open-source cryptographic software components that appeared frequently in the vulnerable populations.

5.1 Weak entropy and the Linux RNG

We conjectured that the cause for many of the entropy problems we observed began with insufficient randomness provided by the operating system. This led us to take an in-depth look at the Linux random number generator (RNG). We note that not every vulnerable key was generated on Linux; we also observed vulnerable keys on FreeBSD and Windows systems, and similar vulnerabilities to those we describe here have been reported with BSD's `arc4random` [36].

The Linux RNG maintains three entropy pools, each with an associated counter that estimates how much fresh entropy it has available. Fresh entropy from unpredictable kernel sources is periodically mixed into the *Input pool*. When processes read from `/dev/random` or `/dev/urandom`, the kernel extracts the requested amount

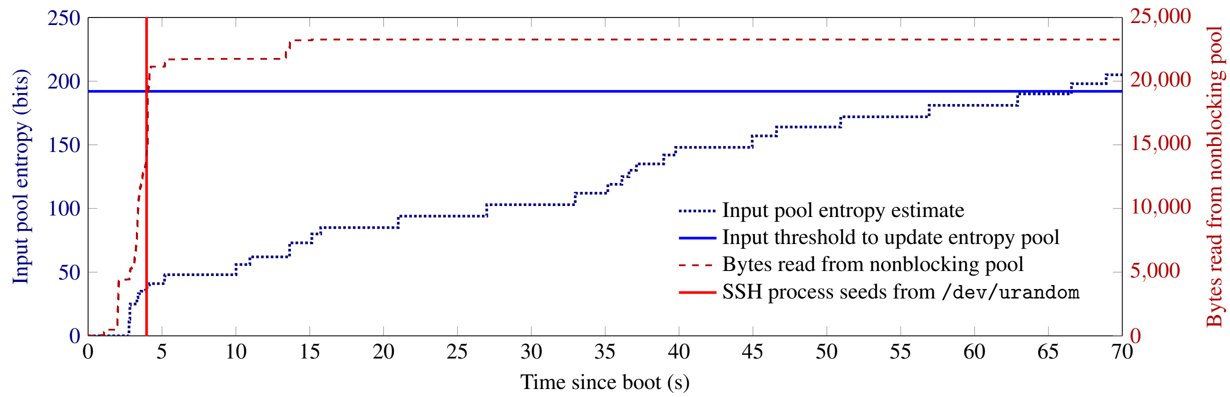


Figure 5: **Linux urandom boot-time entropy hole** — We instrumented an Ubuntu Server 10.04 system to record its estimate of the entropy contained in the Input entropy pool during a typical boot. Linux does not mix Input pool entropy into the Nonblocking pool that supplies `/dev/urandom` until the Input pool exceeds a threshold of 192 bits (blue horizontal line), which occurs here at 66 seconds post-boot. For comparison, we show the cumulative number of bytes generated from the Nonblocking entropy pool; the vertical red line marks the time when OpenSSH seeds its internal PRNG by reading from `urandom`, *well before* this facility is ready for secure use.

of entropy from the Input pool and mixes it into the *Blocking* or *Nonblocking* pool, respectively, and then extracts bytes from the respective pool to return. If the input pool does not contain enough entropy to satisfy the request, the read from the Blocking pool blocks; the Nonblocking pool read is satisfied immediately.

Entropy sources We experimented with the Linux 2.6.35 kernel to exhaustively determine the sources of entropy used by the RNG. To do this, we traced through the kernel source code and systematically disabled entropy sources until the RNG output was repeatable. All of the entropy sources we found are greatly weakened under certain operating conditions.

The explicit entropy sources we observed are the uninitialized contents of the pool buffers when the kernel starts, the startup clock time in nanosecond resolution, input event and disk access timings, and entropy saved across boots to a local file. Surprisingly, modern Linux systems no longer collect entropy from IRQ timings.

