CMSC/Math 456: Cryptography (Fall 2023)

Lecture 15

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Administrative

Midterm this Thursday, Oct. 19.

- In class
- Open book (including textbook), no electronic devices
- Will cover material through Diffie-Hellman and key exchange, but not public key encryption.

Solution sets for PS #6 and last year's midterm are now posted on ELMS.

For last year's midterm, I strongly recommend doing your best to try the midterm problems before looking at the solutions.

Pseudorandomness

Pseudorandom generator G(y)

- One input y, "seed"
- Output looks like a random string when s unknown
- Output should be longer than the seed
- Generally only good for EAV security
- Stream cipher is a more flexible version

Pseudorandom function $F_k(r)$

- Two inputs: k (key) and r
- For fixed but unknown k, looks like a random function of r
- Output can be the same size or smaller than the input
- Useful for CPA security
- Block cipher is a fixed-size version, but must be a permutation (with computable inverse for known k)

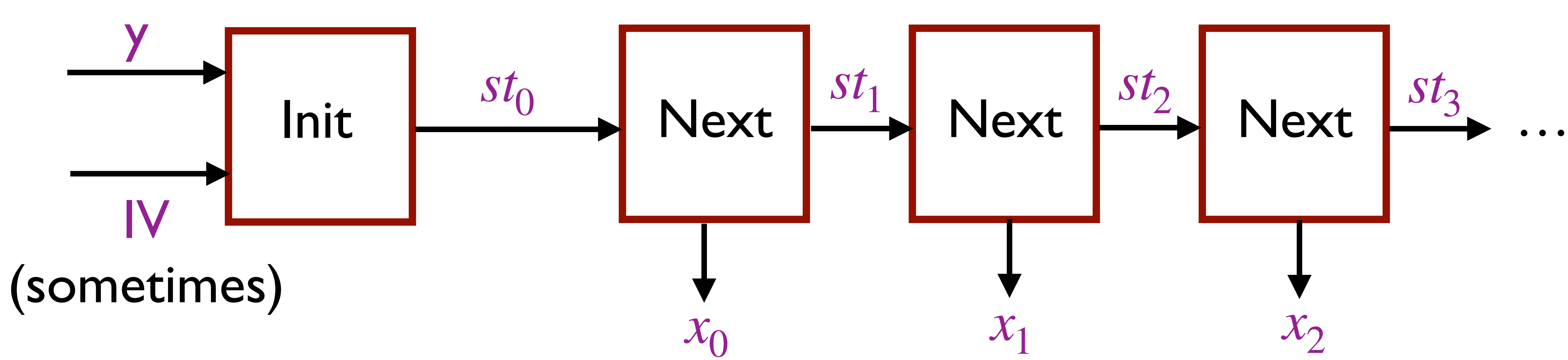
Stream Ciphers

Pseudorandom generator G(y)

- Input only seed y, which has length s
- Single output string G(s) of length $\ell(s)$

Stream cipher

- Input seed x, but *may also* take initial value IV as input
- With IV, can provide CPA security
- Has two component functions Init and Next
- Each time Next is called, the stream cipher outputs a fixed number of bits. I.e., output can be any length
- Function $y \mapsto (x_0, x_1, x_2, \ldots, x_\ell)$ is a pseudorandom generator



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Encryption Using a Stream Cipher

Suppose we have a message $m = m_1, m_2, m_3, \ldots, m_a$ of length a and we wish to encrypt using a stream cipher to get EAV security using key k:

1. Run Init using input k and no IV or fixed IV.
2. Run Next a times to get $x_1, x_2, x_3, \ldots, x_a$.
3. Ciphertext is $c = (m_1 \oplus x_1, m_2 \oplus x_2, \ldots, m_a \oplus x_a)$.

For CPA security:

1. Choose random IV. Run Init using input k and IV.
2. Run Next a times to get $x_1, x_2, x_3, \ldots, x_a$.
3. Ciphertext is $c = (IV, m_1 \oplus x_1, m_2 \oplus x_2, \ldots, m_a \oplus x_a)$.

Block Ciphers

Pseudorandom function $F_k(r)$

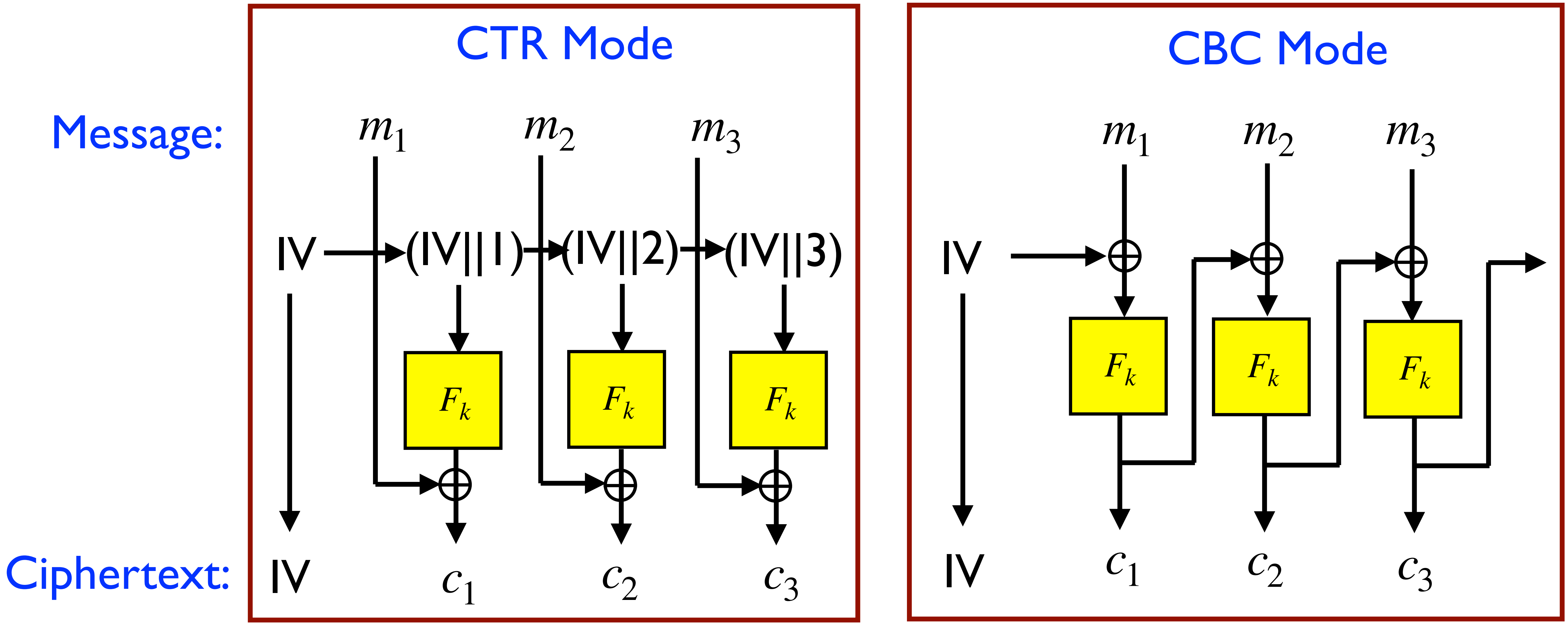
- Inputs k (key) and r have length n, a variable (although k could have a different length than r)
- For fixed but unknown k, looks like a random function of r
- Output can be the same size or smaller than the input

