CMSC/Math 456: Cryptography (Fall 2022)

Lecture 17

Daniel Gottesman

Administrative

This week's problem set will be out by early next week, maybe earlier, and due two weeks from today (Thursday, Nov. 9).

Next week I will be away, and there will be guest lectures by Gorjan Alagic on quantum and post-quantum cryptography. This material is part of the course content and will potentially be on the final exam, but only qualitative aspects of it.

I will not have an office hour next Tuesday, but Mahathi's office hour will be on as usual unless she announces otherwise.

Message Authenticity

Suppose you receive this e-mail:

From: Daniel Eric Gottesman <dgottesm@umd.edu>

Subject: Important assignment

Please review the material on this website today.

Daniel Gottesman

What do you do?

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It seems a bit suspicious. What if you can't reach me to ask if it is real or not?

And if I sent a lot of e-mails like that, it might not even look suspicious.

Message Authentication

Message authentication is a cryptographic solution to this problem: It lets you verify that the message really came from me.

From: Daniel Eric Gottesman <dgottesm@umd.edu>

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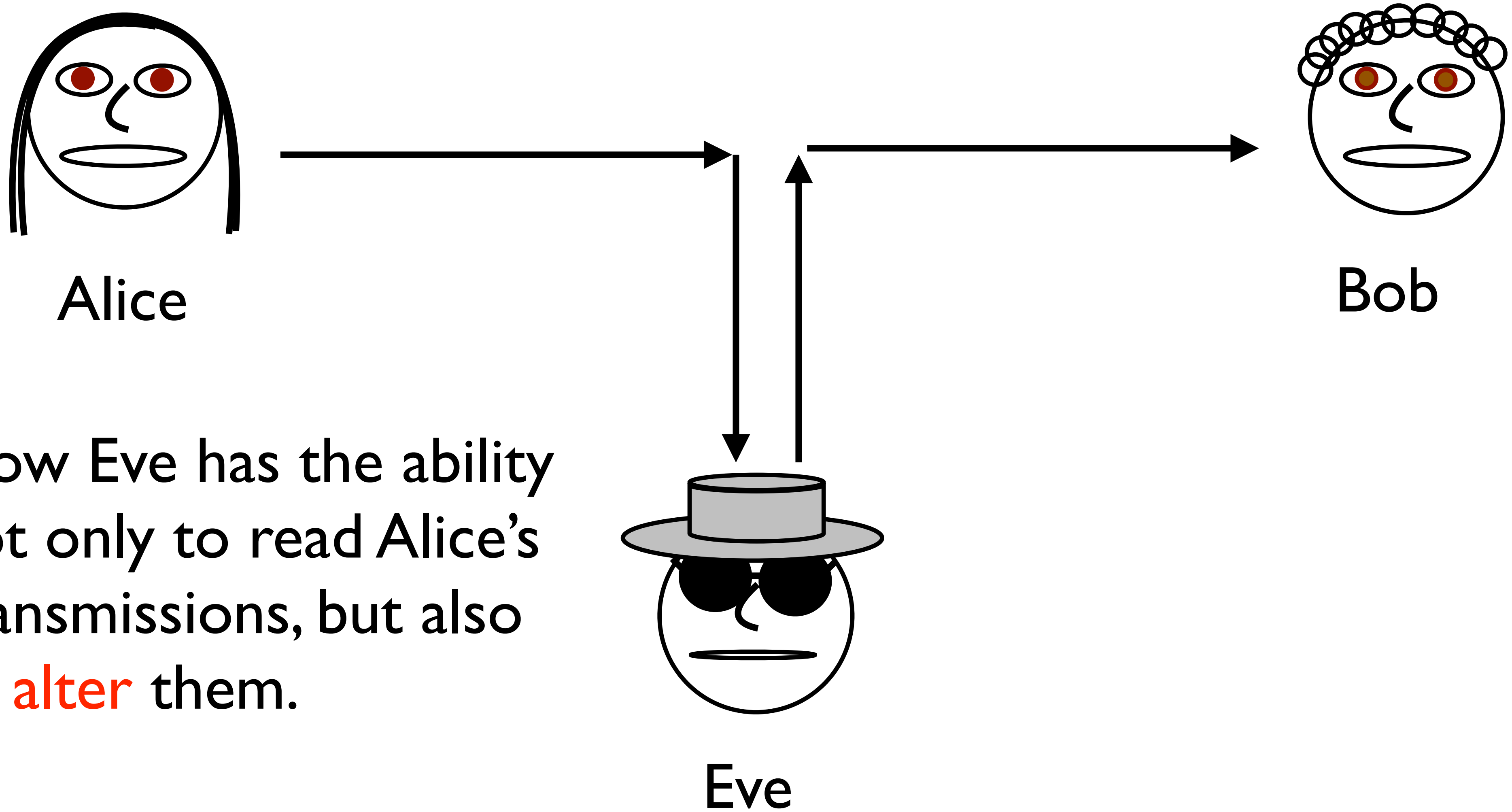
What if the message really did come from me … but it's been altered, perhaps by changing the link? Even a one-character change would point you to a different website which could contain malware. Asking me won't help, but message authentication addresses this too.

New Threat Model

In the threat model for message authentication, Eve is able to change messages sent between Alice and Bob and wants Bob to accept a message that Alice didn't send.



Now Eve has the ability not only to read Alice's transmissions, but also to alter them.

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Encryption Doesn't Help

Note that encryption by itself doesn't solve the problem. Eve can still change the message even if she can't read it.

One-time pad:

Key: 00101000101110101010

Message: 10111110010011001100

Ciphertext: 10010110111101100110

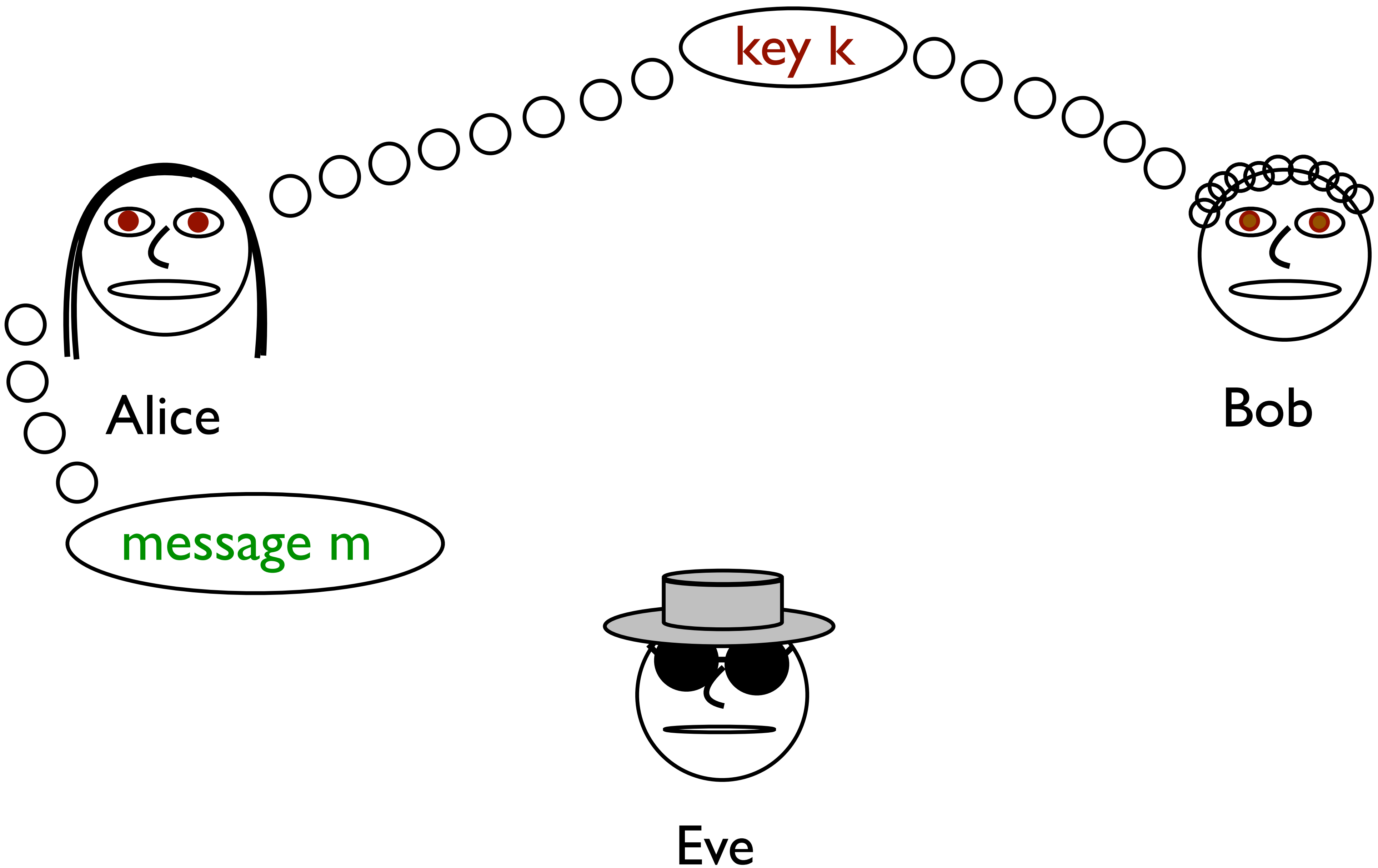
Alteration: 10010110111100100110

Decryption: 10111110010010001100

Changing the ciphertext produces a predictable change in the decrypted plaintext. (The one-time-pad is malleable.)

Message Authentication Code

Encryption is also not necessary. Instead, Alice appends a tag to her message to prove its authenticity. Alice and Bob share a secret key to give them an advantage over Eve.



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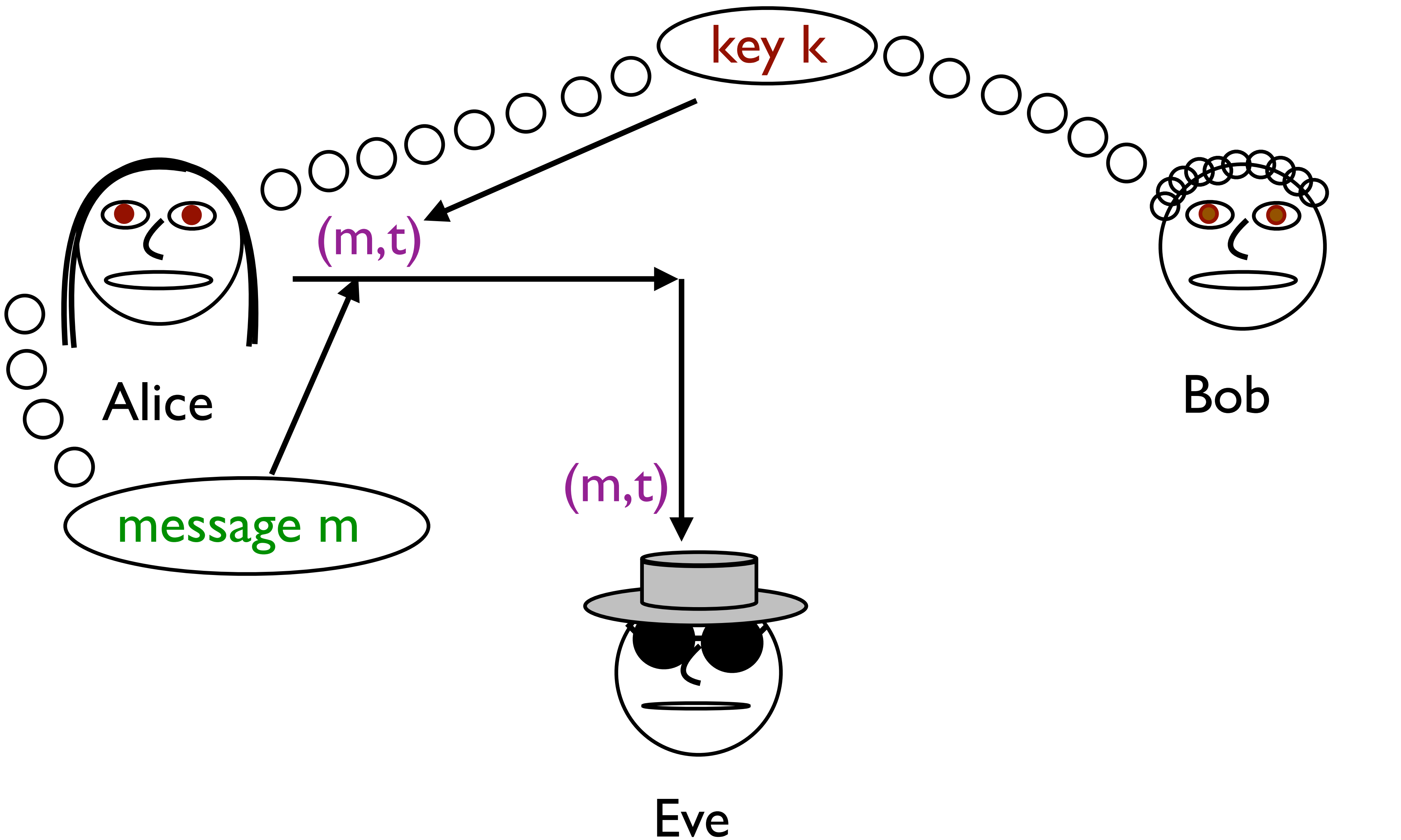

