CMSC 330: Organization of Programming Languages

Functional Programming with Lists

Lists in OCaml

- The basic data structure in OCaml
  - Lists can be of *arbitrary length*
    - Implemented as a linked data structure
  - Lists must be *homogeneous*
    - All elements have the same type

- Operations
  - Construct lists
  - Destruct them via pattern matching

Constructing Lists: Syntax

Syntax

- `[]` is the empty list (pronounced “nil”)
- ***e1*** `::` ***e2*** prepends element ***e1*** to list ***e2***
  - Operator `::` is pronounced "cons"
  - ***e1*** is the head, ***e2*** is the tail
- `[`***e1***`;`***e2***`;…;`***en***`]` is *syntactic sugar* for ***e1***`::`***e2***`::…::`***en***`::[]`

Both *cons* and *nil* are terms from LISP

Examples

```
3::[]           (* The list [3] *)
2::(3::[])      (* The list [2; 3] *)
[1; 2; 3]       (* The list 1::(2::(3::[])) *)
```

Beware:
[1,2,3] is **not a list**!
[1;2;3] is.
Using the former may lead to confusing error messages.

Constructing Lists: Evaluation

Evaluation

- `[]` is a value
- To evaluate `[e1;…;en]`
  - evaluate `e1` to a value `v1`,
  - …,
  - evaluate `en` to a value `vn`,
  - and return `[v1;…;vn]`
- *Desugaring*: evaluate `e1::e2`
  - evaluate `e1` to a value `v1`,
  - evaluate `e2` to a (list) value `v2`,
  - and return `v1::v2`

Remember: Evaluation order in OCaml is **right to left** (not left to right);

Constructing Lists: Examples

```
# let y = [1; 1+1; 1+1+1] ;;
val y : int list = [1; 2; 3]

# let x = 4::y ;;
val x : int list = [4; 1; 2; 3]

# let z = 5::y ;;
val z : int list = [5; 1; 2; 3]

# let m = “hello”::”bob”::[];;
val m : string list = [“hello”; “bob”]
```

Constructing Lists: Typing

Polymorphic type:
like a generic type in Java

Nil:

`[]`: `'a list`

i.e., empty list has type *`t`* `list` for any type *`t`*

Cons:

If *`e1`* : *`t`* and *`e2`* : *`t`* `list` then *`e1`* `::` *`e2`* : *`t`* `list`

With parens for clarity:

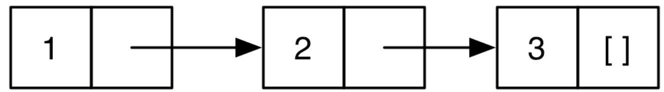
If *`e1`* : *`t`* and *`e2`* : (*`t`* `list`) then (*`e1`* `::` *`e2`*) : (*`t`* `list`)

Examples

```
# let x = [1;"world"] ;;
This expression has type string but an expression was
  expected of type int

# let m = [[1];[2;3]];;
val y : int list list = [[1]; [2; 3]]

# let y = 0::[1;2;3] ;;
val y : int list = [0; 1; 2; 3]

# let w = [1;2]::y ;;
This expression has type int list but is here used with
  type int list list
```

- The left argument of :: is an element, the right is a list
- Can you construct a list y such that [1;2]::y makes sense?