The final and most interesting entropy source was one that we have not seen documented elsewhere. The developers chose not to put a lock around the mixing procedure when entropy is extracted from the pool, and, as a result, if two threads extract entropy concurrently, the pool contents may change anywhere in the middle of the hash computation, resulting in the introduction of significant (but uncredited) entropy to the pool.

The removal of IRQs as an entropy source has likely exacerbated RNG problems in headless and embedded devices, which often lack human input devices, disks, and multiple cores. If they do, the only source of entropy—if there are any at all—may be the time of boot.

Experiment To test whether Linux's `/dev/urandom` can produce repeatable output in conditions resembling the initial boot of a headless or embedded networked device, we modified the 2.6.35 kernel to add instrumentation to the RNG and disable certain entropy sources to simulate a cold boot on a low-end machine without a working clock.

We experimented with this kernel on a Dell Optiplex 980 system using a fresh installation of Ubuntu server 10.04.4. The machine was configured with a Core i7 CPU, 4 GB RAM, a 32 GB SSD, and a USB keyboard. It was attached to a university office LAN and obtained an IP address using DHCP. With this configuration, we performed 1,000 unattended boots. Each time, we read 32 bytes from `urandom` at the point in the initialization process where the SSH server would normally start. Under these conditions, we found that the output of `/dev/urandom` was entirely predictable and repeatable.

The kernel maintains a reserve threshold for the Input pool, and no data is copied into the Nonblocking pool until the Input pool has been credited with at least that much entropy (192 bits, for our kernel). Figure 5 shows the cumulative amount of entropy credited to the Input pool during a typical bootup from our tests. (Note that none of the entropy sources we disabled would have resulted in more entropy being *credited* to the pool.) The credited entropy does not cross this reserve threshold until more than a minute after boot, well after the SSH server and other startup processes have launched. Although Ubuntu tries to restore entropy saved during the last shutdown, this happens slightly *after* the point when `sshd` first reads from `urandom`. With no entropic inputs, `urandom` produces a deterministic output stream.

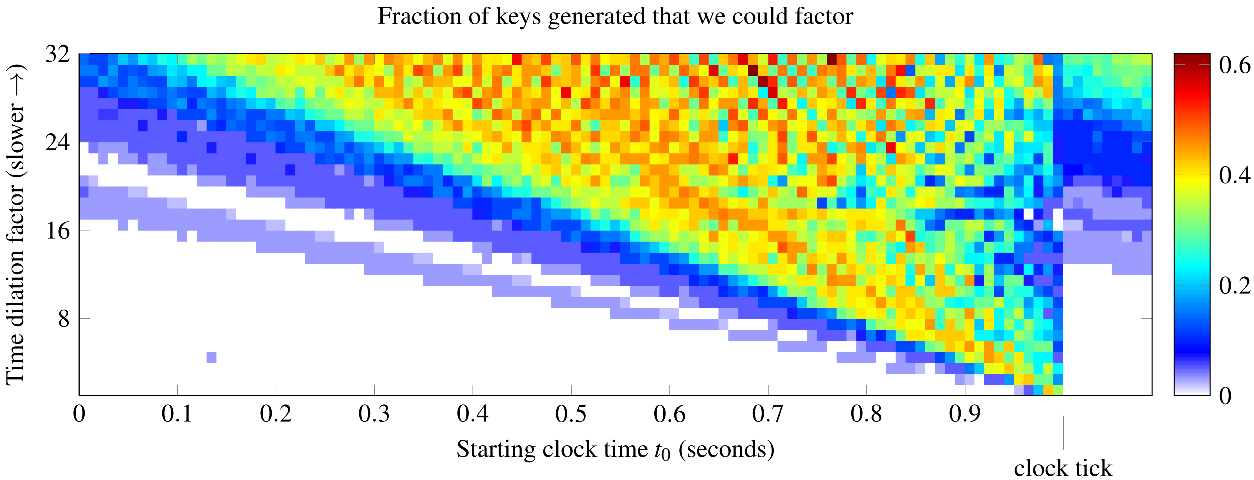


Figure 6: **OpenSSL generating factorable keys** — We hypothesized that OpenSSL can generate factorable keys under low-entropy conditions due to slight asynchronicity between the key generation process and the real-time clock, we generated 14 million RSA keys using controlled entropy sources for a range of starting clock times. Each square in the plot indicates the fraction of 100 generated keys that could we could factor. In many cases (white), keys are repeated but never share primes. After a sudden phase change, factorable keys occur during a range leading up to the second boundary, and that range increases as we simulate machines with slower execution speeds.