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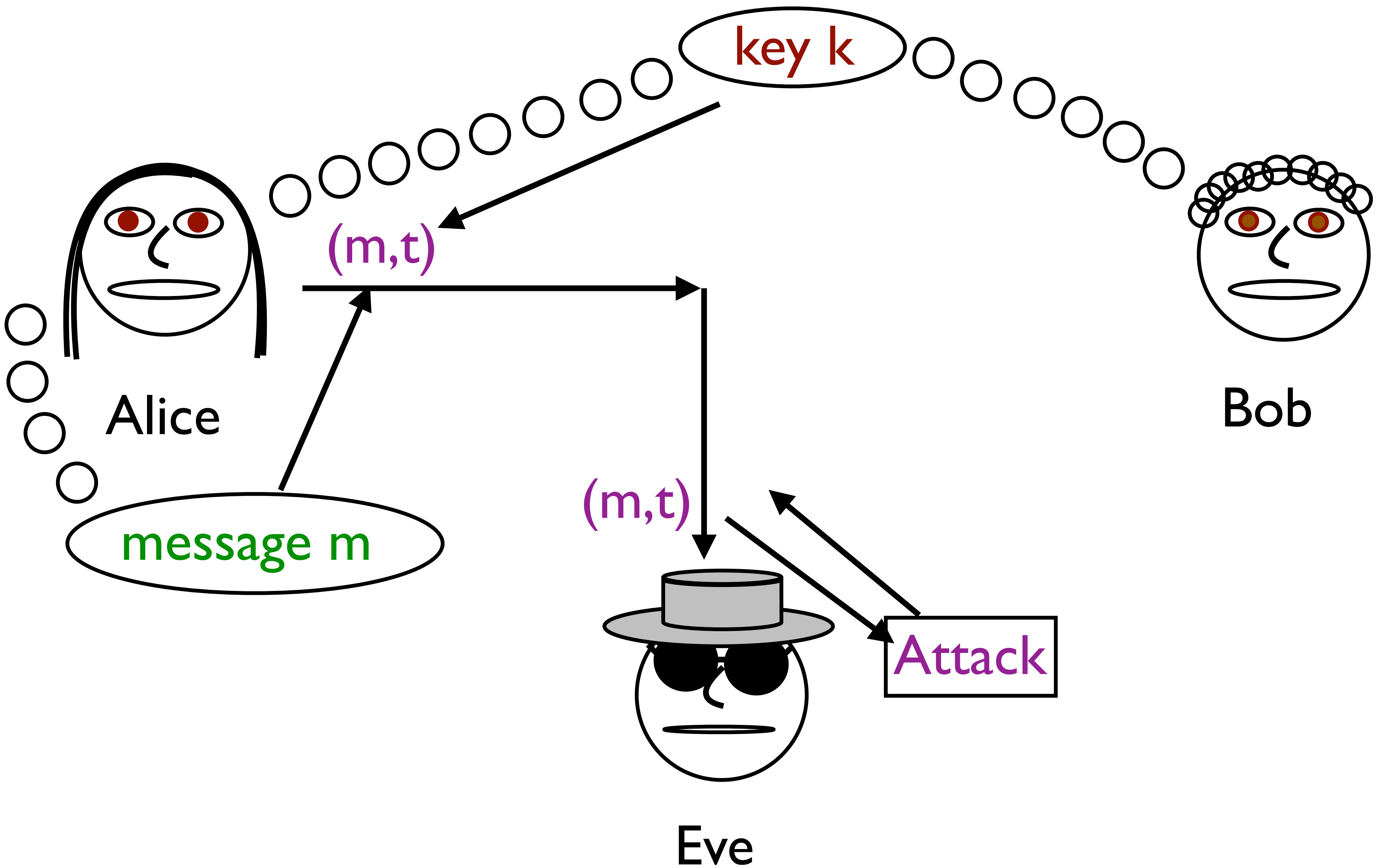

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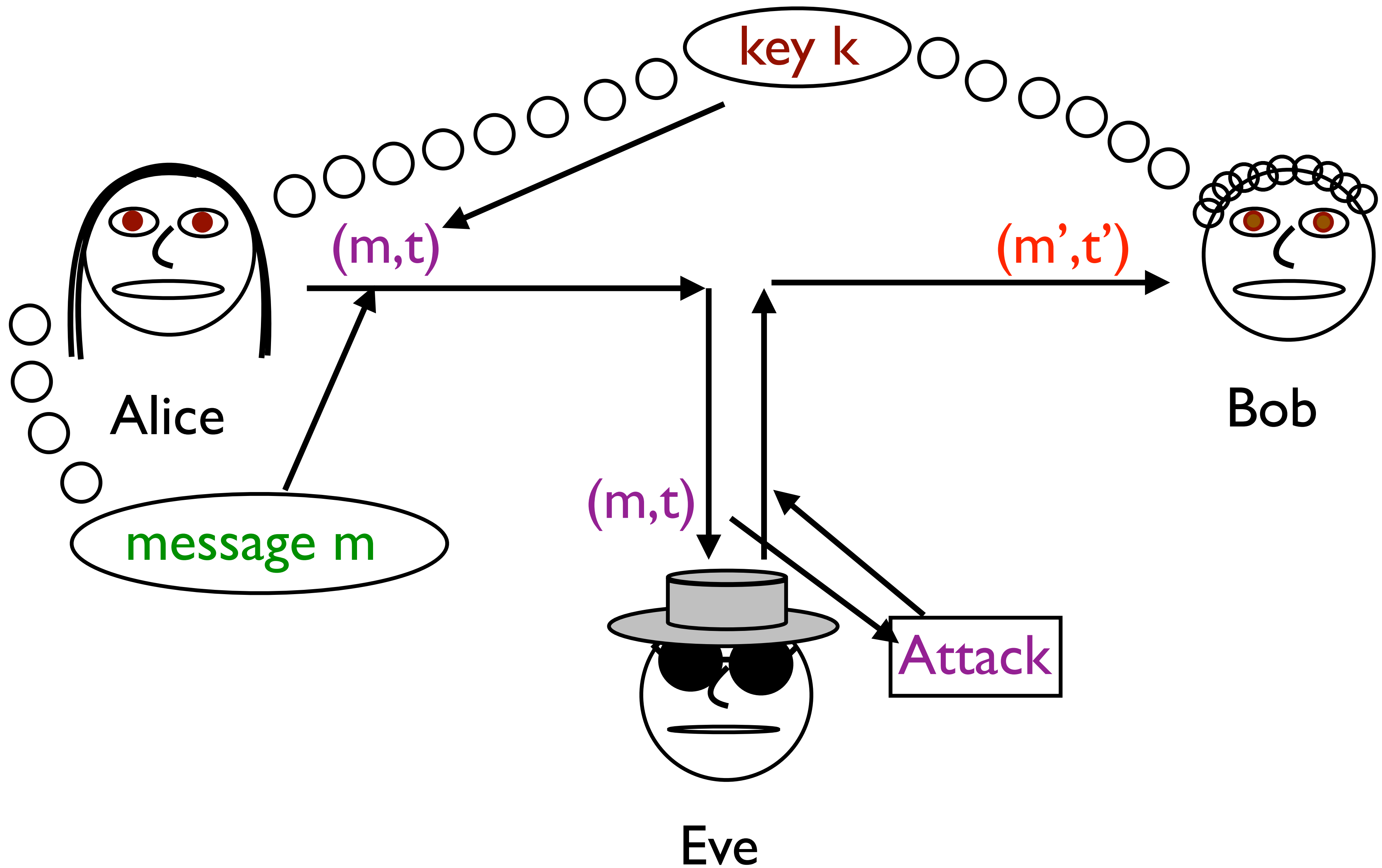

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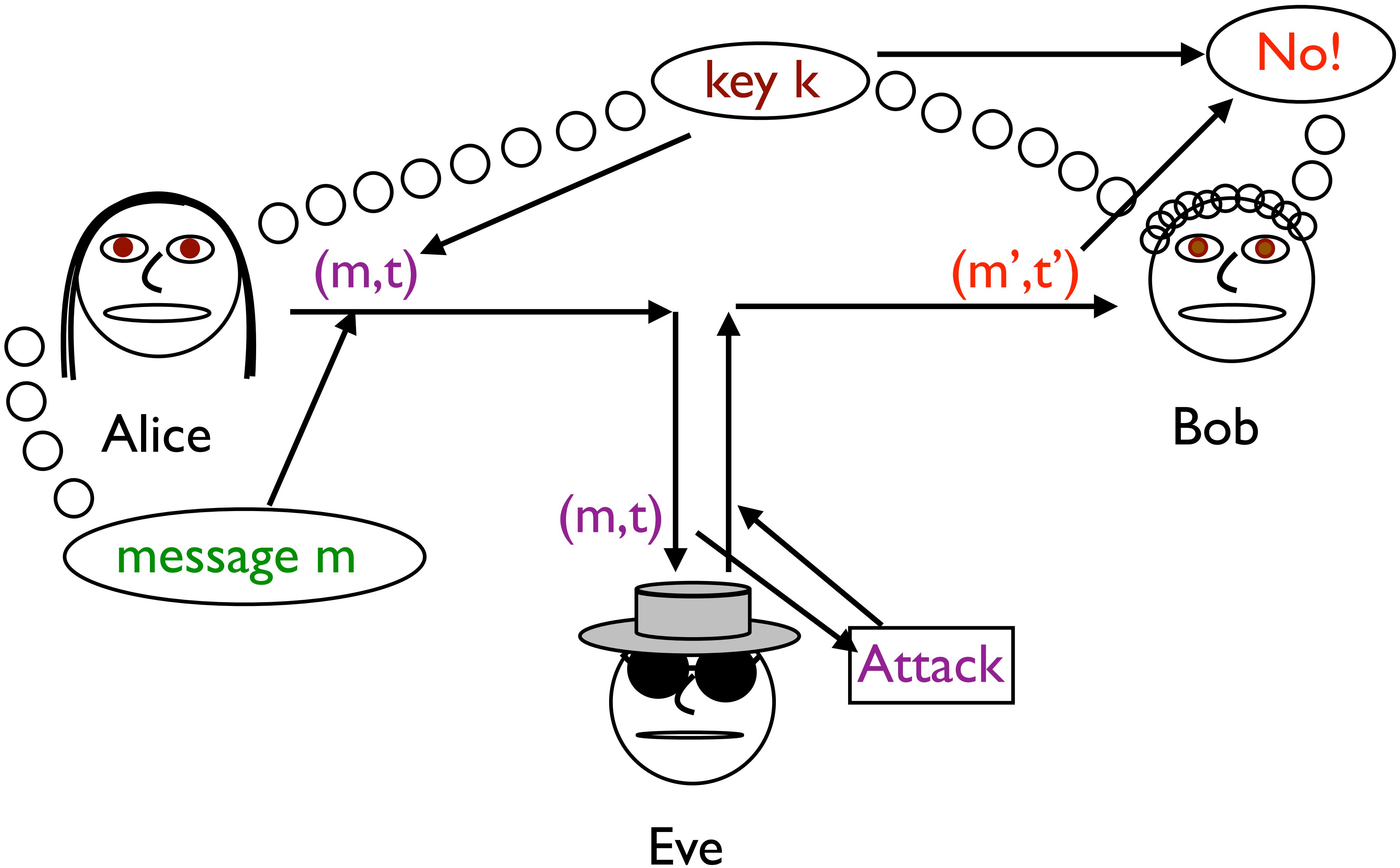

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MAC Definition

Definition: A message authentication code (MAC) is a set of three probabilistic polynomial-time algorithms (Gen, Mac, Vrfy):

Gen is the key generation algorithm. It takes as input s, the security parameter, and outputs a private key $k \in \{0,1\}^*$ of length poly(s).