Block cipher $F_k(r)$

- Also inputs k (key) and r but now they are fixed length
- For fixed but unknown k, looks like a random function of r
- Output must be same size as r is
- Is a permutation: has computable inverse for known k

Block Cipher Modes of Operation

Break up message m into blocks of fixed size: $m_1, m_2, m_3, \ldots$.



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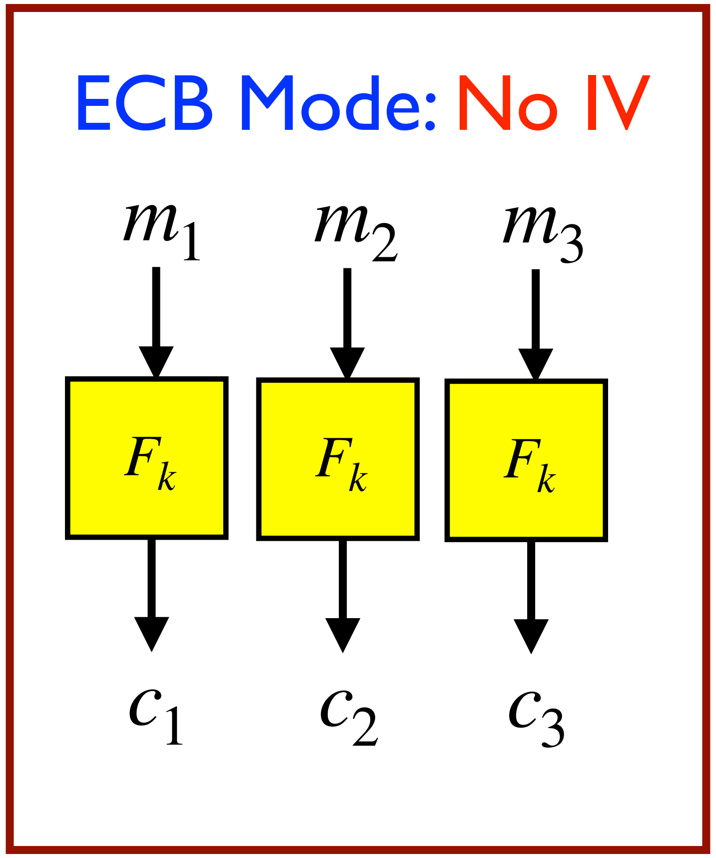
Block Cipher Modes

- CTR and CBC modes are both CPA-secure.
- IV is needed to provide randomness as part of the encoding so that the same message doesn't always have the same ciphertext.
- IV must be sent as part of the ciphertext so that Bob can decode.
- Another mode, ECB mode (below) is insecure, even against EAV attacks.



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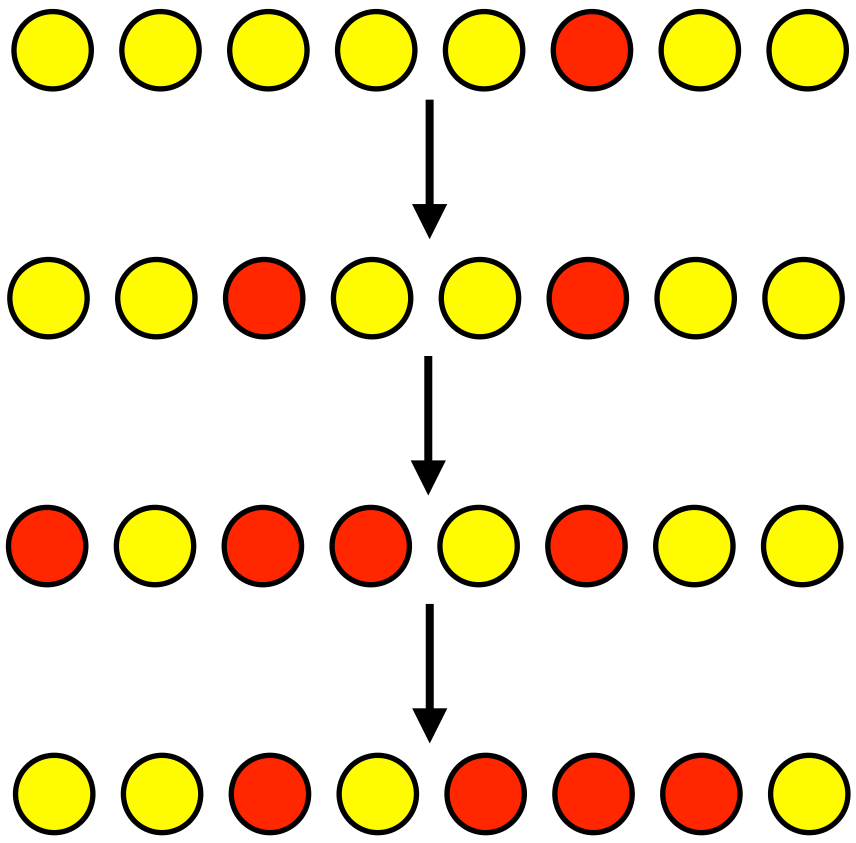
Goals of Block Cipher Design