Boot-time entropy hole The entropy sources we disabled would likely be missing in some headless and embedded systems, particularly on first boot. This means that there is a window of vulnerability—a boot-time entropy hole—during which Linux's `urandom` may be entirely predictable, at least for single-core systems. If processes generate long-term cryptographic keys or maintain their own entropy pools seeded only with entropy gathered during this window, those keys are likely to be vulnerable. The risk is particularly high for unattended systems that ship with preconfigured operating systems and generate SSH or TLS keys the first time the respective daemons start during the initial boot.

On stock Ubuntu systems, these risks are somewhat mitigated: TLS keys must be generated manually, and OpenSSH host keys are generated during package installation, which is likely to be late in the install process, giving the system time to collect sufficient entropy. However, on the Fedora, Red Hat Enterprise Linux (RHEL), and CentOS Linux distributions, OpenSSH is installed by default, and host keys are generated on first boot. We experimented further with RHEL 5 and 6 to determine whether host keys on these systems might be compromised, and observed that sufficient entropy had been collected at the time of key generation by a slim margin. We believe that most server systems running these distributions are safe, particularly since they likely have multiple cores and gather additional entropy from physical concurrency. However, it is possible that other distributions and customized installations do not collect sufficient entropy on startup and generate weak keys on first boot.

5.2 Factorable RSA keys and OpenSSL

One interesting question raised by our vulnerability results is why factorable RSA keys occur at all. A naïve implementation of RSA key generation would simply seed a PRNG from the operating system's entropy pool and then use it to generate p and q. In this approach, we would expect to see duplicate keys if the OS provided the same seed, but factorable keys would be extremely unlikely. What we see instead is that some devices seem prone to generating keys with common factors. Another curious feature is that although some of the most common prime factors divided hundreds of different moduli, in nearly all of these cases the second prime factor did not divide any other keys.

One explanation for this pattern is that implementations updated their entropy pools in the middle of key generation. In this case, the entropy pool states might be identical as distinct key generation processes generate the first prime p and diverge while generating the second prime q. In order to explore this theory, we studied the source code of OpenSSL [14], one of the most widely used open-source cryptographic libraries. OpenSSL is not the only software library responsible for the problems we observed, but we chose to examine it because the source code is freely available and because of its popularity.

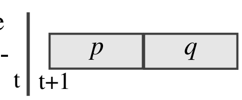
OpenSSL RSA key generation OpenSSL's built-in RSA key generator relies on an internal entropy pool maintained by OpenSSL. The entropy pool is seeded on first use with (on Linux) 32 bytes read from `/dev/urandom`, the process ID, user ID, and the current time in seconds.

OpenSSL provides a function named `bnrand()` to generate cryptographically-sized integers from the entropy pool, which, on each call, mixes into the entropy pool the current time in seconds, the process ID, and the possibly uninitialized contents of a destination buffer.

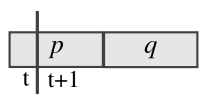
The RSA key generation algorithm generates the primes p and q using a randomized algorithm. During this process, OpenSSL extracts entropy from the entropy pool dozens to hundreds of times. Since we observed many keys with one prime factor in common, we can conjecture that multiple systems are starting with `urandom` and the time in the same state and that the entropy pool states diverge due to the addition of time during intermediate steps of the key generation process.

We hypothesized that this process is hypersensitive to small variations in where the *boundary between seconds* falls. Slight variations in execution speed might cause the wall clock tick to fall between different calls to `bnrand()`, resulting in different execution paths. This can result in several different behaviors, with three simple cases:

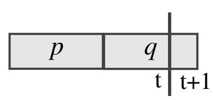
If the second never changes while computing p and q, every execution will generate identical keys.