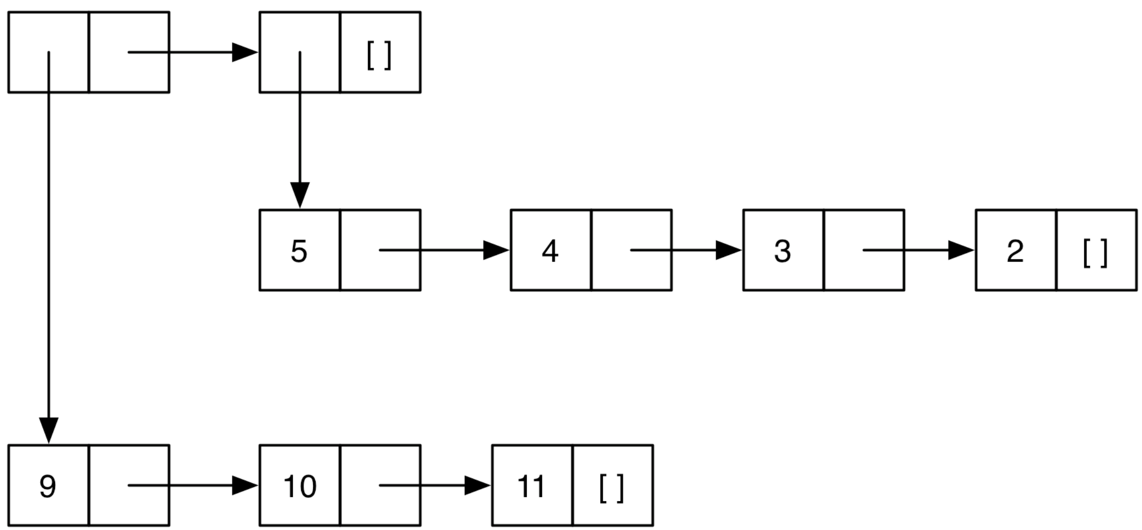
Lists in Ocaml are Linked



- [1;2;3] is represented as shown above
  - A nonempty list is a pair (element, rest of list)
  - The element is the head of the list
  - The pointer is the tail or *rest* of the list
    - ...which is itself a list!
- Thus in math (i.e., inductively) a list is either
  - The empty list [ ]
  - Or a pair consisting of an element and a list
    - This recursive structure will come in handy shortly

Lists of Lists

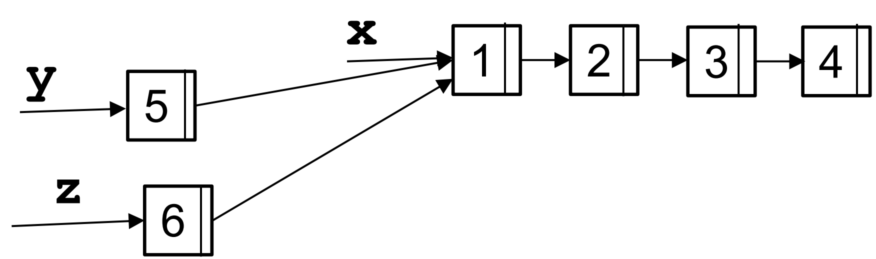
- Lists can be nested arbitrarily
  - Example: `[ [9; 10; 11]; [5; 4; 3; 2] ]`
    - Type `int list list`, also written as (`int list`) `list`



Lists are Immutable

- No way to *mutate* (change) an element of a list
- Instead, build up new lists out of old, e.g., using **::**

```
let x = [1;2;3;4]
let y = 5::x
let z = 6::x
```



Quiz 1

What is the type of the following expression?

```
[1.0; 2.0; 3.0; 4.0]
```

A. **`array`**
B. **`list`**
C. **`float list`**
D. **`int list`**

Quiz 1

What is the type of the following expression?

```
[1.0; 2.0; 3.0; 4.0]
```

A. `array`

B. `list`

C. `float list`

D. `int list`

Quiz 2

What is the type of the following expression?

```
10::[20]
```

A. `int`

B. `int list list`

C. `int list`

D. `error`

Quiz 2

What is the type of the following expression?

```
10::[20]
```

A. `int`

B. `int list list`

C. `int list`

D. `error`

Quiz 3

What is the type of the following definition?

```
let f x = "alien"::[x]
```

A. `string -> string`

B. `string list`

C. `string list -> string list`

D. `string -> string list`

Quiz 3

What is the type of the following definition?

```
let f x = "alien"::[x]
```

A. `string -> string`

B. `string list`

C. `string list -> string list`

D. `string -> string list`

Pattern Matching

- To pull lists apart, use the **`match`** construct
- Syntax

```
match e with
/ p1 -> e1
| …
| pn -> en
```

- ***p1***...***pn*** are *patterns* made up of **`[]`**, **`::`**, constants, and *pattern variables* (which are normal OCaml variables)
- ***e1***...***en*** are *branch expressions* in which pattern variables in the corresponding pattern are bound

Pattern Matching: Evaluation

- To pull lists apart, use the **`match`** construct
- Syntax

```
match e with
| p1 -> e1
| …
| pn -> en
```