Mac is the tag-generation algorithm. It takes as input k and a message $m \in \{0,1\}^*$ and outputs a tag $t \in \{0,1\}^*$.

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Often Vrfy just runs Mac(k,m) to get a tag t' and outputs "valid" if t=t'.

Private Key vs. Public Key

A MAC is a *private key* protocol. That means that Alice and Bob must share the private key and can only use it to authenticate messages to each other.

If you want a *public key* authentication protocol, use a digital signature, which is basically a public key version of a MAC. However, they have some useful additional properties by virtue of being public key.

We will discuss digital signatures later in the course.

First Try

Suppose we let t be the parity of the bits in the message m (i.e., the XOR of all bits, 0 if m has an even number of 1s).

m = 001101011 ⇨ t=1

Vote: Does this work? (Yes/no/unknown)

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Answer: No. It has a number of problems.

- Eve knows the procedure and can easily make her own tags.

E.g.: m = 000000000, t=0

We need to involve a key somehow.

Second Try

What if we instead have a key k and the tag is $t = m \cdot k$?

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Answer: Still no.

- The tag space is too small. Eve can just guess the tag with 50% chance of success.
- The all 0's message always has the same tag, so Eve can forge that:

  m = 000000000, t=0

- With multiple messages, Eve can quickly determine k and then she can forge messages easily.

Pseudorandom Functions

What we would really want is that the tag t = Mac(k,m) is a random string unrelated to t' = Mac(k,m'). That way, seeing some tags won't help in forging different messages.

That means we want a random function for Mac.

But random functions need a long key and can't be computed efficiently. So we will use the next-best thing: a pseudorandom function.

$\mathrm{Mac}(k, m) = F_k(m)$ for pseudorandom $F_k(m)$.

E.g.: $F_k(m)$ could be AES.

Note that we don't need a pseudorandom *permutation* here, only a pseudorandom *function*. Mac does not need to be invertible.

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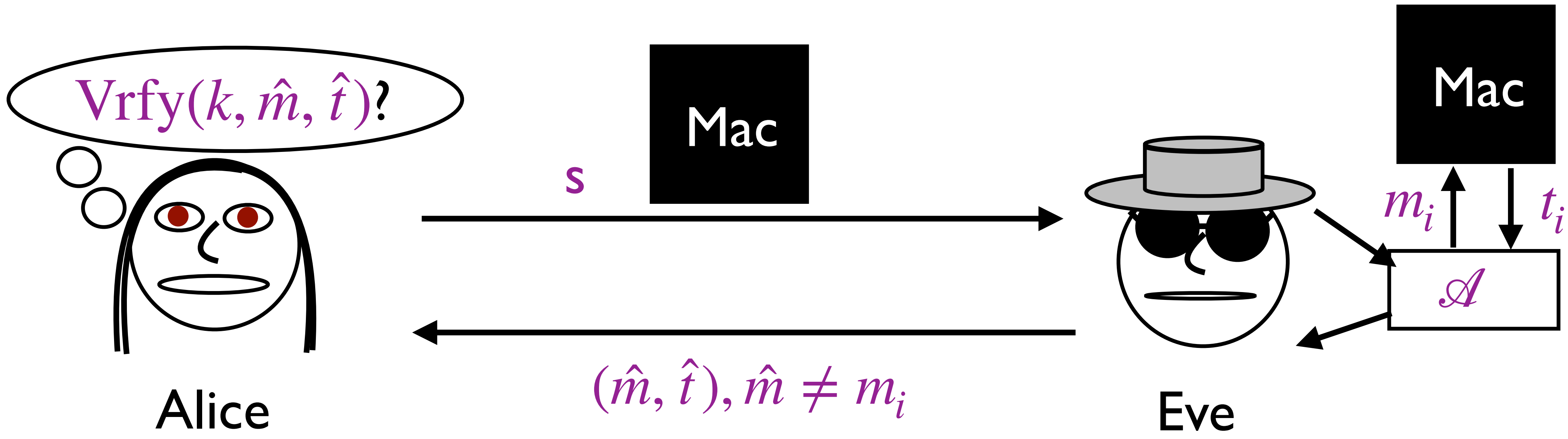
Time for another game!

MAC Security Game



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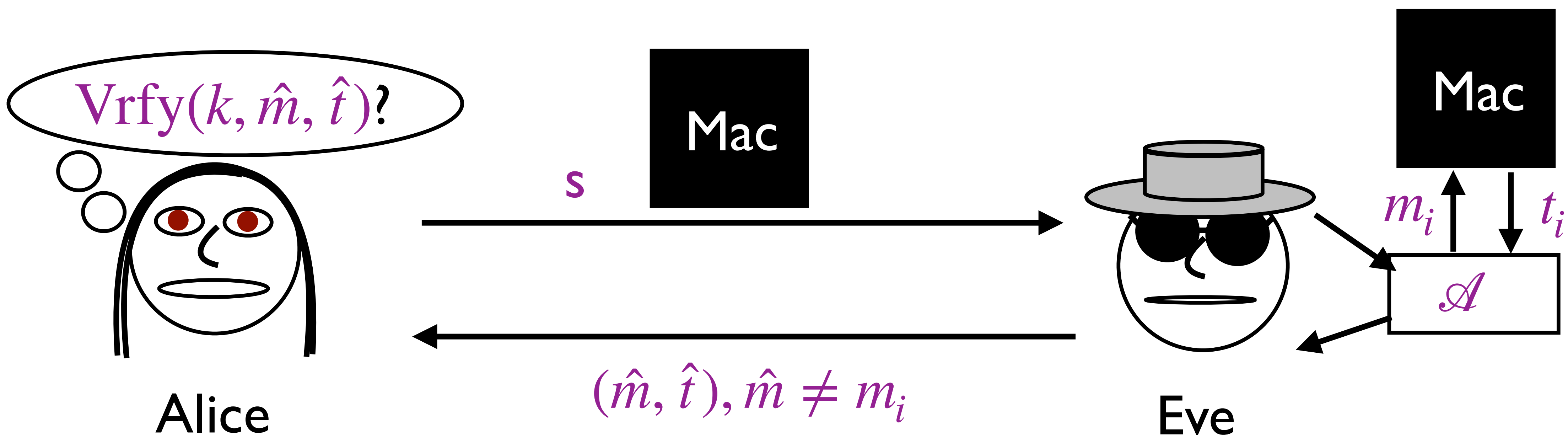

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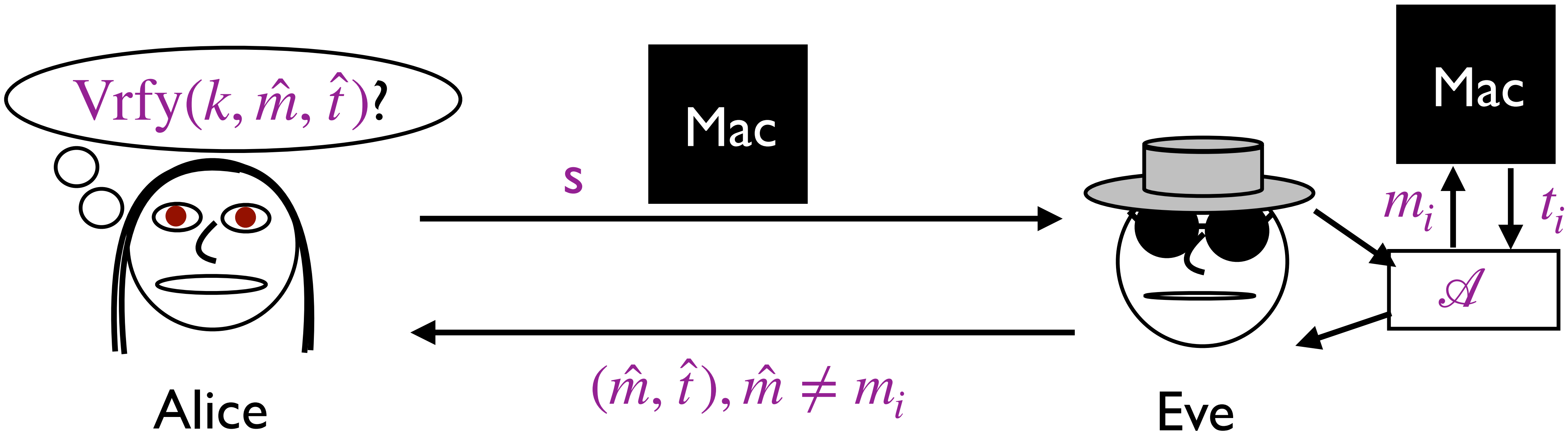

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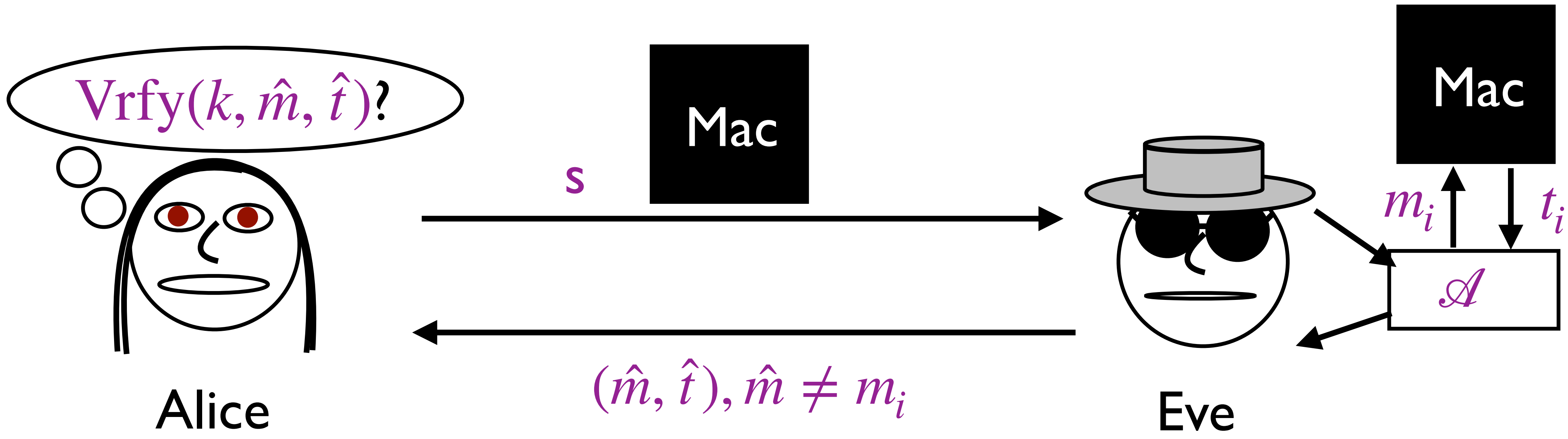

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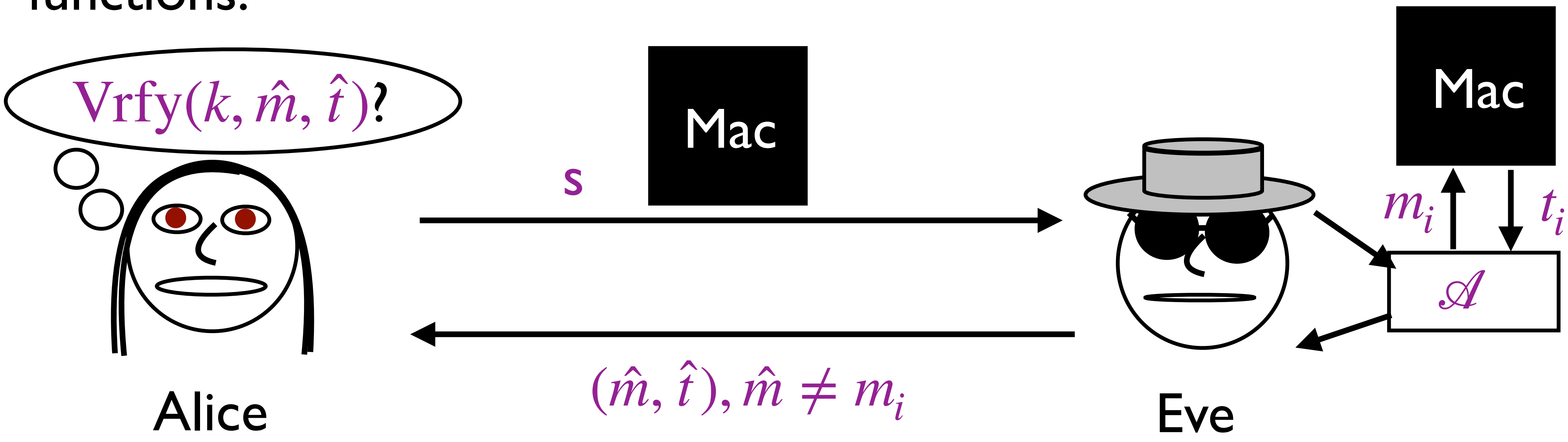
3. Eve wins if $\mathrm{Vrfy}(k, \hat{m}, \hat{t}) =$ valid **and** $\hat{m} \neq m_i$ for any m_i which she used to query the oracle.