If the clock ticks while generating p, both p and q diverge, yielding distinct keys with no shared factors.



If instead the clock advances to the next second during the generation of the second prime q, then two executions will generate identical primes p but can generate distinct primes q based on exactly when the second changes.



Experiment To test our hypothesis, we modified OpenSSL 1.0.0g to control all the entropy inputs used during key generation, generated a large number of RSA keys, and determined how many were identical or factorable. To simulate the effects of slower clock speeds, we dilated the clock time returned by `time()` and repeated the experiment using dilation multipliers of 1–32. In all, we generated 14 million keys. We checked for common factors within each batch of 100 keys.

The results we obtained, illustrated in Figure 6, are consistent with our hypothesis. No factorable keys are generated for low starting offsets, as both p and q are generated before the second changes. As the initial offset increases, there is a rapid phase change to generating factorable keys, as generation of q values begins to overlap the second boundary. Eventually, the fraction of factorable keys falls as the second boundary occurs during the generation of more p values, resulting in distinct moduli with no common factors.

5.3 DSA signature weaknesses and Dropbear

The DSA signature vulnerabilities we observed indicate that entropy problems impact not only key generation but also the continued runtime behavior of server software during normal usage. This is somewhat surprising, since we might expect the operating system to collect sufficient entropy *eventually*, even in embedded devices. We investigated Dropbear, a popular light-weight SSH implementation. It maintains its own entropy pools seeded from the operating system at launch, on Linux with 32 bytes read from `urandom`. This suggests a possible explanation for the observed problems: the operating system had not collected enough entropy when the SSH server started, and, from then on, even though the system entropy pool may have had further entropy, the running SSH daemon did not.

To better understand why these programs produce vulnerable DSA signatures, we examined the source code for the current version of Dropbear, 0.55. The ephemeral key is generated as output from an internal entropy pool. Whenever Dropbear extracts data from its entropy pool, it increments a static counter and hashes the result into the pool state. No additional randomness is added until the counter (a 32-bit integer) overflows. This implies that, if two Dropbear servers are initially seeded with the same value from `urandom`, they will provide identical signature randomness as long as their counters remain synchronized and do not overflow.

(We note that Dropbear contains a routine to generate k in a manner dependent on the message to be signed, which would ensure that distinct messages are always signed with distinct k values and protect against the vulnerability that we explore here. However, that code is disabled by default.)

We looked for evidence of synchronized sequences of ephemeral keys in the wild by making further SSH requests to a handful of the Dropbear hosts from our scan. We chose two hosts with the SSH version string `dropbear-0.39` that had used identical DSA public keys and r values and found that the signatures followed an identical sequence of r values. We could advance the sequence of one host by making several SSH requests, then cause the other host to catch up by making the same number of requests. When probed again an hour later, both hosts remained in sync. This suggests that the Dropbear code is causing vulnerabilities on real hosts in the manner we predicted.

Several other implementations, including hosts identifying OpenSSH and the Siemens Gigaset routers displayed similar behavior when rescanned. Because OpenSSL adds the current clock time to the entropy pool before extracting these random values, this suggests that some of these devices do not have a working clock at all.

6 Discussion

6.1 RSA vs. DSA in the face of low entropy

We believe that the DSA signature vulnerabilities pose more cause for concern than the RSA factorization vulnerability. The RSA key factorization vulnerability that we investigated occurs only for certain patterns of key generation implementations in the presence of low entropy. In contrast, the DSA signature vulnerability can compromise any DSA private key—no matter how well generated—if there is ever insufficient entropy at the time the key is used for signing. It is not necessary to search for a collision, as we did; it suffices for an attacker to be able to guess the ephemeral private key k. The most analogous attacks against RSA of which we are aware show that some types of padding schemes can allow an attacker to discover the encrypted plaintext or forge signatures [10]. We are unaware of any attacks that use compromised RSA signatures to recover the private key.