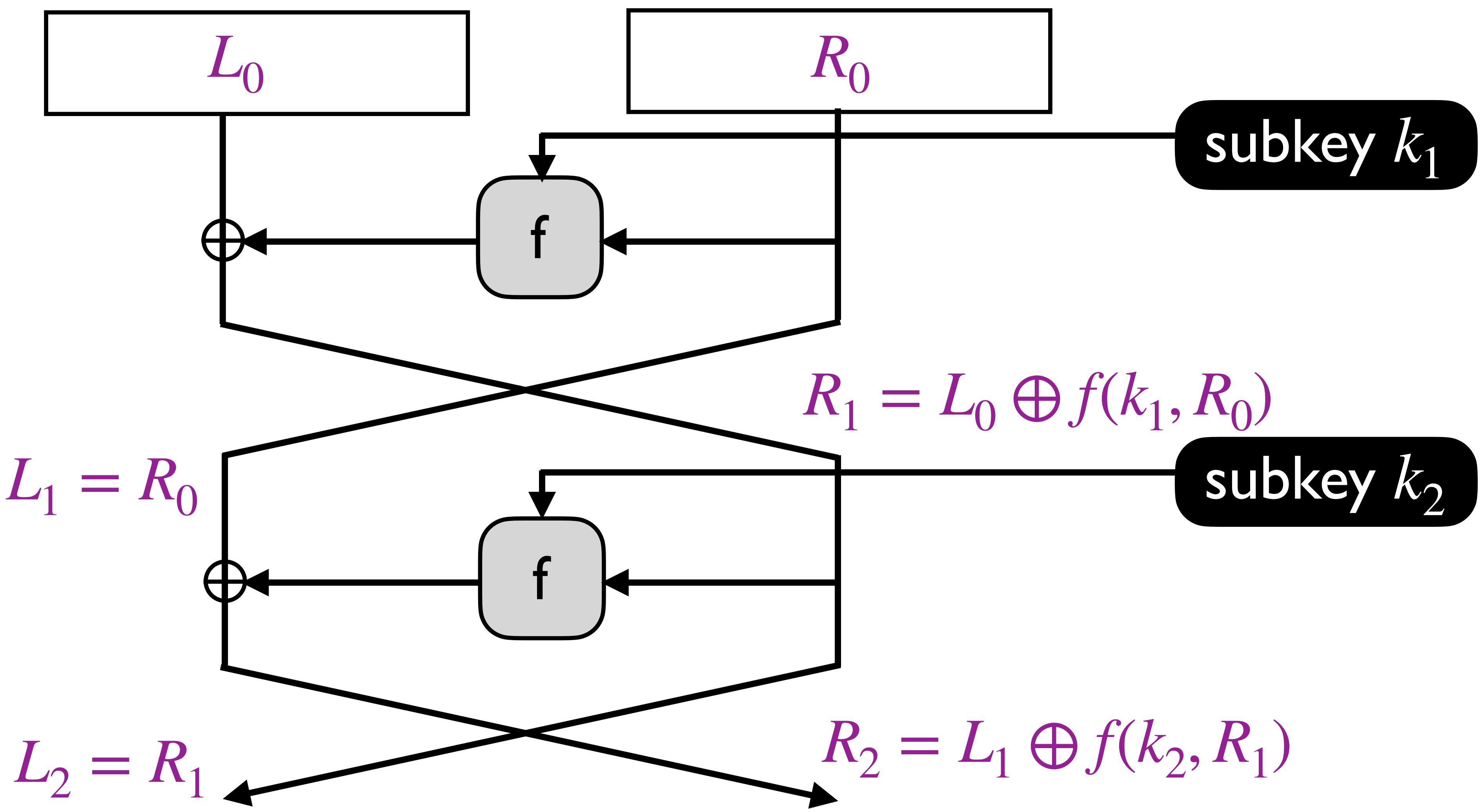
- Must be invertible to use with CBC mode (i.e., pseudorandom permutation rather than pseudorandom function).
- Even when the inputs are related, the outputs should be very different.

In particular, the change of even a single bit of the input should result in a totally different output. This is known as the "avalanche effect." It is often achieved by having multiple rounds, each of which magnifies small changes.



Feistel Network

A Feistel network consists of a sequence of rounds sequentially acting on the message, which is split into a left and right half.



In each round, the current right half is fed into a round function f with a key for the round and then XORed with the left half. The modified left half and old right half are then switched.