MAC Security Definition

Definition: A MAC (Gen, Mac, Vrfy) with security parameter s is secure (against an adaptive chosen-message attack) if, for any polynomial-time attack $\mathscr{A}$ with oracle access to $M_k(m) = \mathrm{Mac}(k, m)$, where $\mathscr{A}$ outputs $(\hat{m}, \hat{t})$ such that $\mathscr{A}$ never queried the oracle for $m = \hat{m}$,

$$\Pr(\mathrm{Vrfy}(k, \hat{m}, \hat{t}) = \mathrm{valid}) \leq \epsilon(s)$$

where $\epsilon(s)$ is a negligible function and the probability is averaged over k generated by Gen and the randomness used in any of the functions.



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Security of MAC w/ PRF

Theorem: If $F_k(m)$ is a pseudorandom function then the following MAC is secure:

Gen chooses a uniform random bit string k.

$\mathrm{Mac}(k, m) = F_k(m)$.

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Then show, via reduction, that if we have an attack against the protocol with a pseudorandom function, then we can distinguish $F_k(m)$ from a random function.

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Which, by the definition of pseudorandom function, implies the protocol is secure.

MAC with Random Function

Lemma: The following MAC is secure:

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She chooses an $\hat{m} \neq m_i$. But f(m) is uncorrelated with f(m') for $m \neq m'$. In particular, $f(\hat{m})$ is a random string uncorrelated with all $f(m_i)$ and all of Eve's other data. Therefore, the probability that her attack outputs $\hat{t} = f(\hat{m})$ is

$$\Pr(\mathscr{A}^f = (\hat{m}, \hat{t} = f(\hat{m}))) = 2^{-s}$$

where f has outputs of length s bits.

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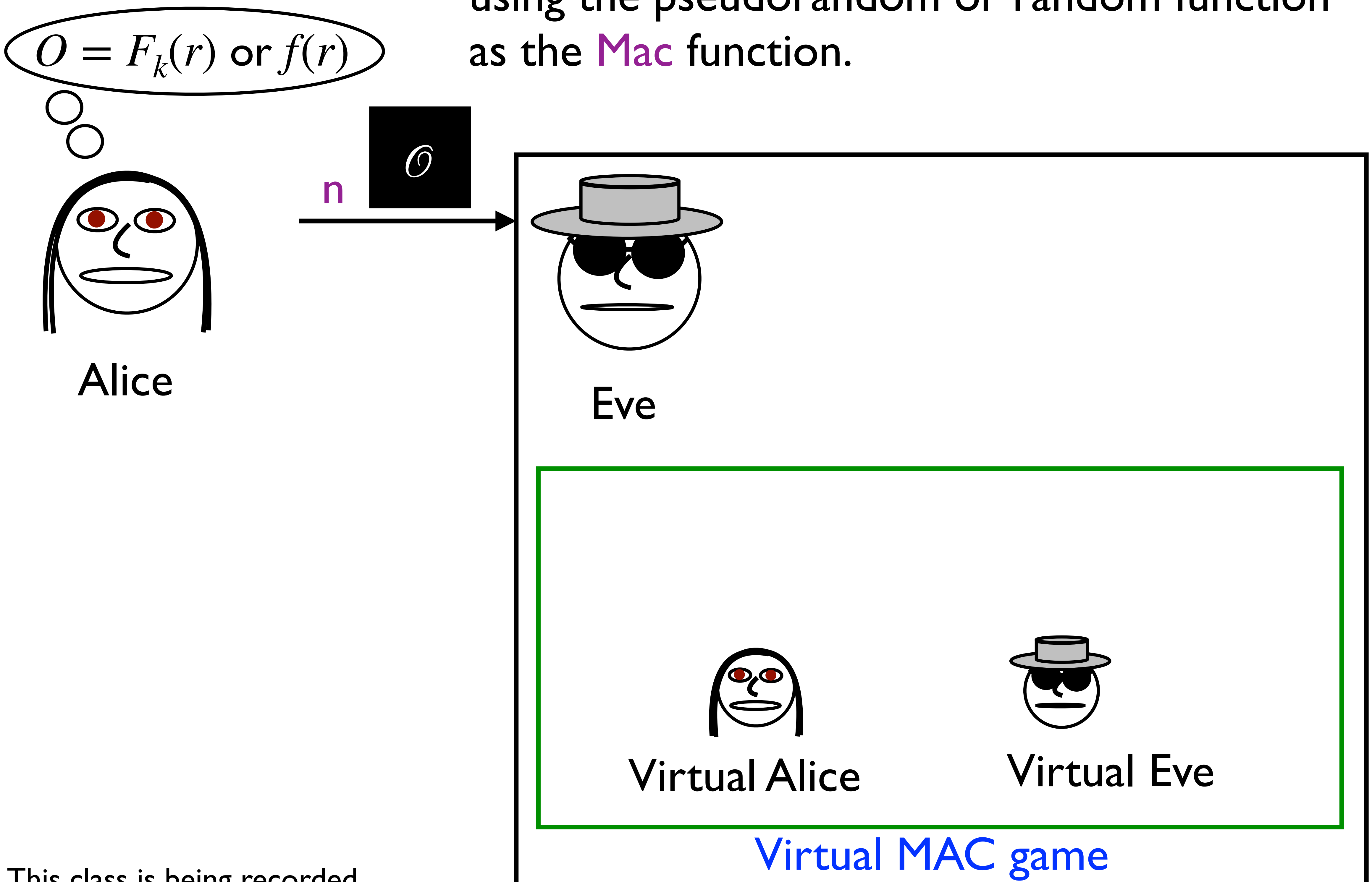
- Given function f, we run a virtual MAC security game.
- If f is random, the MAC is secure by the lemma, so Virtual Eve is likely to lose. Thus, if Eve loses: guess random.
- If $f = F_k$, the attack on the MAC succeeds, so Eve is likely to win. Thus, if Eve wins: guess pseudorandom.

Reduction to Pseudorandom Function

The reduction runs a virtual MAC protocol using the pseudorandom or random function as the Mac function.



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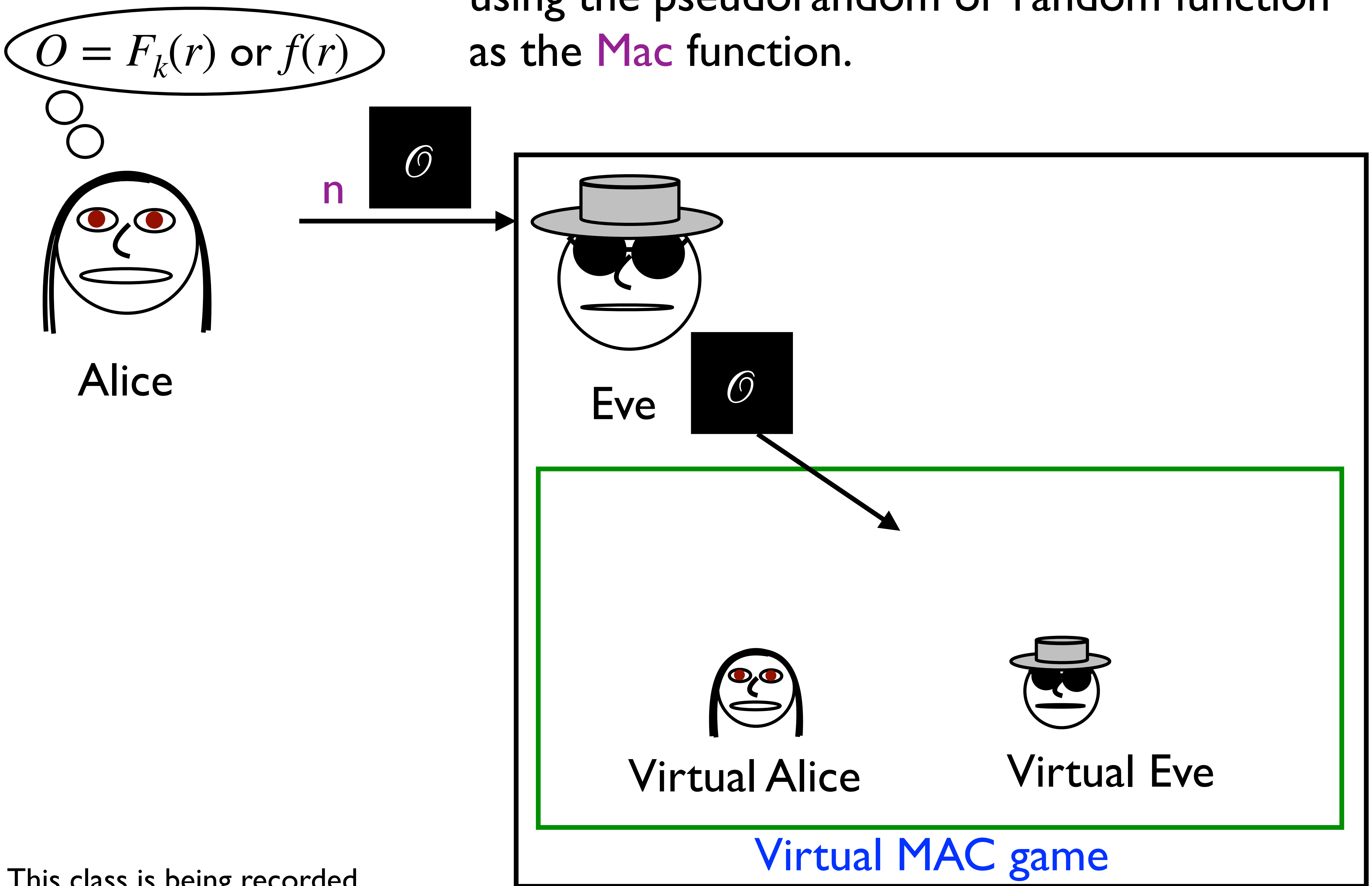