We note that our findings show a larger fraction of SSH hosts are compromised by the DSA vulnerability than by factorable RSA keys, even though our scanning techniques have likely only revealed a small fraction of the hosts prone to repeating DSA signature randomness. In contrast, the factoring algorithm we used has found all of the repeated RSA primes in our sample of keys.

There are specific countermeasures that implementations can use to protect against these attacks. If both prime factors of an RSA modulus are generated from a PRNG without adding additional randomness during key generation, then low entropy would result in repeated but not factorable keys. These are more readily observable, but may be trickier to exploit, because they do not immediately reveal the private key to a remote attacker. To prevent DSA randomness collisions, the randomness for each signature can be generated as a function of the message and the pseudorandom input. (It is very important to use a cryptographically secure PRNG for this process [4].) Of course, the most important countermeasure is for implementations to use sufficient entropy during cryptographic operations that require randomness, but defense-in-depth remains the prudent course.

6.2 /dev/(u)random as a usability failure

The Linux documentation states that "[a]s a general rule, `urandom` should be used for everything except long-lived GPG/SSL/SSH keys" [1]. However, all the open-source implementations we examined used `urandom` to generate keys by default. Based on a survey of developer mailing lists and forums, it appears that this choice is motivated by two factors: `random`'s extremely conservative behavior and the mistaken perception that `urandom`'s output is secure enough.

As others have noted, Linux is very conservative at crediting randomness added to the entropy pool [23], and `random` further insists on using *freshly collected* randomness that has not already been mixed into the output PRNG. The blocking behavior means that applications that read from `random` can hang unpredictably, and, in a headless device without human input or disk entropy, there may never be enough input for a read to complete. While blocking is intended to be an indicator that the system is running low on entropy, `random` often blocks even though the system has collected more than enough entropy to produce cryptographically strong PRNG output—in a sense, `random` is often "crying wolf" when it blocks.

Our experiments suggest that many of the vulnerabilities we observed may be due to the output of `urandom` being used to seed entropy pools before any entropic inputs have been mixed in. Unfortunately, the existing interface to `urandom` gives the operating system no means of alerting applications to this dangerous condition. Our recommendation is that Linux should add a secure RNG that blocks until it has collected adequate seed entropy and thereafter behaves like `urandom`.

6.3 Are we seeing only the tip of the iceberg?

Nearly all of the vulnerable hosts that we were able to identify were headless or embedded devices. This raises the question of whether the problems we found appear *only* in these types of devices, or if instead we are merely seeing the tip of a much larger iceberg.

Based on the experiments described in Section 5.1, we conjecture that there may exist further classes of vulnerable keys that were not visible to our methods, but could be compromised with targeted attacks. The first class is composed of embedded or headless devices with an accurate real-time clock. In these cases, keys generated during the boot-time entropy hole may appear distinct, but depend only on a configuration-specific state and the boot time. These keys would not appear vulnerable in our scanning, but an attacker may be able to enumerate some or all of such a reduced key space for targeted implementations.

A more speculative class of potential vulnerability consists of traditional PC systems that automatically generate cryptographic keys on first boot. We observed in Section 5.1 that an experimental machine running RHEL 5 and 6 did collect sufficient entropy in time for SSH key generation, but that the margin of safety was slim. It is conceivable that some lower-resource systems may be vulnerable.

Finally, our study was only able to detect vulnerable DSA ephemeral keys under specific circumstances where a large number of systems shared the same long-term key and were choosing ephemeral keys from the same small set. There may be a larger set of hosts using ephemeral keys that do not repeat across different systems but are nonetheless vulnerable to a targeted attack.

We found no evidence suggesting that RSA keys from standard implementations that were generated interactively or subsequent to initial boot are vulnerable.