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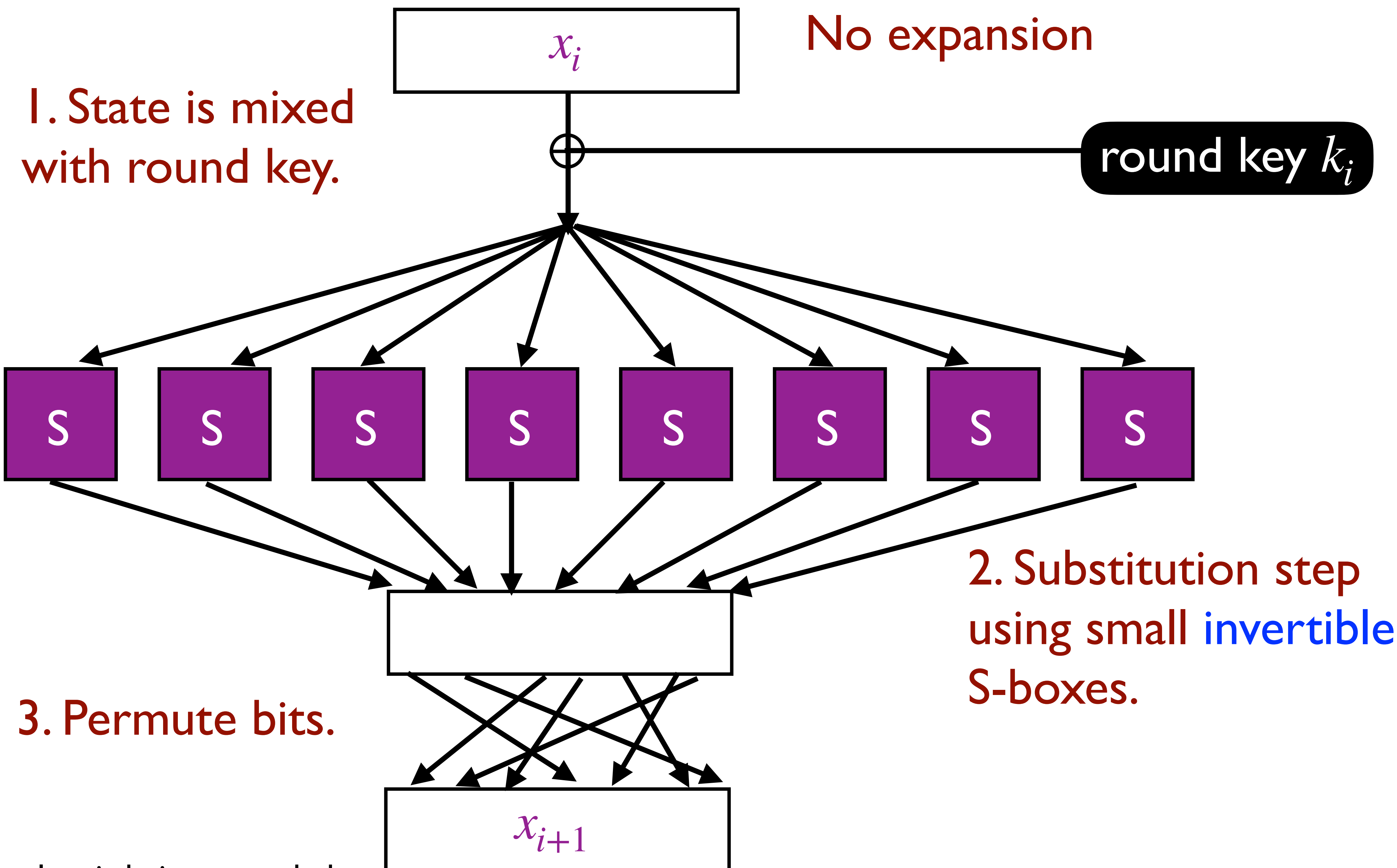
Substitution-Permutation Networks

Variants of substitution-permutation networks are used for both the DES mangler function and for AES.



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Confusion-Diffusion

The S-boxes introduce confusion: They change their inputs into totally different strings and magnify single-bit changes. However, the S-box is small and acts on only a few bits, so the confusion is only local.

Then the permutation step causes diffusion: whatever local confusion was introduced by the S-boxes spreads out to many different locations.

> Multiple rounds of substitution and permutation cause the confusion to be magnified further and continue to spread around.

We need both to get an avalanche effect.

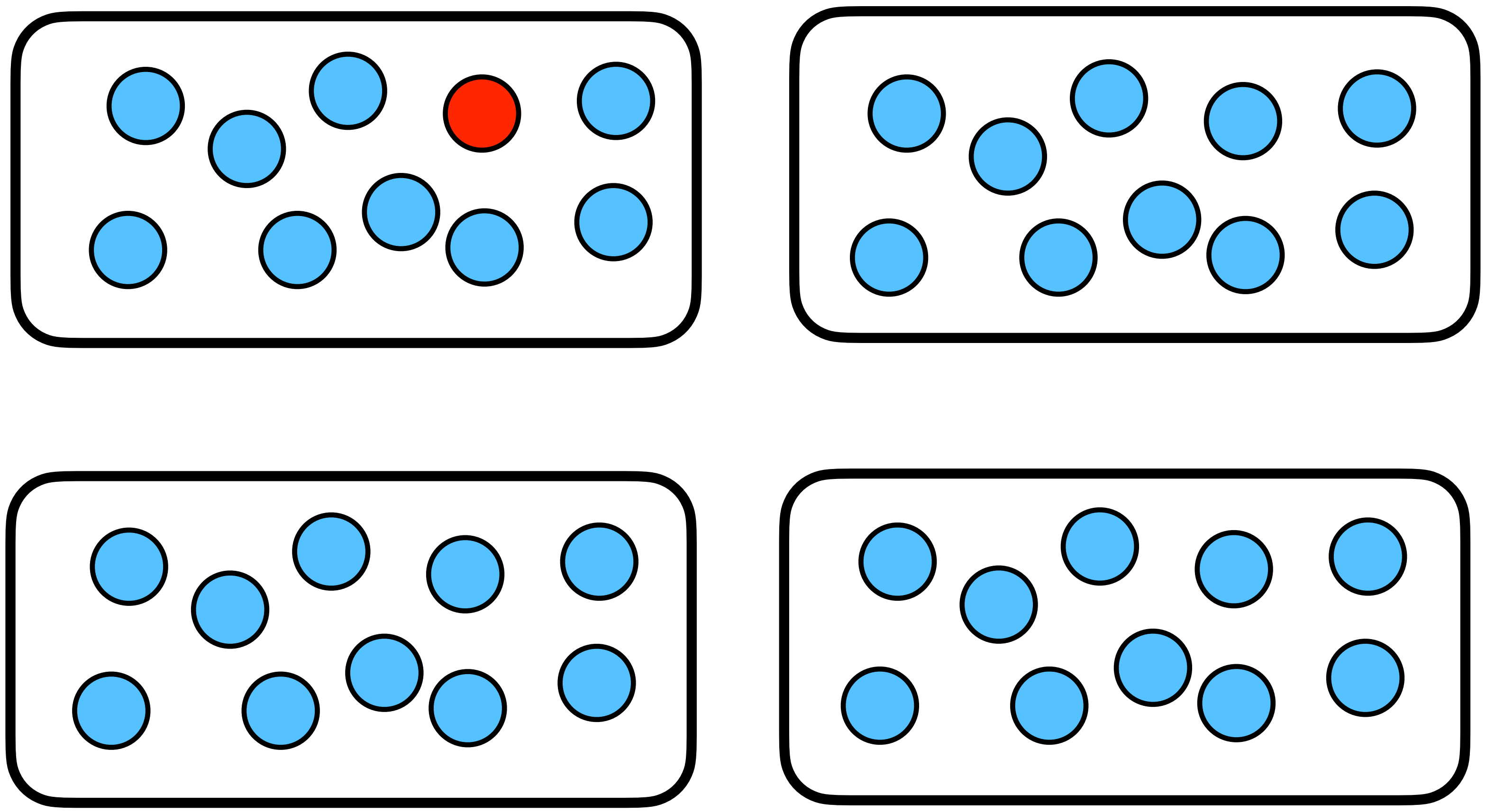
You also need key mixing: This is a permutation, and without the key, Eve can just trace the permutation backwards to get the input.