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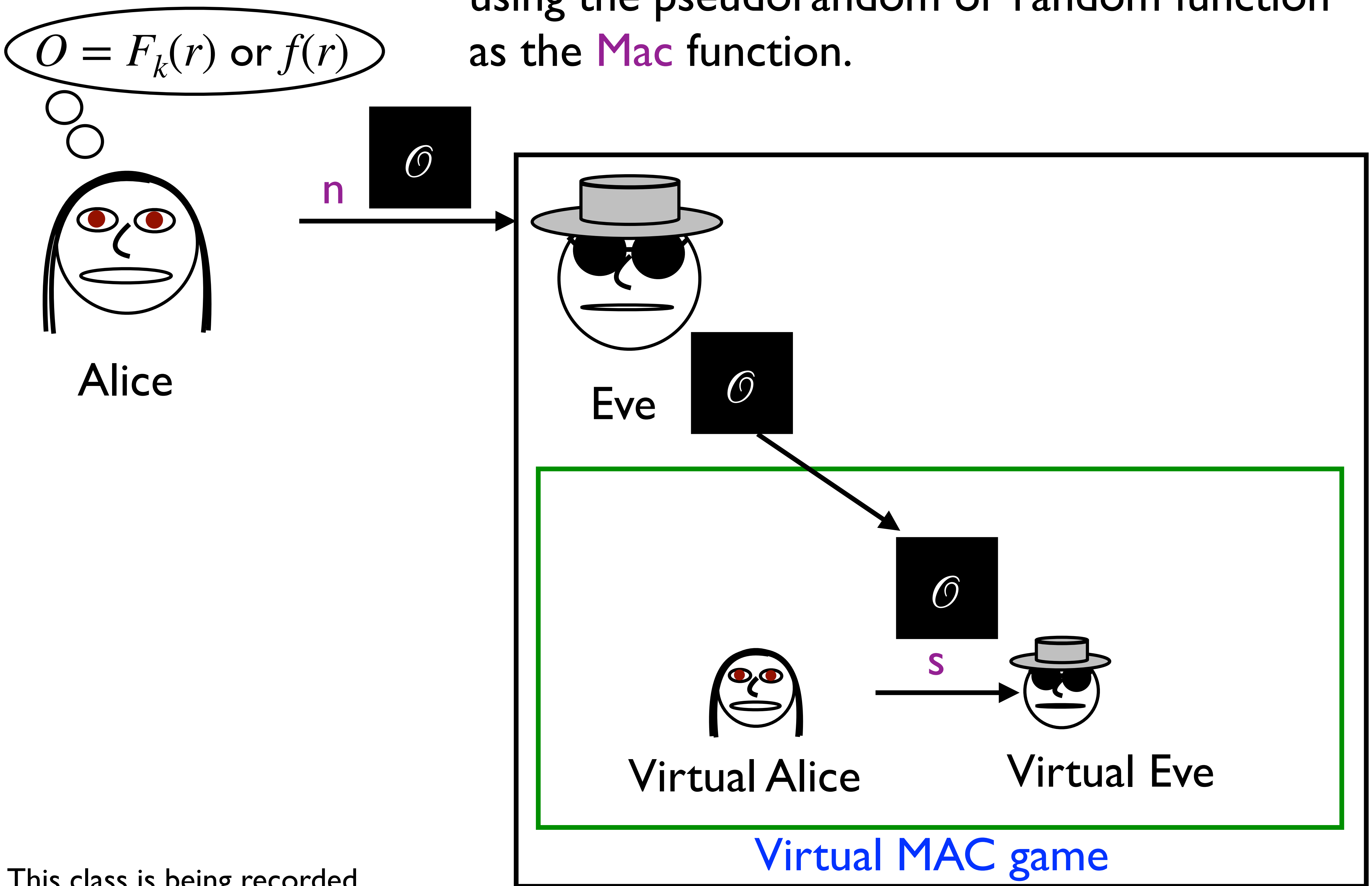

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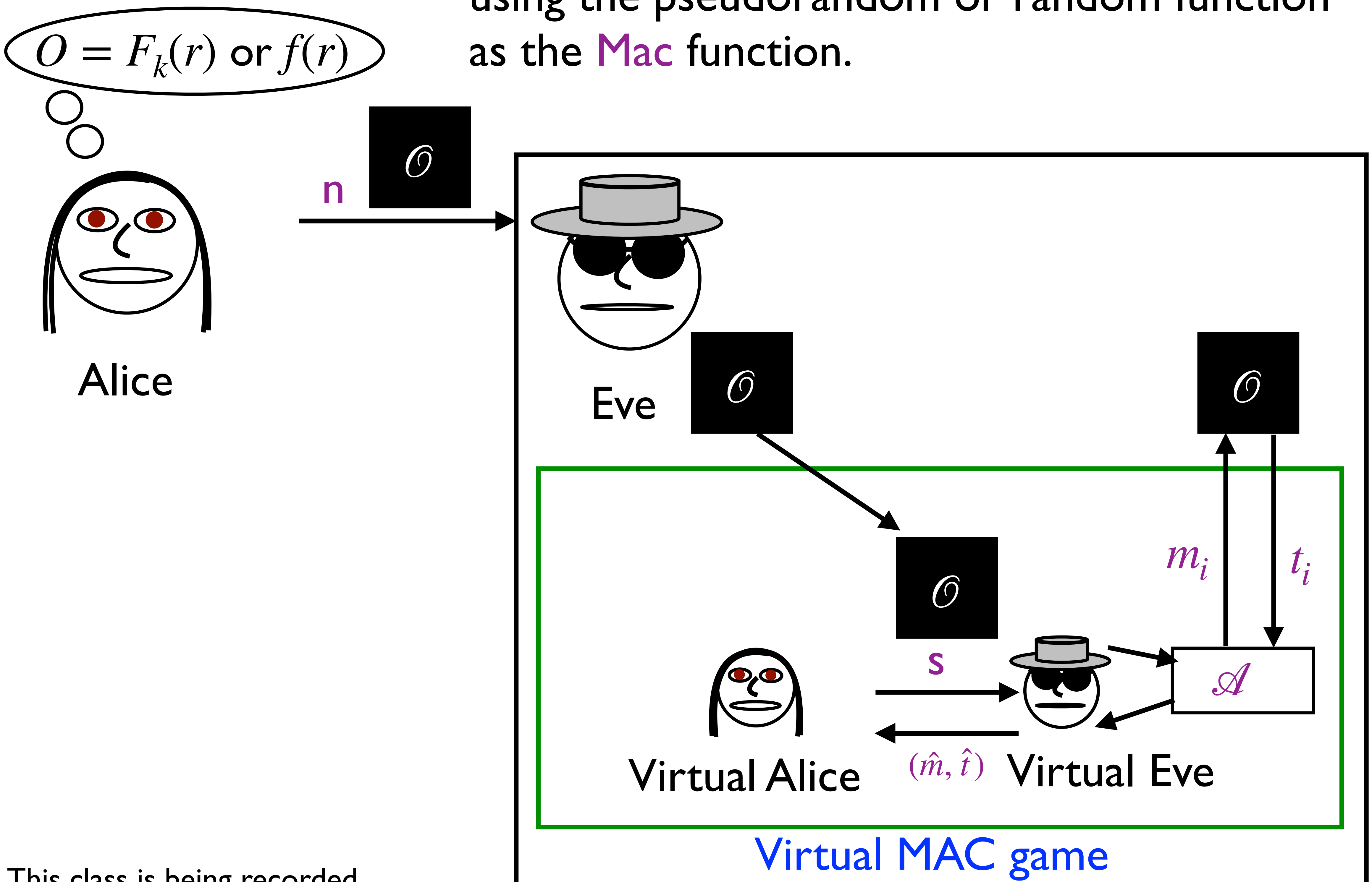

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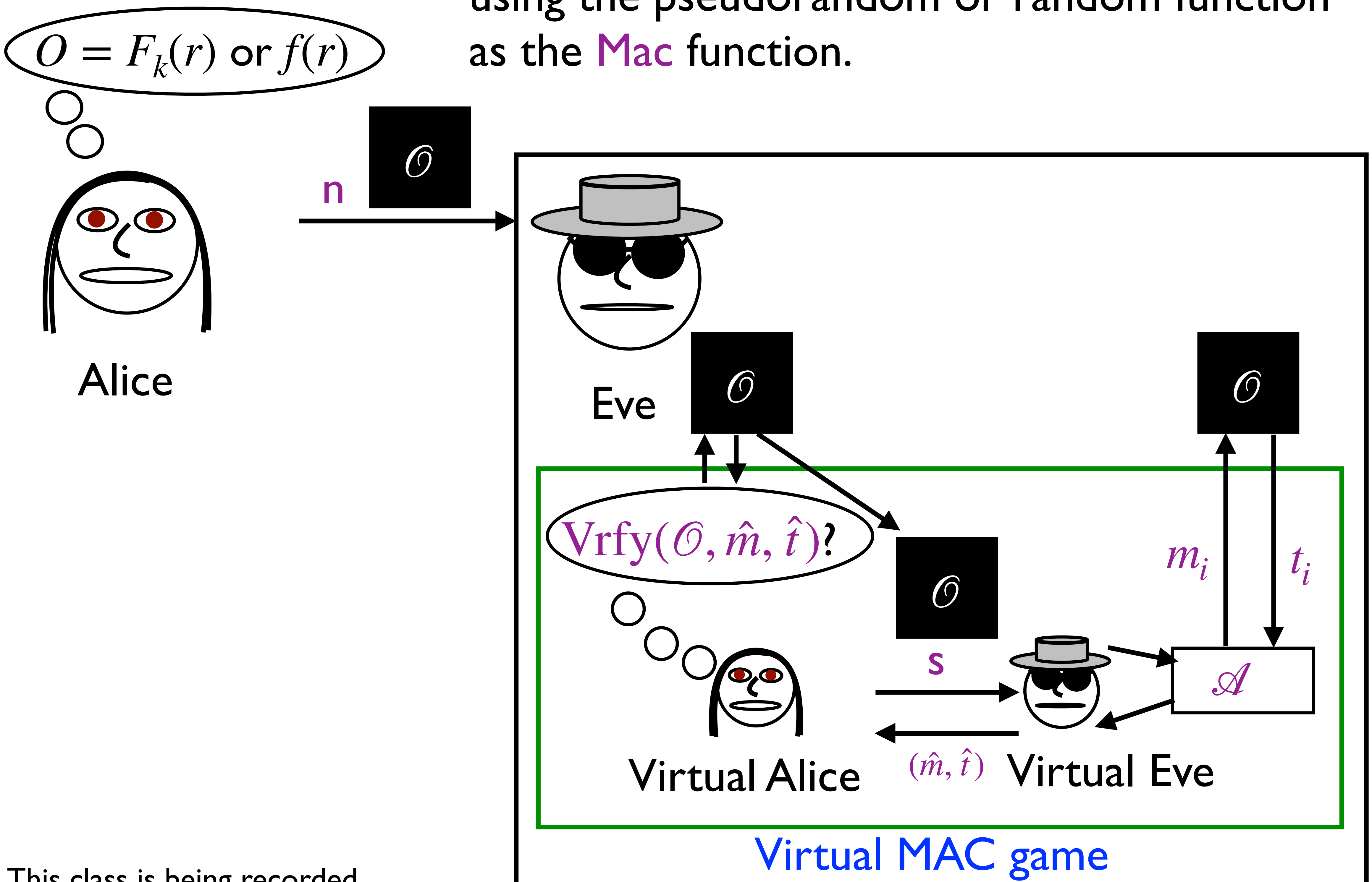

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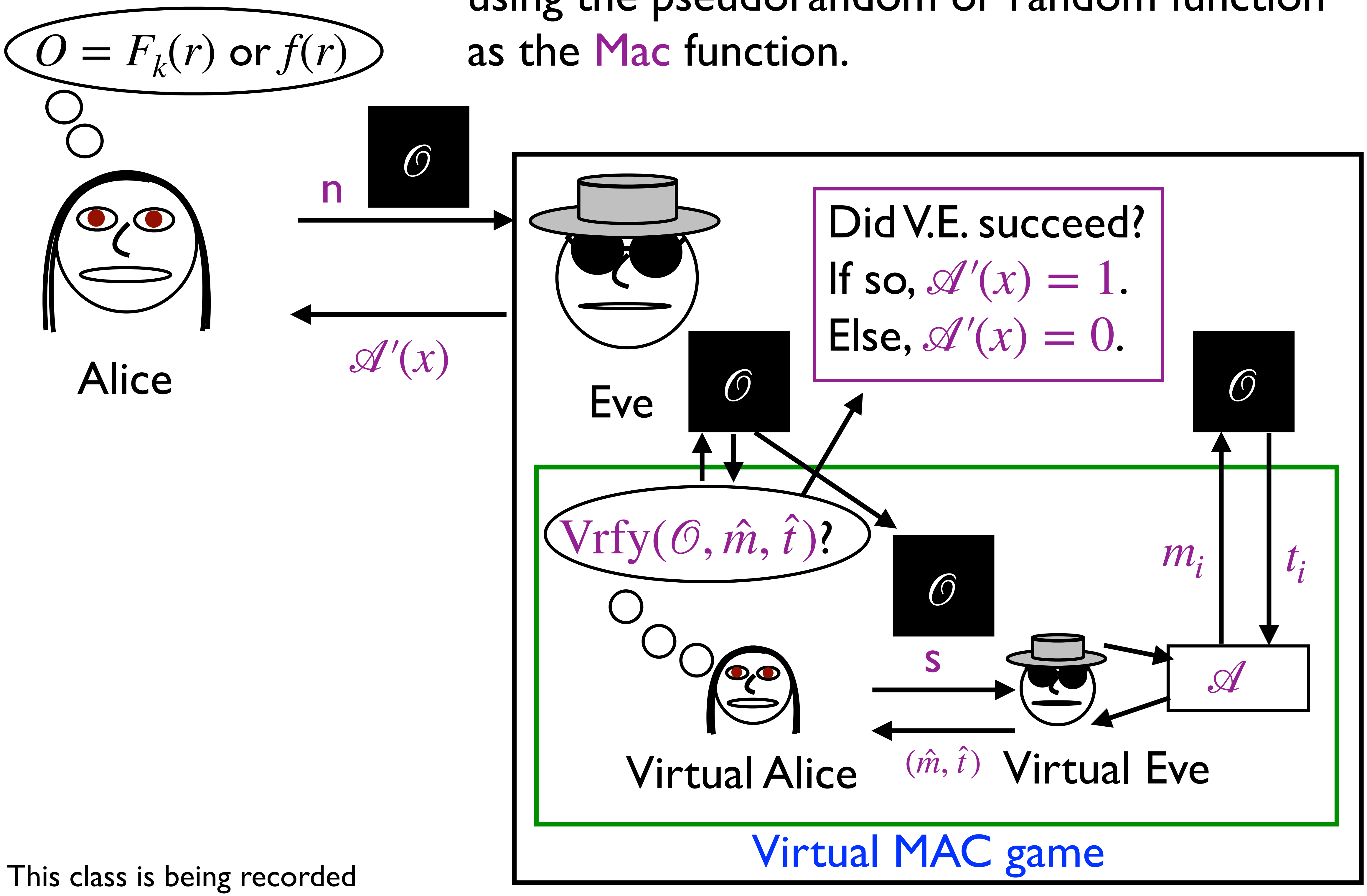