6.4 Directions for future work

In this work, we examined keys from two cryptographic algorithms on two protocols visible via Internet-wide scans of two ports . A natural direction for future work is to broaden the scope of all of these choices. Entropy problems can also affect the choice of Diffie-Hellman key parameters and keying material for symmetric ciphers. In addition, there are many more subtle attacks against RSA, DSA, and ECDSA that we did not search for. We focused on keys, but one might also try to search for evidence of repeated randomness in initialization vectors in ciphertext or salts in cryptographic hashes.

We also focused solely on services visible to our scans of the public Internet. Similar vulnerabilities might be found by applying this methodology to keys or other cryptographic data obtained from other resource-constrained devices that perform cryptographic operations, such as smart cards or mobile phones.

The observation that `urandom` can produce predictable output on some types of systems at boot may lead to attacks on other services that automatically begin at boot and depend on good randomness from the kernel. It warrants investigation to determine whether this behavior may undermine other security mechanisms such as address space layout randomization or TCP initial sequence numbers.

7 Defenses and Lessons

The vulnerabilities we have identified are a reminder that secure random number generation continues to be a challenging problem. There is a tendency for developers at each layer of the software stack to silently shift responsibility to other layers; a far better practice would be a defense-in-depth approach where developers at every layer apply careful security design and testing and make assumptions clear. We suggest defensive strategies and lessons for several important groups of stakeholders.

For OS developers:

Provide the RNG interface applications need. Typical security applications require a source of randomness that is guaranteed to be of high quality and has predictable performance; neither Linux's `/dev/random` nor `/dev/urandom` strikes this balance. The operating system should maintain a secure PRNG that refuses to return data until it has been seeded with a minimum amount of true randomness and is continually seeded with fresh entropy collected during operation.

Communicate entropy conditions to applications. The problem with `/dev/urandom` is that it can return data even before it has been seeded with any entropy. The OS should provide an interface to indicate how much entropy it has mixed into its PRNG, so that applications can gauge whether the state is sufficiently secure for their needs.

Test RNGs thoroughly on diverse platforms. Many of the entropy sources that Linux supports are not available on headless or embedded devices. These behaviors may not be apparent to OS developers unless they routinely test the internals of the entropy collection subsystem across the full spectrum on platforms the system supports.

For library developers:

Default to the most secure configuration. Both OpenSSL and Dropbear default to using `/dev/urandom` instead of `/dev/random`, and Dropbear defaults to using a less secure DSA signature randomness technique even though a more secure technique is available as an option. In general, cryptographic libraries should default to using the most secure mechanisms available.

Use RSA and DSA defensively. Crypto libraries can take specific steps to prevent weak entropy from resulting in the immediate leak of private keys due to co-factorable RSA moduli and repeated DSA signature randomness (see Section 6.1).

For application developers:

Generate keys on first use, not on install or first boot. If keys must be generated automatically, it may be better to defer generation until the keys are needed.

Heed warnings from below. If the OS or cryptography library being used raises a signal that insufficient entropy is available (such as blocking), applications should detect this signal and refuse to perform security-critical operations until the system recovers from this potentially vulnerable state. Developers have been known to work around low-entropy states by ignoring or disabling such warnings, with extremely dangerous results [22].

For device manufacturers:

Avoid factory-default keys or certificates. While some defense is better than nothing, default keys and certificates provide only minimal protection.

Seed entropy at the factory. Devices could be initialized with truly random seeds at the factory. Sometimes it is already necessary to configure unique state on the assembly line (such as to set MAC addresses), and entropy could be added at the same time.

Ensure entropy sources are effective. Embedded or headless devices may not have access to sources of randomness assumed by the operating system, such as user-input devices or disk timing. Device makers should ensure that effective entropy sources are present, and that these are being harvested in advance of cryptographic operations.

Use hardware random number generators when possible. Security-critical devices should use a hardware random number generator for cryptographic randomness whenever possible.

For certificate authorities:

Check for repeated, weak, and factorable keys Certificate authorities have a uniquely broad view of keys contained in TLS certificates. We recommend that they repeat our work against their certificate databases and take steps to protect their customers by alerting them to potentially weak keys.