Disease Confusion-Diffusion

Imagine you have a disease spreading. It starts with one patient.

Confusion: The disease infects additional people in the same city as someone who is sick.

Diffusion: Some people travel to different cities.



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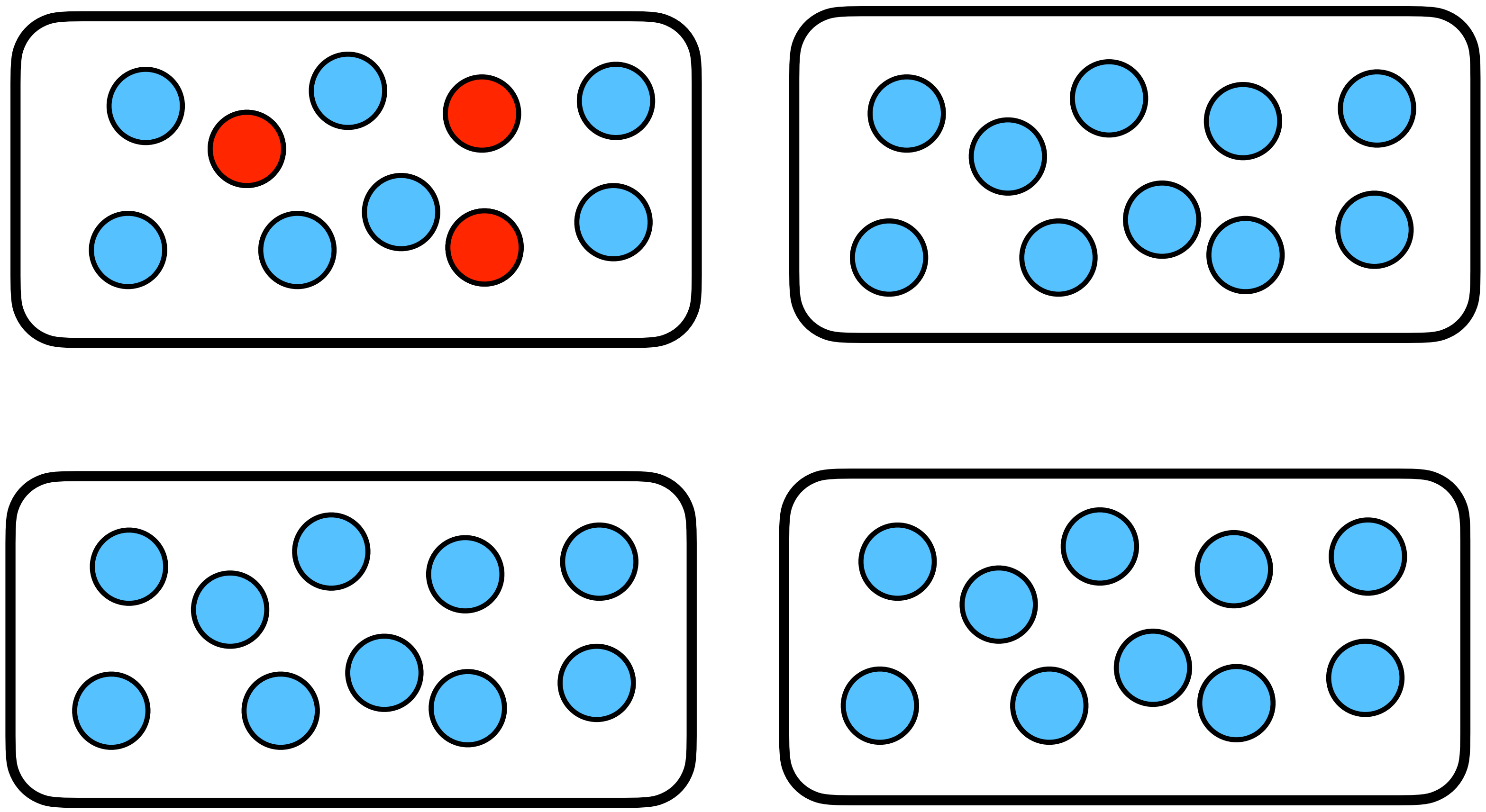