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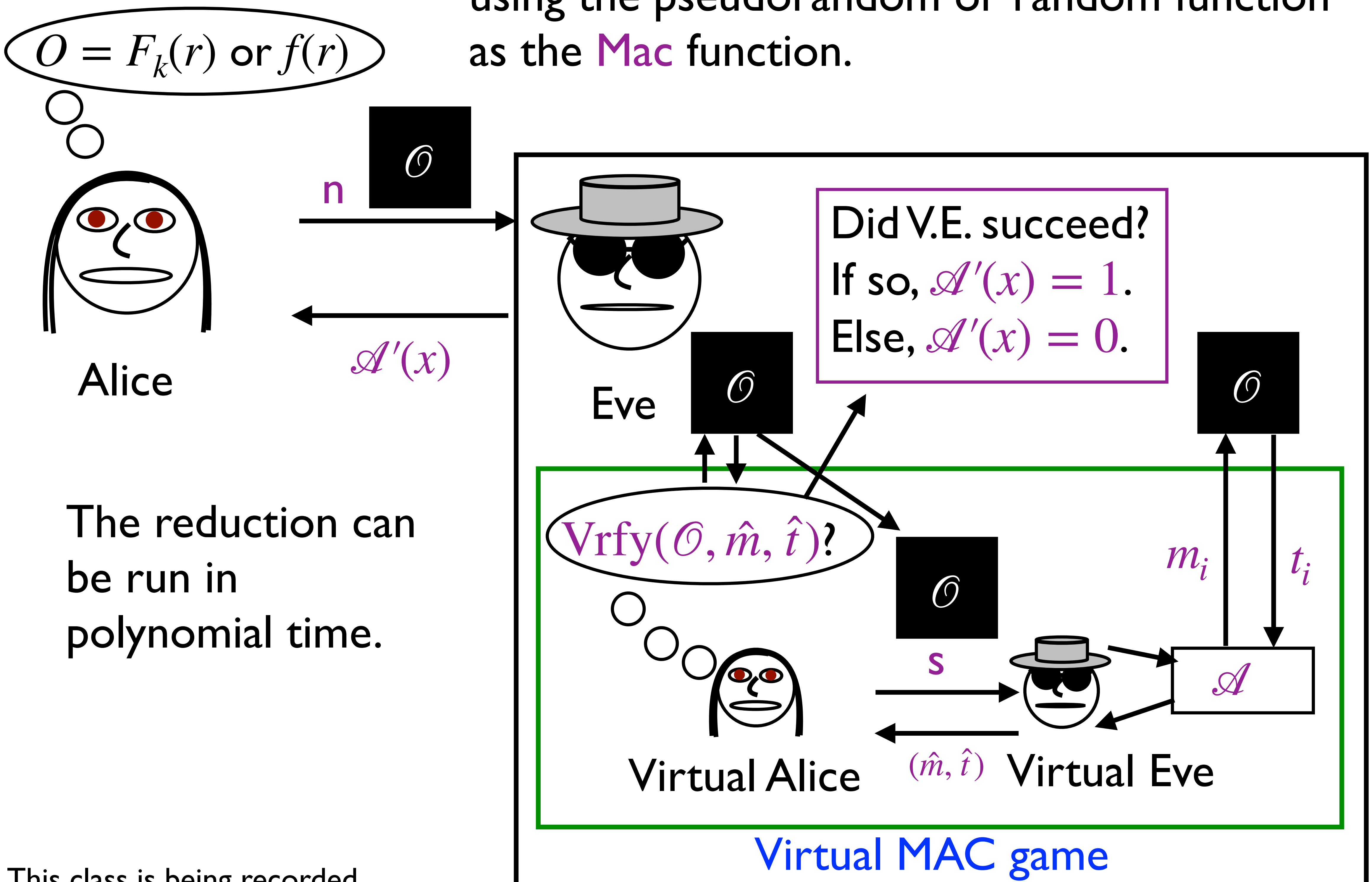

The reduction can be run in polynomial time.

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Thus,

$$|\Pr(\mathscr{A}'^{F_k} = 1) - \Pr(\mathscr{A}'^f = 1)| = |\Pr(\text{Vrfy}(k, \hat{m}, \hat{t}) = \text{valid}) - 2^{-s}|$$

Completing the Proof

In this reduction, the real Eve outputs $\mathscr{A}' = 1$ iff virtual Eve succeeds in outputting a correct message-tag pair $(\hat{m}, \hat{t})$.

- If the function is random, this happens with probability 2^{-s} by the lemma. That is,

$$\Pr(\mathscr{A}'^{f} = 1) = 2^{-s}$$

- If the function is pseudorandom, virtual Eve succeeds with probability $\Pr(\text{Vrfy}(k, \hat{m}, \hat{t}) = \text{valid})$:

$$\Pr(\mathscr{A}'^{F_k} = 1) = \Pr(\text{Vrfy}(k, \hat{m}, \hat{t}) = \text{valid})$$

Thus,

$$|\Pr(\mathscr{A}'^{F_k} = 1) - \Pr(\mathscr{A}'^{f} = 1)| = |\Pr(\text{Vrfy}(k, \hat{m}, \hat{t}) = \text{valid}) - 2^{-s}|$$

But, by the definition of a pseudorandom function,

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Longer Messages

Standardized block ciphers have a fixed size. So how do we authenticate longer messages?

Break m up into blocks: $m = m_0 \| m_1 \| m_2 \| \ldots$ and authenticate each one separately: $(m_0, t_0), (m_1, t_1), (m_2, t_2), \ldots$

Vote: Secure? (Yes/No/Unknown)

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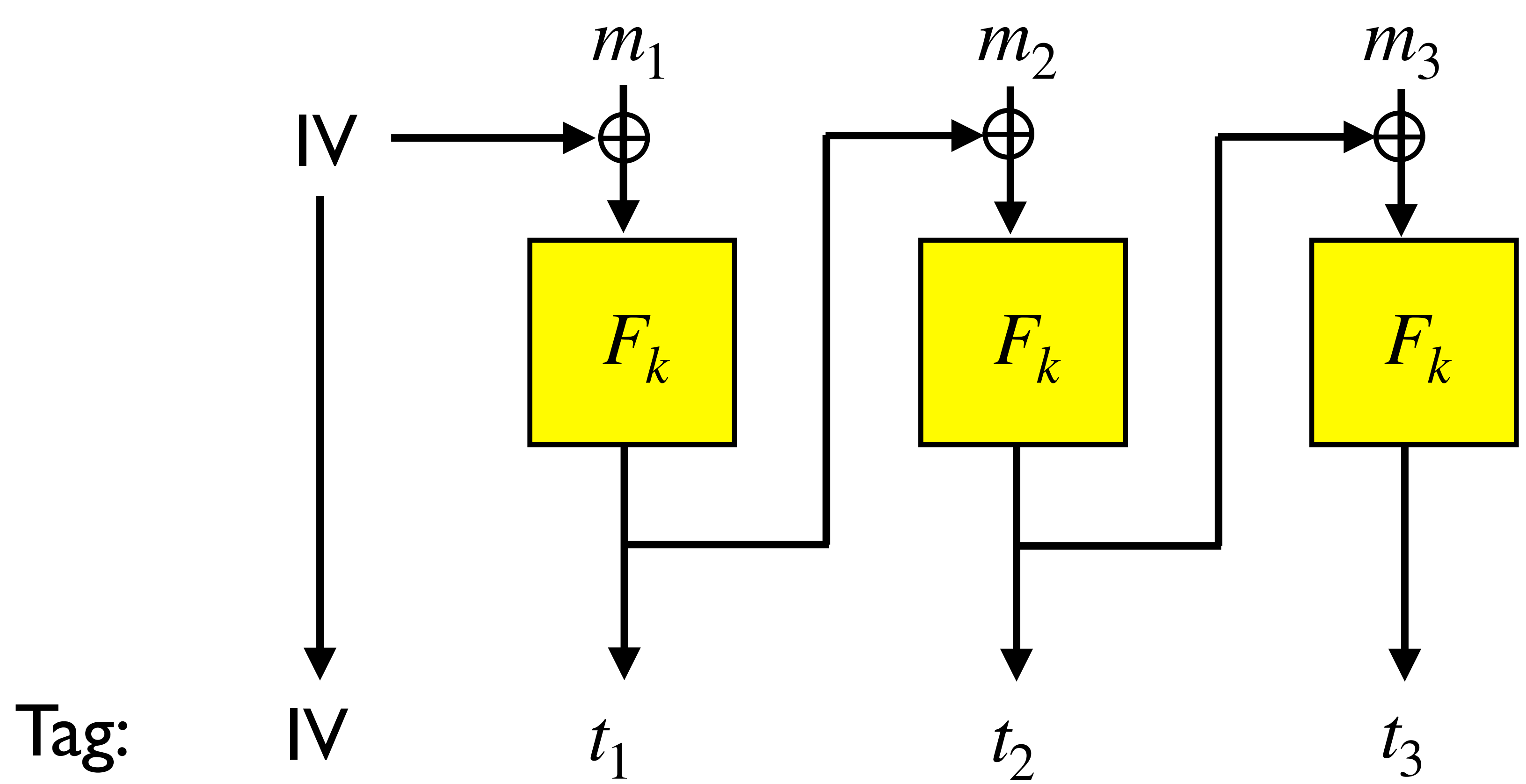
Answer: No. Eve has various attacks.

- Could change the order: $(m_2, t_2), (m_1, t_1), (m_0, t_0), \ldots$ is a valid set of tags for the message $m_2 \| m_1 \| m_0 \| \ldots$
- Could truncate: (m_0, t_0) by itself is a valid tag for the message m_0.
- Could switch blocks from multiple messages: Given (m_i, t_i) and (m_i', t_i'), she could send $(m_0, t_0), (m_1', t_1'), (m_2, t_2), \ldots$, which is a valid set of tags for $m_0 \| m_1' \| m_2 \| \ldots$ This is different from either of the original messages.