- Evaluate ***e*** to a value ***v***
- If ***p1*** matches ***v***, eval ***e1*** to ***v1*** and return it

…

- Else if ***pn*** matches ***v***, evaluate ***en*** to ***vn*** and return it
- Else, no patterns match: raise **`Match_failure`** exception

When evaluating branch expression ***ei***, any **pattern variables in *pi* are bound in *ei***, i.e., they are in scope

Pattern Matching Example

```
let is_empty l =
  match l with
    [] -> true
  | (h::t) -> false
```

- Example runs
  - `is_empty []    (* evaluates to true  *)`
  - `is_empty [1]   (* evaluates to false *)`
  - `is_empty [1;2] (* evaluates to false *)`

Pattern Matching Example (cont.)

```
let hd l =
  match l with
   (h::t) -> h
```

- Example runs
  - `hd [1;2;3] (* evaluates to 1 *)`
  - `hd [2;3]   (* evaluates to 2 *)`
  - `hd [3]     (* evaluates to 3 *)`
  - `hd []        (* Exception: Match_failure *)`

Quiz 4

To what does the following expression evaluate?

```
match [1;2;3] with
   [] -> [0]
 | h::t -> t
```

A. `[]`
B. `[0]`
C. `[1]`
D. `[2;3]`

Quiz 4

To what does the following expression evaluate?

```
match [1;2;3] with
   [] -> [0]
 | h::t -> t
```

A. `[]`
B. `[0]`
C. `[1]`
D. `[2;3]`

"Deep" pattern matching

- You can nest patterns for more precise matches
  - **`a::b`** matches lists with **at least one** element
    - Matches **`[1;2;3]`**, binding **`a`** to **`1`** and **`b`** to **`[2;3]`**
  - **`a::[]`** matches lists with **exactly one** element
    - Matches **`[1]`**, binding **`a`** to **`1`**
    - Could also write pattern **`a::[]`** as **`[a]`**
  - **`a::b::[]`** matches lists with **exactly two** elements
    - Matches **`[1;2]`**, binding **`a`** to **`1`** and **`b`** to **`2`**
    - Could also write pattern **`a::b::[]`** as **`[a;b]`**
  - **`a::b::c::d`** matches lists with **at least three** elements
    - Matches **`[1;2;3]`**, binding **`a`** to **`1`**, **`b`** to **`2`**, **`c`** to **`3`**, and **`d`** to **`[]`**
    - *Cannot* write pattern as **`[a;b;c]::d`** (why?)

Pattern Matching – Wildcards

- An underscore _ is a wildcard pattern
  - Matches anything
  - But doesn't add any bindings
  - Useful to hold a place but discard the value
    - i.e., when the variable does not appear in the branch expression

- In previous examples
  - Many values of `h` or `t` ignored
  - Can replace with wildcard _

Pattern Matching – Wildcards (cont.)

- Code using _
  - `let is_empty l = match l with`
    `[] -> true | (_::_) -> false`
  - `let hd l = match l with (h::_) -> h`
  - `let tl l = match l with (_::t) -> t`
- Outputs
  - `is_empty[1](* evaluates to false *)`
  - `is_empty[ ](* evaluates to true *)`
  - `hd [1;2;3] (* evaluates to 1 *)`
  - `hd [1] (* evaluates to 1 *)`
  - `tl [1;2;3] (* evaluates to [2;3] *)`
  - `tl [1] (* evaluates to [ ] *)`

Quiz 5

To what does the following expression evaluate?

```
match [1;2;3] with
  | 1::[]     -> [0]
  | _::_      -> [1]
  | 1::_::[] -> []
```

A. `[]`
B. `[0]`
C. `[1]`
D. `[2;3]`

Quiz 5

To what does the following expression evaluate?

```
match [1;2;3] with
   | 1::[]     -> [0]
   | _::_      -> [1]
   | 1::_::[] -> []
```

A. `[]`
B. `[0]`
C. `[1]`
D. `[2;3]`

Pattern Matching – An Abbreviation

- `let f` ***p*** `=` ***e***, where ***p*** is a pattern
  - is shorthand for `let f x = match x with` ***p*** `->` ***e***
- Examples
  - `let hd (h::_) = h`
  - `let tl (_::t) = t`
  - `let f (x::y::_) = x + y`
  - `let g [x; y] = x + y`
- Useful if there's only one acceptable input