1. Confusion

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Disease Confusion-Diffusion

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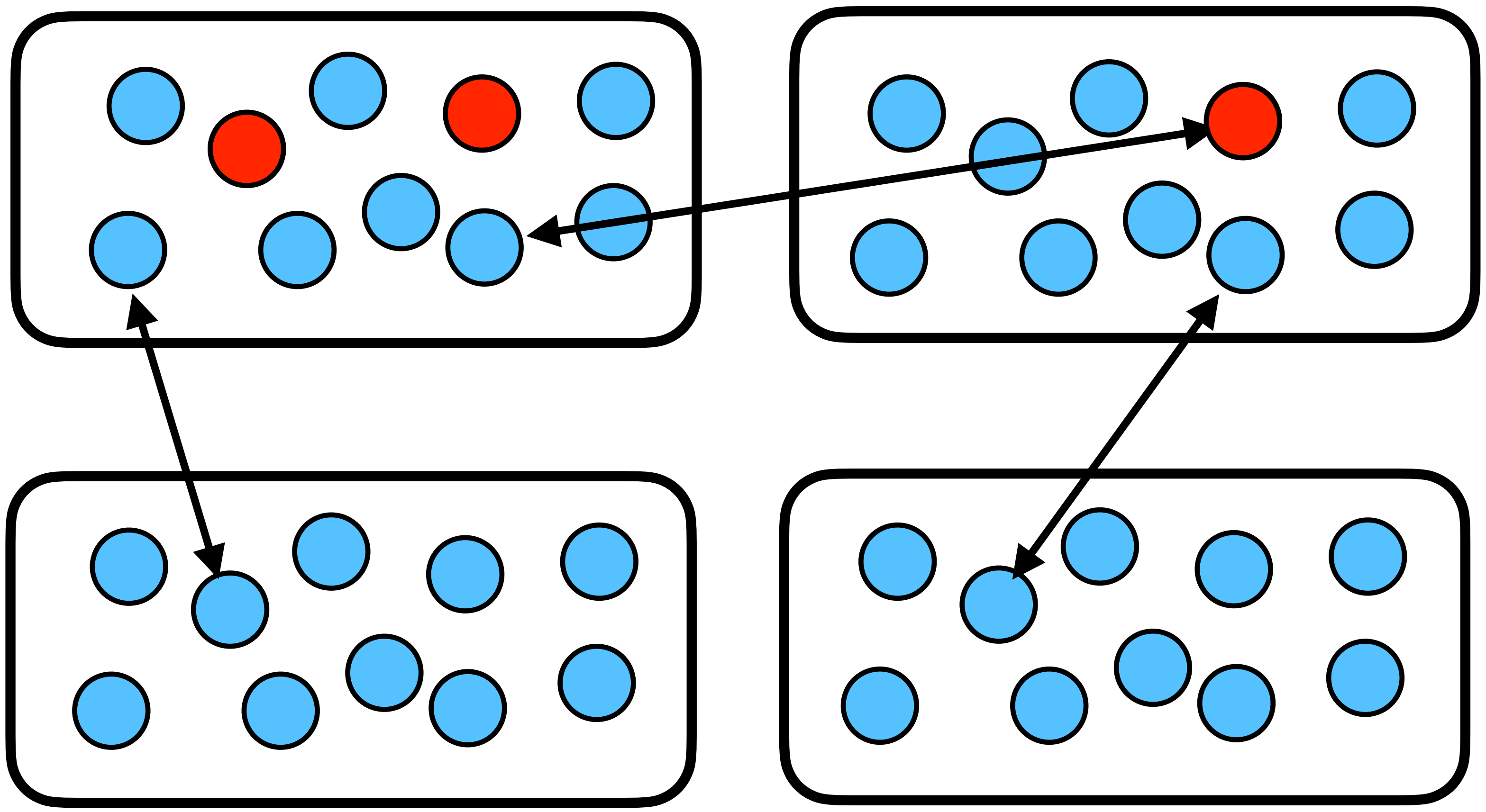

1. Confusion
2. Diffusion

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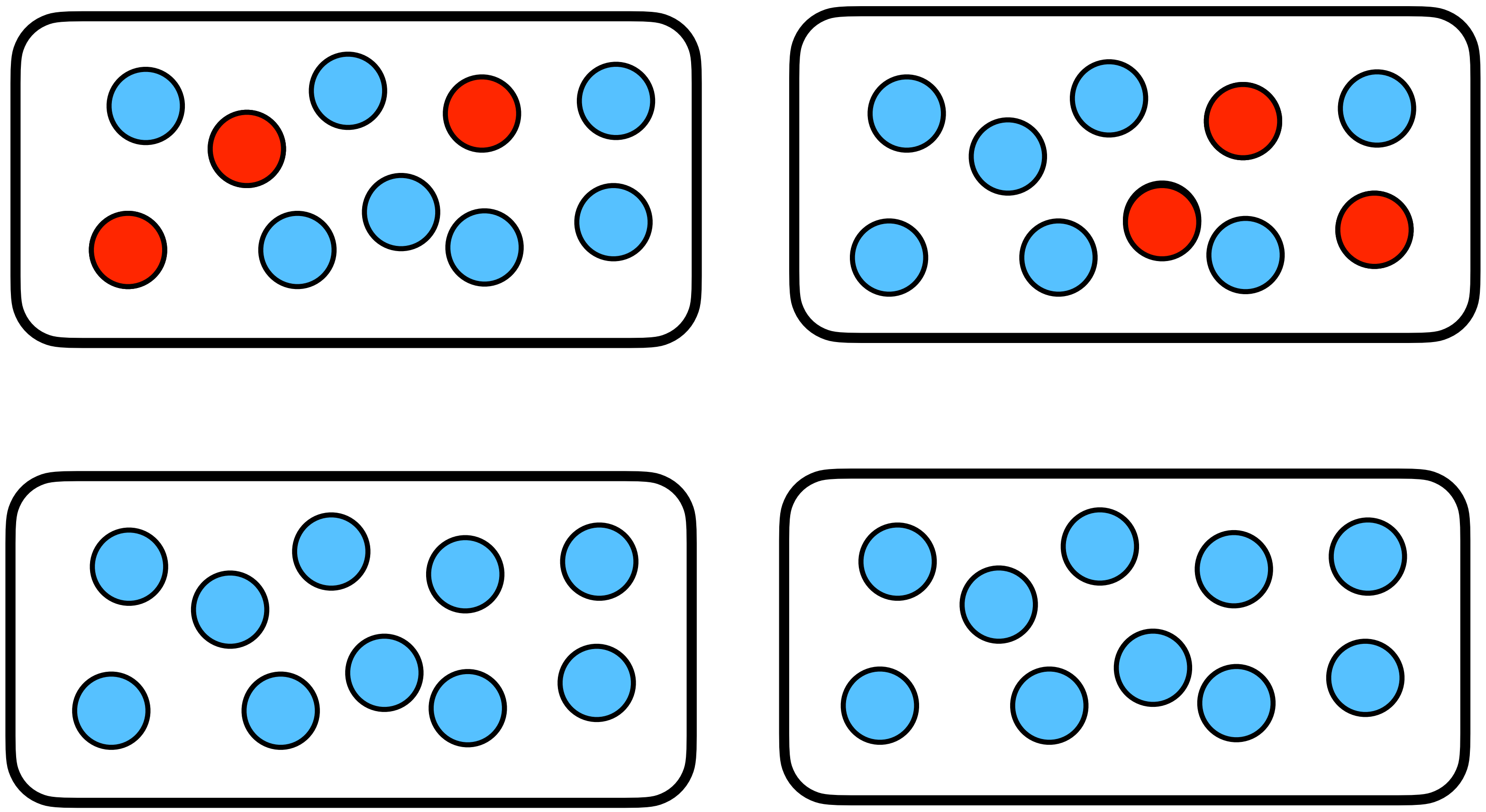