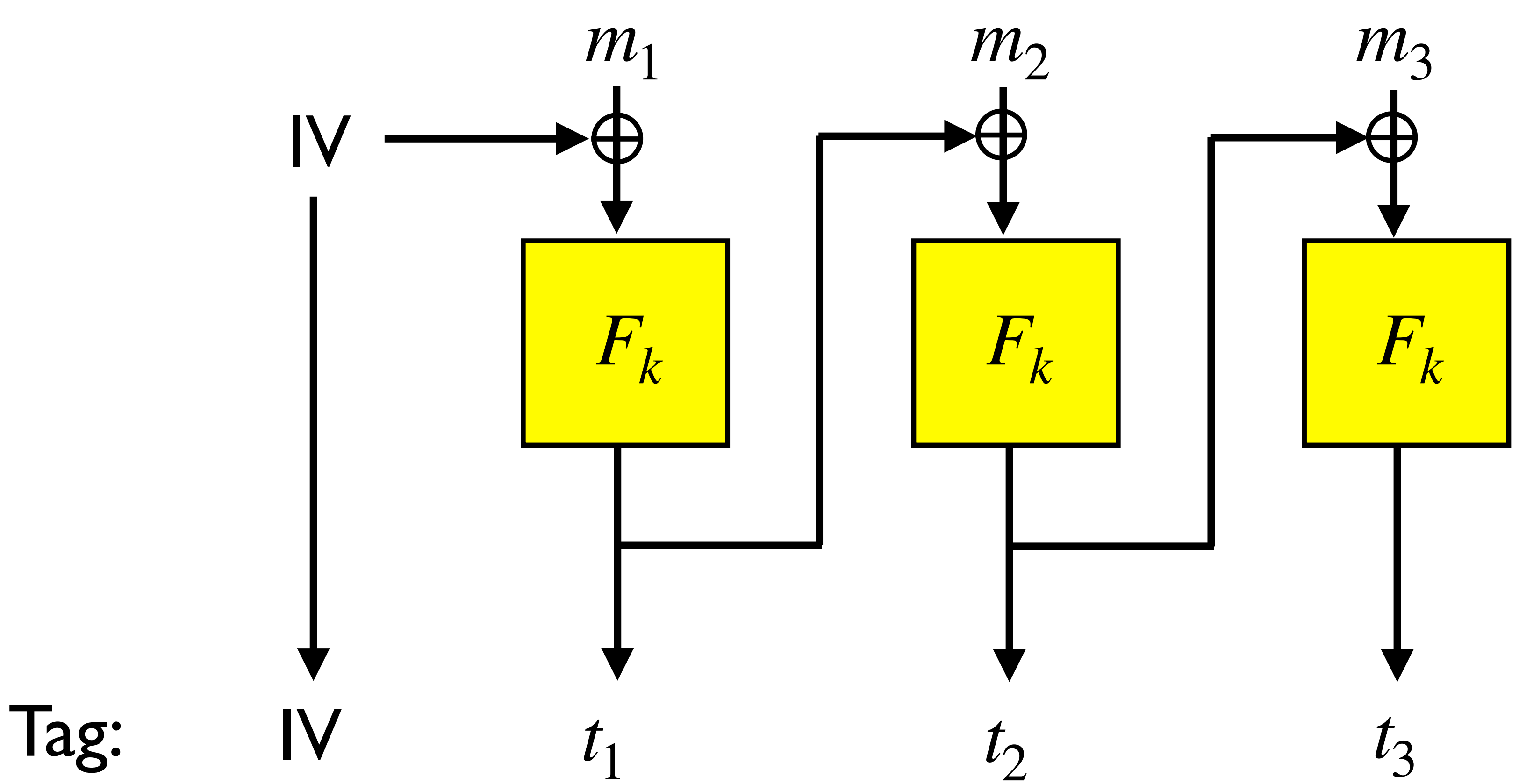
CBC Mode for MACs

What about CBC mode for MACs?



Vote: Secure? (Yes/No/Unknown)

CBC Doesn't Work for MACs



Not secure. Eve can change the IV *and* m_1 with it to leave the tags the same:

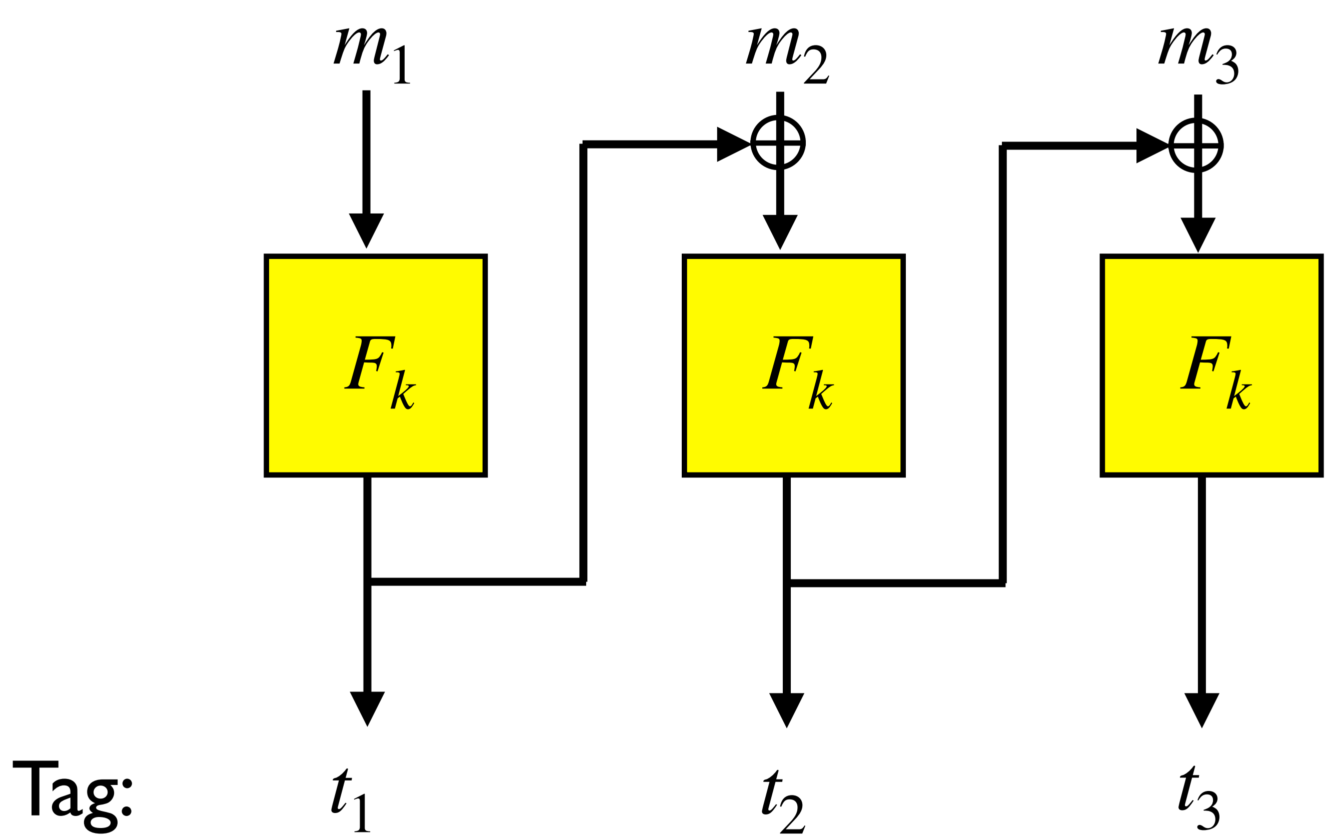
E.g.: IV = 010100, $m_1 = 110000$, so $IV \oplus m_1 = 100100$

But if IV = 110001, $\hat{m}_1 = 010101$, then it is still true that $IV \oplus \hat{m}_1 = 100100$, so the message $\hat{m}_1 \| m_2 \| m_3$ has the same tag as the original message $m_1 \| m_2 \| m_3$.

This class is being recorded

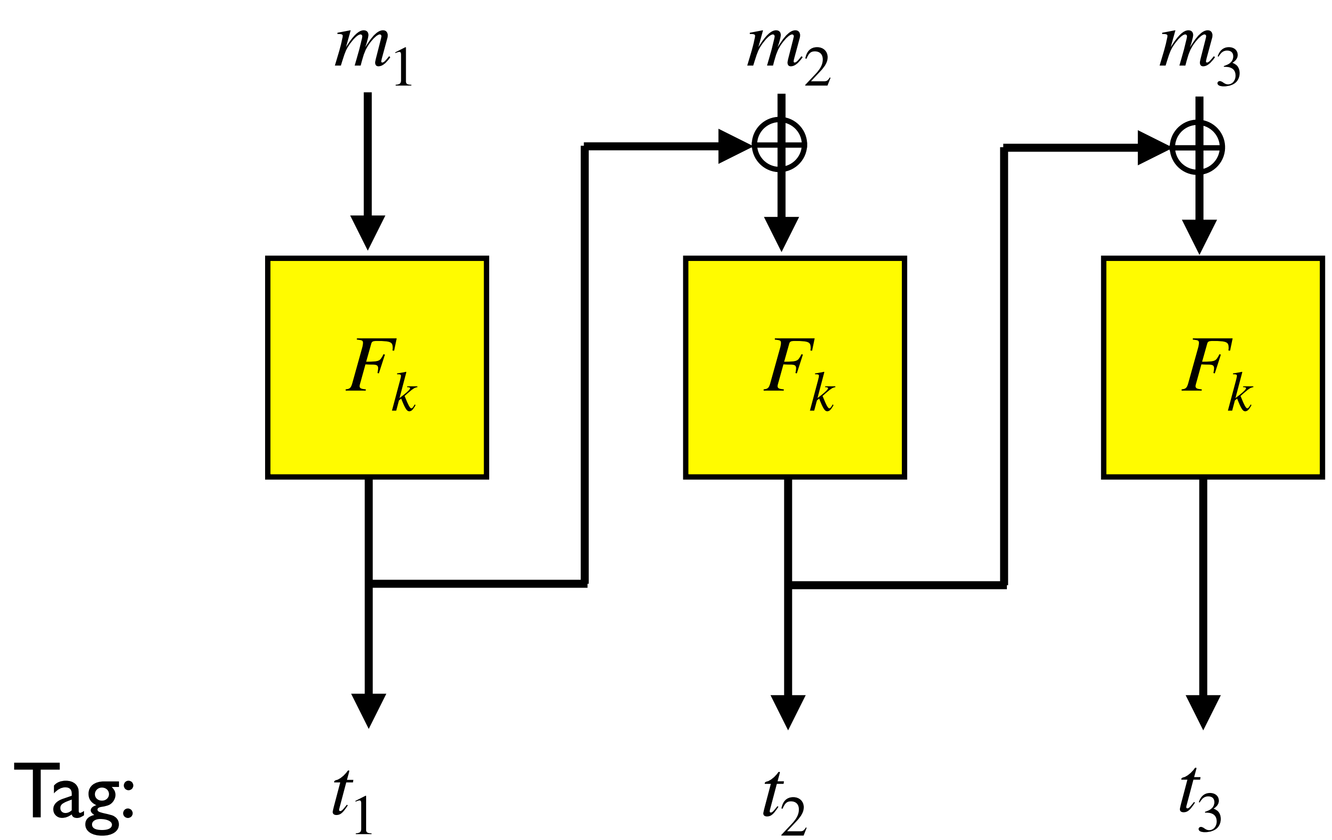
CBC for MACs with no IV

The IV is causing this problem since Eve can change it and m_1 together. But we don't need it: For encryption, we needed randomness to ensure CPA security, but MACs don't need that.



Vote: Secure? (Yes/No/Unknown)

It Still Doesn't Work



Not secure. Eve can still combine messages to make a new one:

E.g.: Given $(m_1, m_2, \ldots), (t_1, t_2, \ldots)$ and $(m_1', m_2', \ldots), (t_1', t_2', \ldots)$, Eve knows $t_2' = F_k(t_1' \oplus m_2')$.

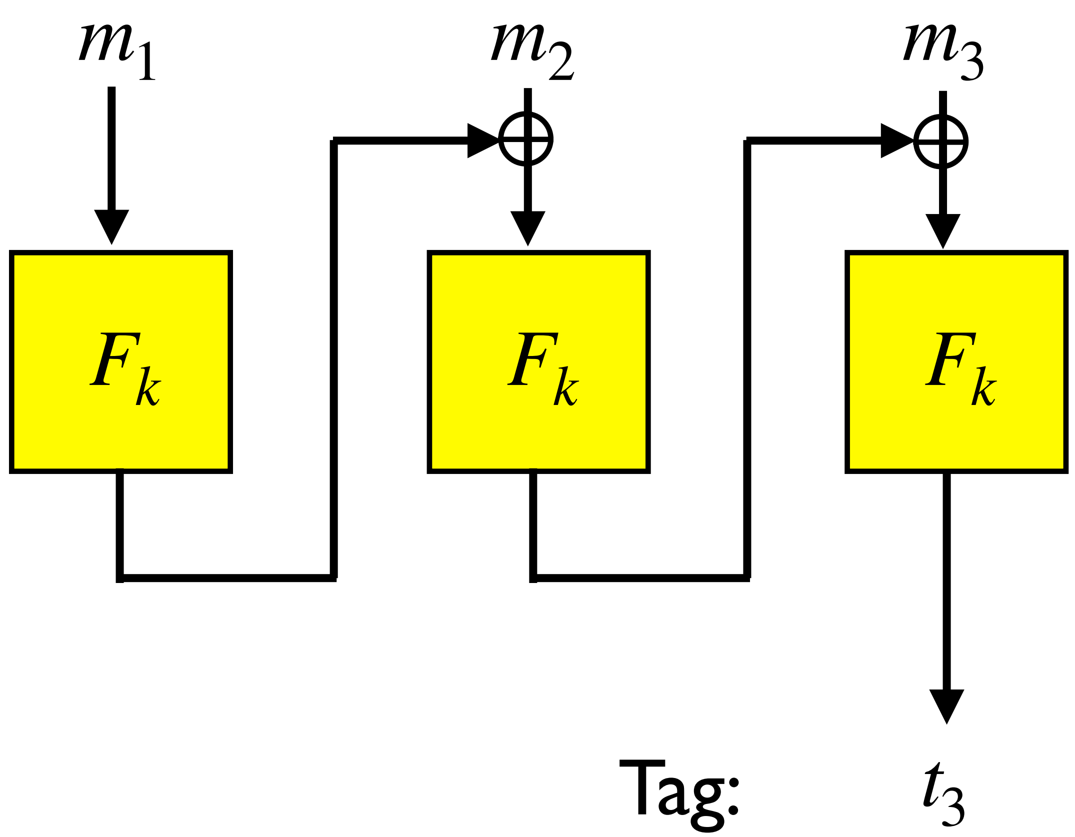
Let $m_2'' = t_1 \oplus t_1' \oplus m_2'$. Then $t_1 \oplus m_2'' = t_1' \oplus m_2'$, which means $F_k(t_1 \oplus m_2) = t_2'$ and the pair