Pattern Matching Typing

```
match e with
| p1 -> e1
| …
| pn -> en
```

- If ***e*** and ***p1***, …, ***pn*** each have type ***ta***
- and ***e1***, …, ***en*** each have type ***tb***
- Then entire **`match`** expression has type ***tb***

- Examples

type: *'a list -> 'a*

ta = 'a list

```
let hd l =
  match l with
  (h::_) -> h
```

tb

tb = 'a

type: *int list -> int*

```
let rec sum l =
  match l with
    [] -> 0
   | (h::t) -> h+sum t
```

tb

ta = int list

tb = int

Polymorphic Types

- The **`sum`** function works only for **`int lists`**
- But the **`hd`** function works for *any type of list*
  - `hd [1; 2; 3]                (* returns 1 *)`
  - `hd ["a"; "b"; "c"]          (* returns "a" *)`
- OCaml gives such functions polymorphic types
  - `hd : 'a list -> 'a`
  - this says the function takes a list of *any* element type `'a`, and returns something of that same type
- These are basically generic types in Java
  - `'a list` is like `List<T>`

Examples Of Polymorphic Types

- `let tl (_::t) = t`
```
# tl [1; 2; 3];;
- : int list = [2; 3]
# tl [1.0; 2.0];;
- : float list = [2.0]
(* tl : 'a list -> 'a list *)
```

- `let fst x y = x`
```
# fst 1 "hello";;
- : int = 1
# fst [1; 2] 1;;
- : int list = [1; 2]
(* fst : 'a -> 'b -> 'a *)
```

Examples Of Polymorphic Types

- `let eq x y = x = y`   `(* let eq x y = (x = y) *)`

```
# eq 1 2;;
- : bool = false
# eq “hello” “there”;;
- : bool = false
# eq “hello” 1     -- type error
(* eq : 'a -> ’a -> bool *)
```

Quiz 6

What is the type of the following function?

```
let f x y =
  if x = y then 1 else 0
```

A. `'a -> 'b -> int`

B. `'a -> 'a -> bool`

C. `'a -> 'a -> int`

D. `int`

Quiz 6

What is the type of the following function?

```
let f x y =
  if x = y then 1 else 0
```

A. `'a -> 'b -> int`

B. `'a -> 'a -> bool`

C. `'a -> 'a -> int`

D. `int`

Missing Cases

- Exceptions for inputs that don't match any pattern
  - OCaml will warn you about non-exhaustive matches
- Example:

```
# let hd l = match l with (h::_) -> h;;
Warning: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched:
[]

# hd [];;
Exception: Match_failure ("", 1, 11).
```

Pattern matching is ***AWESOME***

1. You can’t forget a case
   – Compiler issues inexhaustive pattern-match warning
2. You can’t duplicate a case
   – Compiler issues unused match case warning
3. You can’t get an exception
   – Can’t do something like `List.hd []`
4. Pattern matching leads to elegant, concise, beautiful code

Lists and Recursion

- Lists have a recursive structure
  - And so most functions over lists will be recursive

```
let rec length l = match l with
    [] -> 0
  | (_::t) -> 1 + (length t)
```

  - This is just like an inductive definition
    - *The length of the empty list is zero*
    - *The length of a nonempty list is 1 plus the length of the tail*
  - Type of length?
    - **`a list -> int**

More Examples

- `sum l (* sum of elts in l *)`

```
let rec sum l = match l with
    [] -> 0
  | (x::xs) -> x + (sum xs)
```

- `negate l (* negate elements in list *)`

```
let rec negate l = match l with
    [] -> []
  | (x::xs) -> (-x) :: (negate xs)
```

- `last l  (* last element of l *)`

```
let rec last l = match l with
    [x] -> x
  | (x::xs) -> last xs
```

More Examples (cont.)

(* return a list containing all the elements in the list l followed by all the elements in list m *)

- append l m

```
let rec append l m = match l with
    [] -> m
  | (x::xs) -> x::(append xs m)
```

- rev l (* reverse list; hint: use append *)

```
let rec rev l = match l with
     [] -> []
   | (x::xs) -> append (rev xs) (x::[])
```

- **rev** takes $O(n^2)$ time. Can you do better?