For end users:

Regenerate default or automatically generated keys. Cryptographic keys and certificates that were shipped with the device or automatically generated at first boot should be manually regenerated. Ideally, certificates and keys should be generated on another device (such as a desktop system) with access to adequate entropy.

Check for known weak keys. We have created a key-check service that individuals can use to check their TLS certificates and SSH host keys against our database of keys we have identified as vulnerable.

For security and crypto researchers:

Secure randomness remains unsolved in practice. The fact that all major operating systems now provide cryptographic RNGs might lead security experts to believe that any entropy problems that still occur are the fault of developers taking foolish shortcuts. Our findings suggest otherwise: entropy-related vulnerabilities can result from complex interaction between hardware, operating systems, applications, and cryptographic primitives. We have yet to develop the engineering practices and principles necessary to make predictably secure use of unpredictable randomness across the diverse variety of systems where it is required.

Primitives should fail gracefully under weak entropy. Cryptographic primitives are usually designed to be secure under ideal conditions, but practice will subject them to conditions that are less than idea. We find that RSA and DSA, with surprising frequency, are used in practice under weak entropy scenarios where, due to the design of these cryptosystems, the private keys are totally compromised. More attention is needed to ensure that future primitives degrade gracefully under likely failure modes such as this.

8 Related Work

HTTPS surveys The HTTPS public-key infrastructure has been a focus of attention in recent years, and researchers have performed several large-scale scans to measure TLS usage and CA behavior. In contrast, our study addresses problems that are mostly separate from the CA ecosystem.

In 2010, the Electronic Frontier Foundation (EFF) and iSEC Partners debuted the SSL Observatory project [18] and released the largest public repository of TLS certificates. The authors used their data to analyze the CA infrastructure and noted several vulnerabilities. We owe the inspiration for our work to their fascinating dataset, in which we first identified several of the entropy problems we describe; however, we ultimately performed our own scans to have more up-to-date and complete data.

In 2011, Holz et al. [26] scanned the Alexa top 1 million domains and observed TLS sessions passing through the Munich Scientific Research Network (MWN). Their study recorded 960,000 certificates and was the largest academic study of TLS data at the time. They report many statistics gathered from their survey, mainly focusing on the state of the CA infrastructure. We note that they examined repeated keys and dismissed them as "curious, but not very frequent." Yilek et al. [37] performed daily scans of 50,000 TLS servers over several months to track replacement time for certificates affected by the Debian weak key bug. Our count of Debian certificates provides another data point on this subject.

Problems with random number generation Several significant vulnerabilities relating to weak random number generation have been found in widely used software. In 1996, the Netscape browser's SSL implementation was found to use fewer than a million possible seeds for its PRNG [19]. In May 2008, Bello discovered that the version of OpenSSL included in the Debian Linux distribution contained a bug that caused keys to be generated with only 15 bits of entropy [5]. The problem caused only 294,912 distinct keys to be generated per key size during a two year period before the error was found [37].

Gutmann [22] draws lessons about secure software design from the example of developer responses to an OpenSSL update intended to ensure that the entropy pool was properly seeded before use. He observes that many developers responded by working around the safety checks in ways that supplied no randomness whatsoever. The root cause, according to Gutmann, was that the OpenSSL design left the difficult job of supplying sufficient entropy to library users. He concludes that PRNGs should handle entropy-gathering themselves.

Gutterman, Pinkas, and Reinmann analyzed the Linux random number generator in 2006 [23]. In contrast to our analysis, which focuses on empirical measurement of an instrumented Linux kernel, theirs was based mainly on a review of the LRNG design. They point out several weaknesses from a cryptographic perspective, some of which have since been remedied. In a brief experimental section, they observe that the only entropy source used by the OpenWRT Linux distribution was network interrupts..

Weak entropy and cryptography In 2004, Bauer and Laurie [2] computed the pairwise GCDs of 18,000 RSA keys from the PGP web of trust and discovered a pair with a common factor of 9, demonstrating that the keys had been generated with broken (or omitted) primality testing.