$$(m_1, m_2'', m_3', \ldots), (t_1, t_2', t_3', \ldots) \quad \text{is valid.}$$

This class is being recorded

Another Try

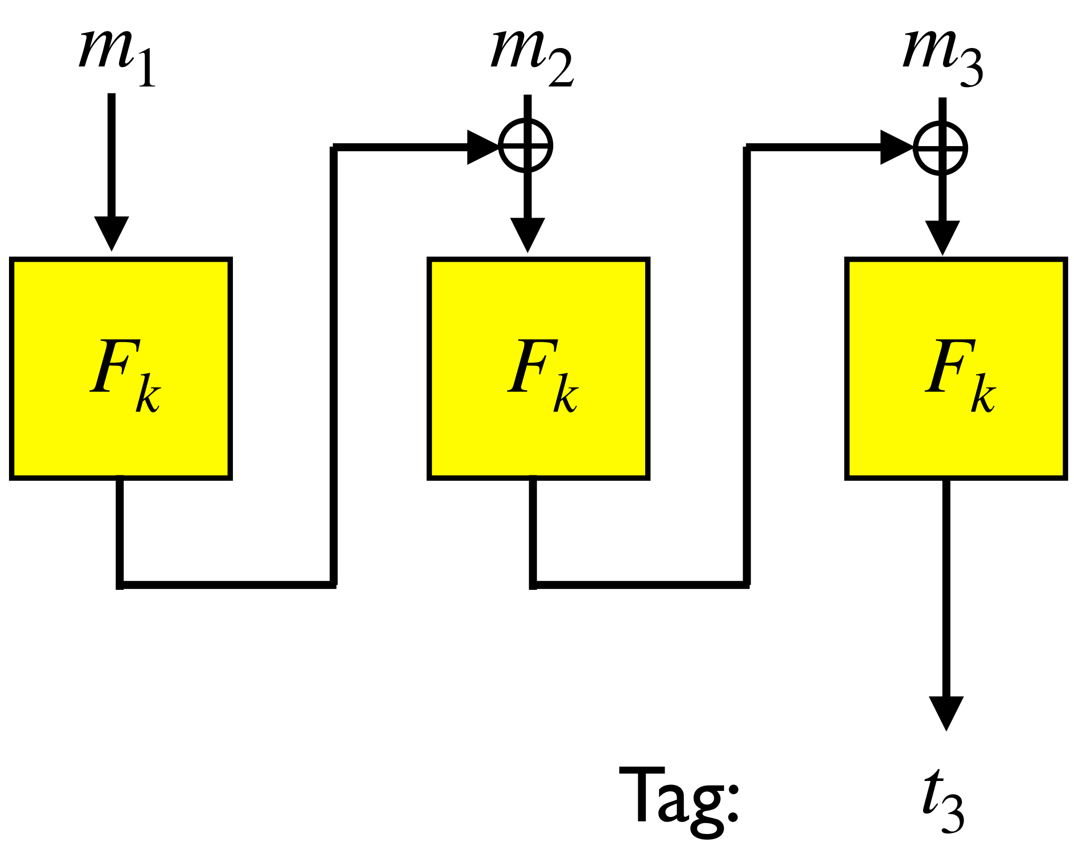
The problem seems to be the intermediate tags t_1, t_2 give Eve too much information. Maybe we should get rid of them:



Only output the final tag.

Vote: Secure? (Yes/No/Unknown)

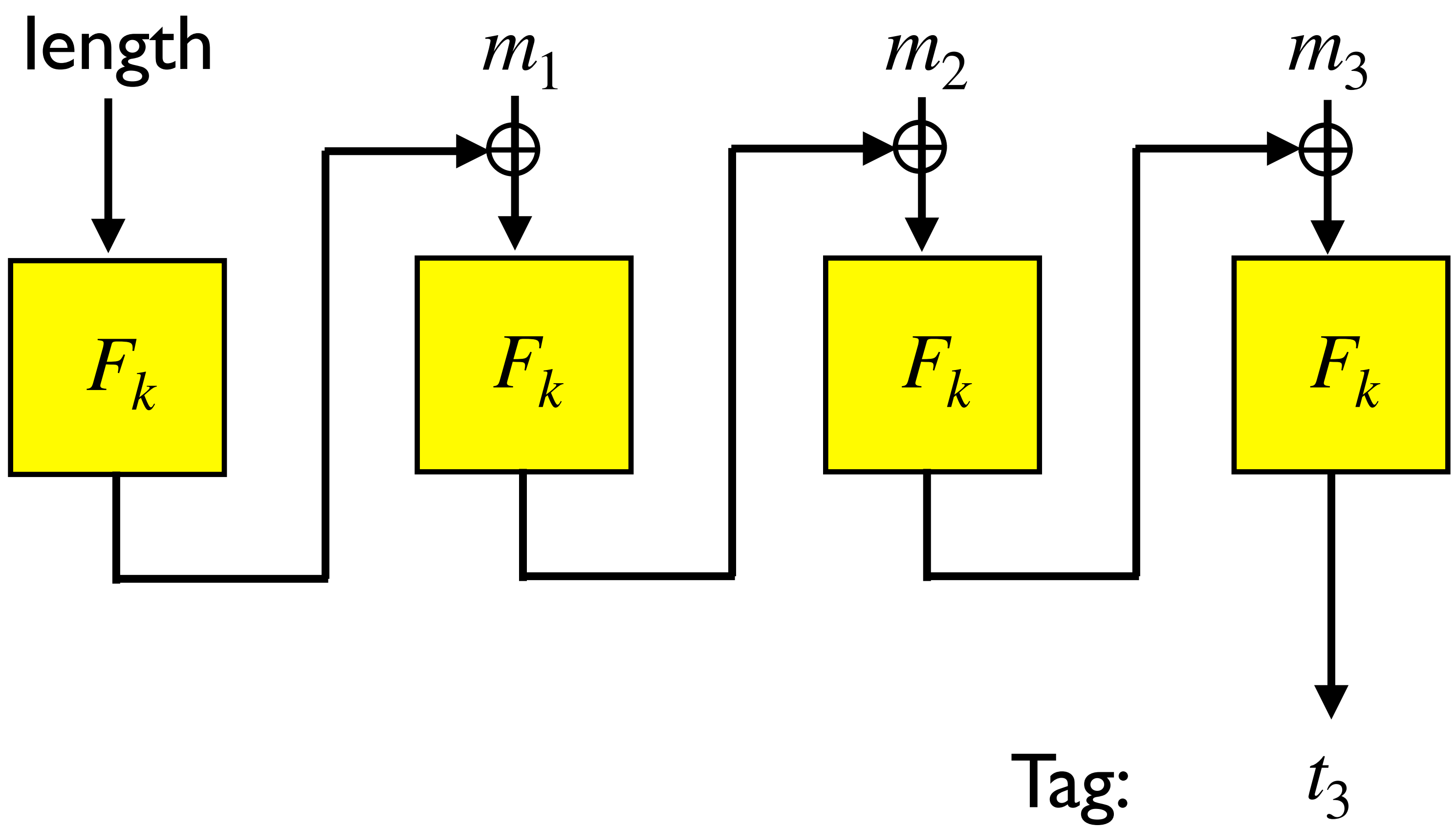
Almost There



Secure only if length fixed. Otherwise, Eve can get the tags t_1, t_2, etc. by querying the oracle for messages m_1, then $m_1 \| m_2$, etc.

Then Eve can proceed as before.

CBC with Length Included



We can authenticate the length as the first part of the message.

Vote: Secure? (Yes/No/Unknown)

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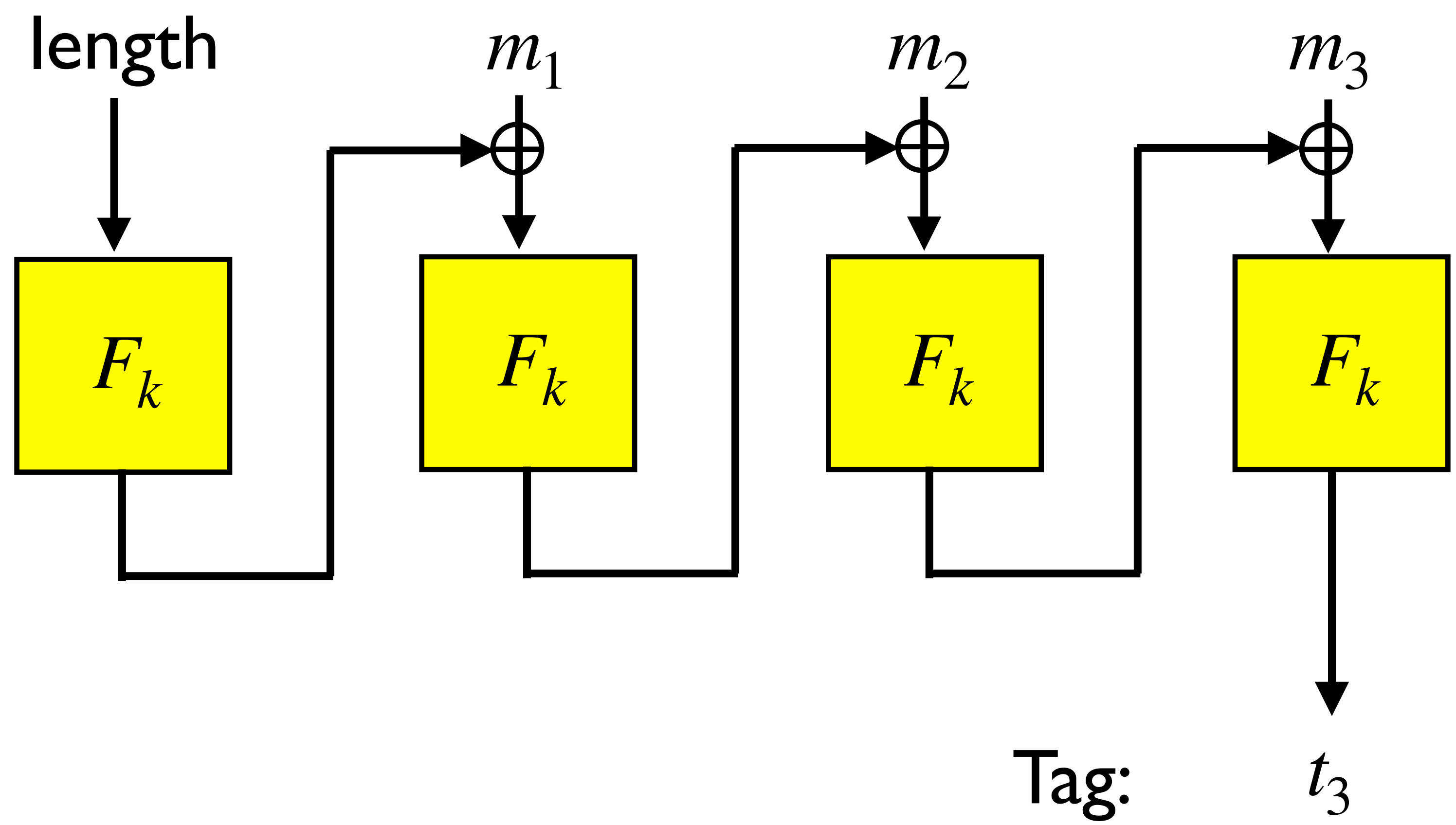

We can authenticate the length as the first part of the message.

Vote: Secure? (Yes/No/Unknown)

Yes! Finally.