1. Confusion
2. Diffusion
3. Confusion

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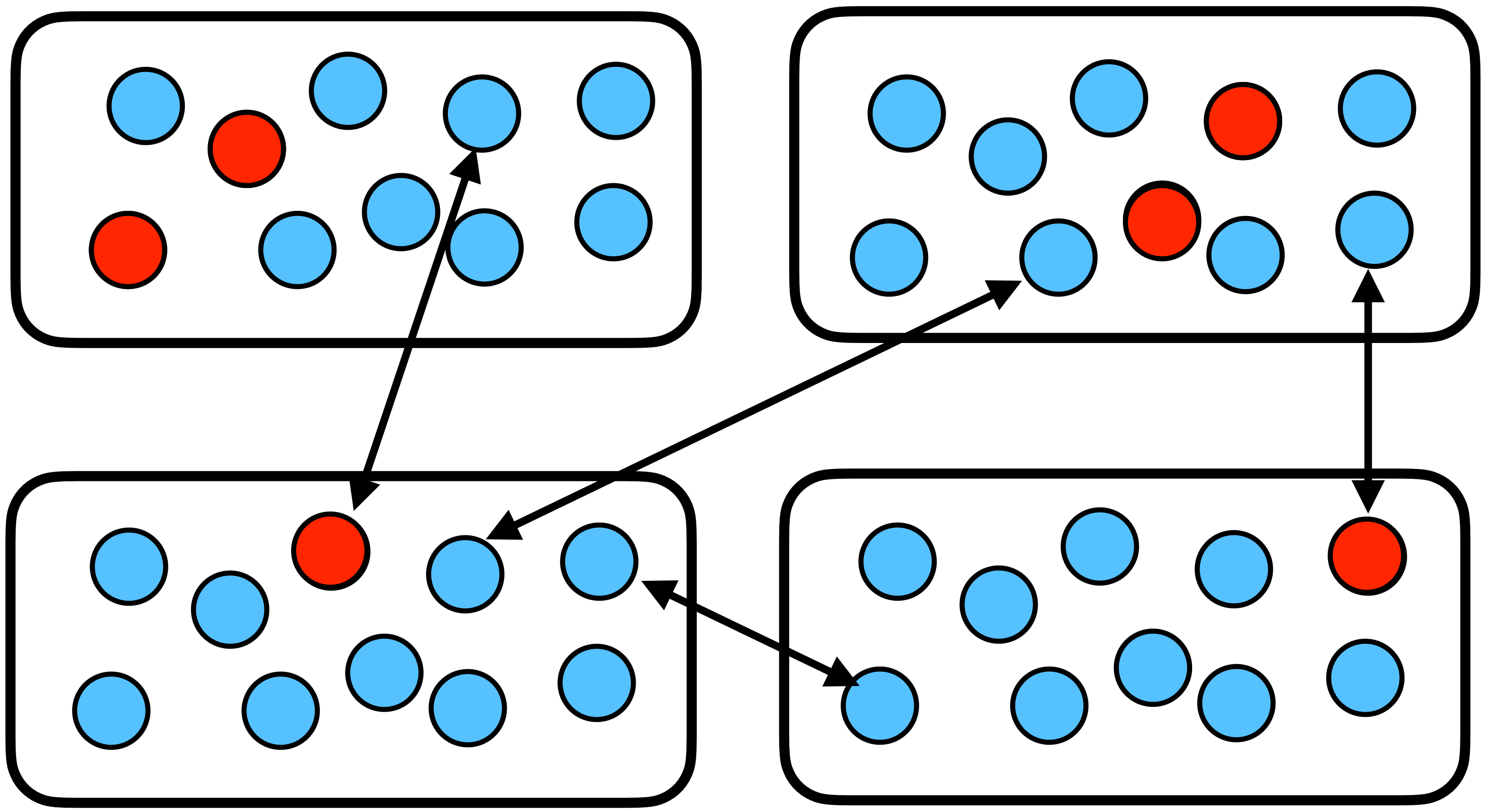