The DSA signature weakness we investigate is well known and appears to be folklore. In 2010, the hacking group fail0verflow computed the ECDSA private key used for code signing on the Sony PS3 after observing that the signatures used repeated ephemeral keys [12]. Several more sophisticated attacks against DSA exist: Bellare, Goldwasser, and Miccancio [4] show that the private key is revealed if the ephemeral key is generated using a linear congruential generator, and Howgrave-Graham and Smart [27] give a method to compute the private key from a fraction of the bits of the ephemeral key.

Ristenpart and Yilek [34] developed "virtual machine reset" attacks in 2010 that induce repeated DSA ephemeral keys after a VM reset, and they implement "hedged" cryptography to protect against this type of randomness failure. Hedged public key encryption was introduced by Bellare et al. in 2009 and is designed to fail as gracefully as possible in the face of bad randomness [3].

As we were preparing this paper for submission, an independent group of researchers uploaded a preprint [31] reporting that they had computed the pairwise GCD of RSA moduli from the EFF SSL Observatory dataset and a database of PGP keys. Their work is concurrent and independent to our own; we were unaware of these authors' efforts before their work was made public. They declined to report the GCD computation method they used. We responded by publishing a blog post [25] describing our GCD computation approach and summarizing some of the key findings we detail in this paper.

The authors of the concurrent work report similar results to our own on the fraction of keys that were able to be factored, and thus the two results provide validation for each other. In their paper, however, the authors draw very different conclusions than we do. They do not analyze the source of these entropy failures, and they conclude that RSA is "significantly riskier" than DSA. In contrast, we performed original scans that targeted SSH as well as TLS, and we looked for DSA repeated signature weaknesses as well as cofactorable RSA keys. We find that SSH DSA private keys are compromised at a higher rate than RSA keys, and we conclude that the fundamental problem is an implementational issue rather than a cryptographic one.

Furthermore, the authors of the concurrent work state that they "cannot explain the relative frequencies and appearance" of the weak keys they observed and report no attempt to determine their source. In this work, we performed extensive investigation to trace the vulnerable keys back to specific devices and software implementations, and we have notified the responsible developers and manufacturers. We find that the weak keys can be explained by specific design and implementation failures at various levels of the software stack, and we make detailed recommendations to developers and users that we hope will lessen the occurrence of these problems in the future.

9 Conclusion

In this work, we investigated the security of random number generation on a broad scale by performing and analyzing the most comprehensive Internet-wide scans of TLS certificates and SSH host keys to date. Using the global view provided by our data, we discovered that insecure RNGs are in widespread use, leading to a significant number of vulnerable RSA and DSA keys.

Our experiences suggest that the type of scanning and analysis we performed can be a useful tool for finding subtle flaws in cryptographic implementations, and we hope it will be applied more broadly in future work. Previous examples of random number generation flaws were found by painstakingly reverse engineering individual devices or implementations, or through luck when a collision was observed by a single user. Our scan data allowed us to essentially mine for vulnerabilities and detect problems in dozens of different devices and implementations in a single shot. Many of the collisions we found were too rare to ever have been observed by a single user but quickly became apparent with a near-global view of the universe of public keys. The results are a reminder to all that vulnerabilities can sometimes be hiding in plain sight.

Acknowledgments

The authors thank Dan Bernstein and Tanja Lange for discussion of batch factorization and OpenSSL, and Hovav Shacham for advice on many aspects of this work. We also thank Jake Appelbaum, Michael Bailey, Kevin Borders, Keith Brautigam, Ransom Briggs, Jesse Burns, Aleksander Durumeric, Prabal Dutta, Peter Eckersley, Andy Isaacson, James Kasten, Ben Laurie, Stephen Schultze, Ron Rivest, and David Robinson.

This material is based upon work supported by the National Science Foundation under Award No. DMS-1103803, the MURI program under AFOSR Grant No. FA9550-08-1-0352, and a National Science Foundation Graduate Research Fellowship.

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