1. Confusion
2. Diffusion
3. Confusion
4. Diffusion

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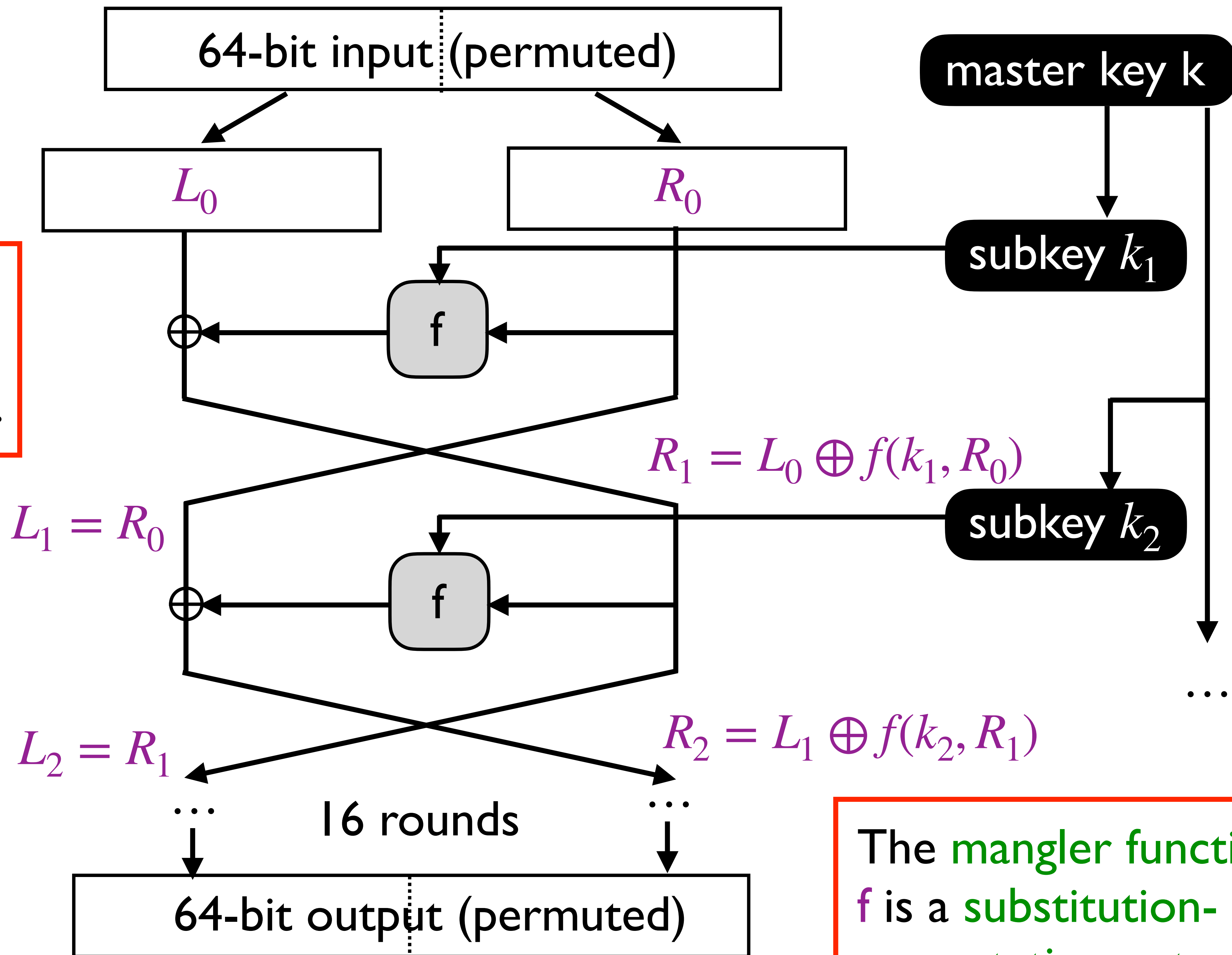
DES Overview



DES is a Feistel network.

The mangler function f is a substitution-permutation network.

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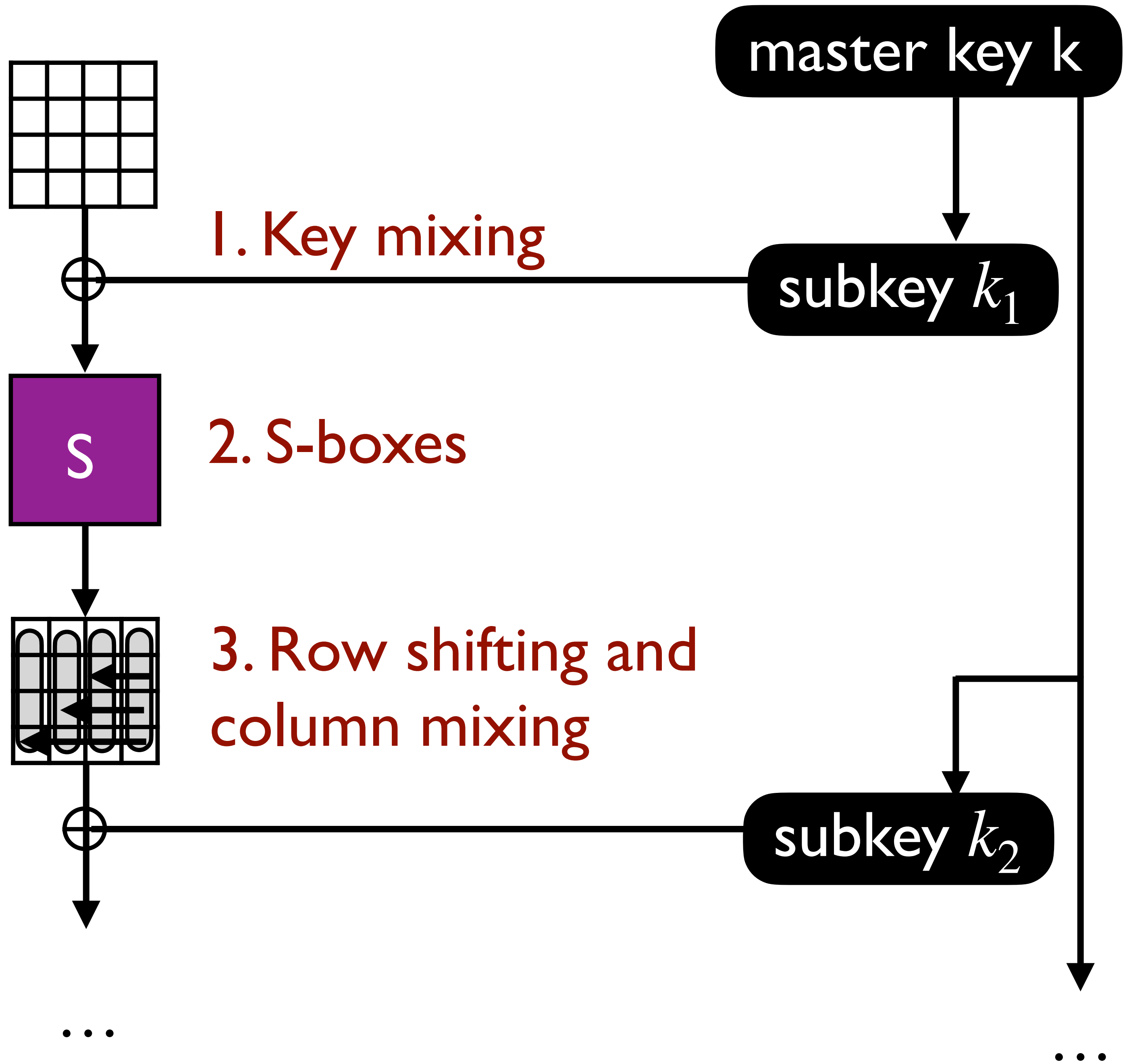
AES Overview

The AES permutation takes a 128-bit input represented as 4 x 4 matrix of bytes:

AES is basically a substitution-permutation network.



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