CMSC/Math 456: Cryptography (Fall 2023)

Lecture 24

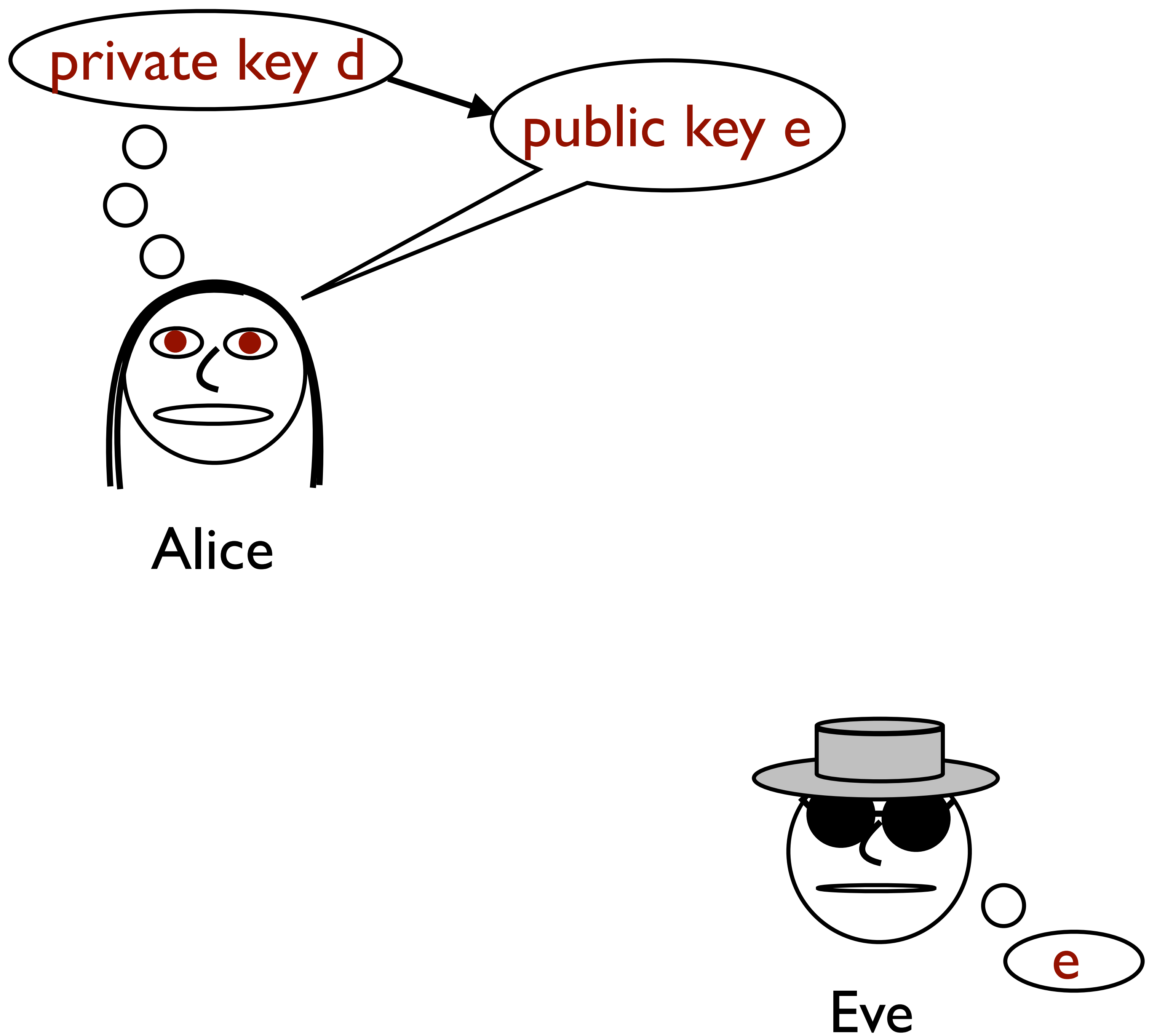
Daniel Gottesman

Administrative

Problem set #9 is due Thursday at noon. There will be one more problem set assigned on Thursday.

Digital Signatures

Digital signatures are a public key version of MACs.



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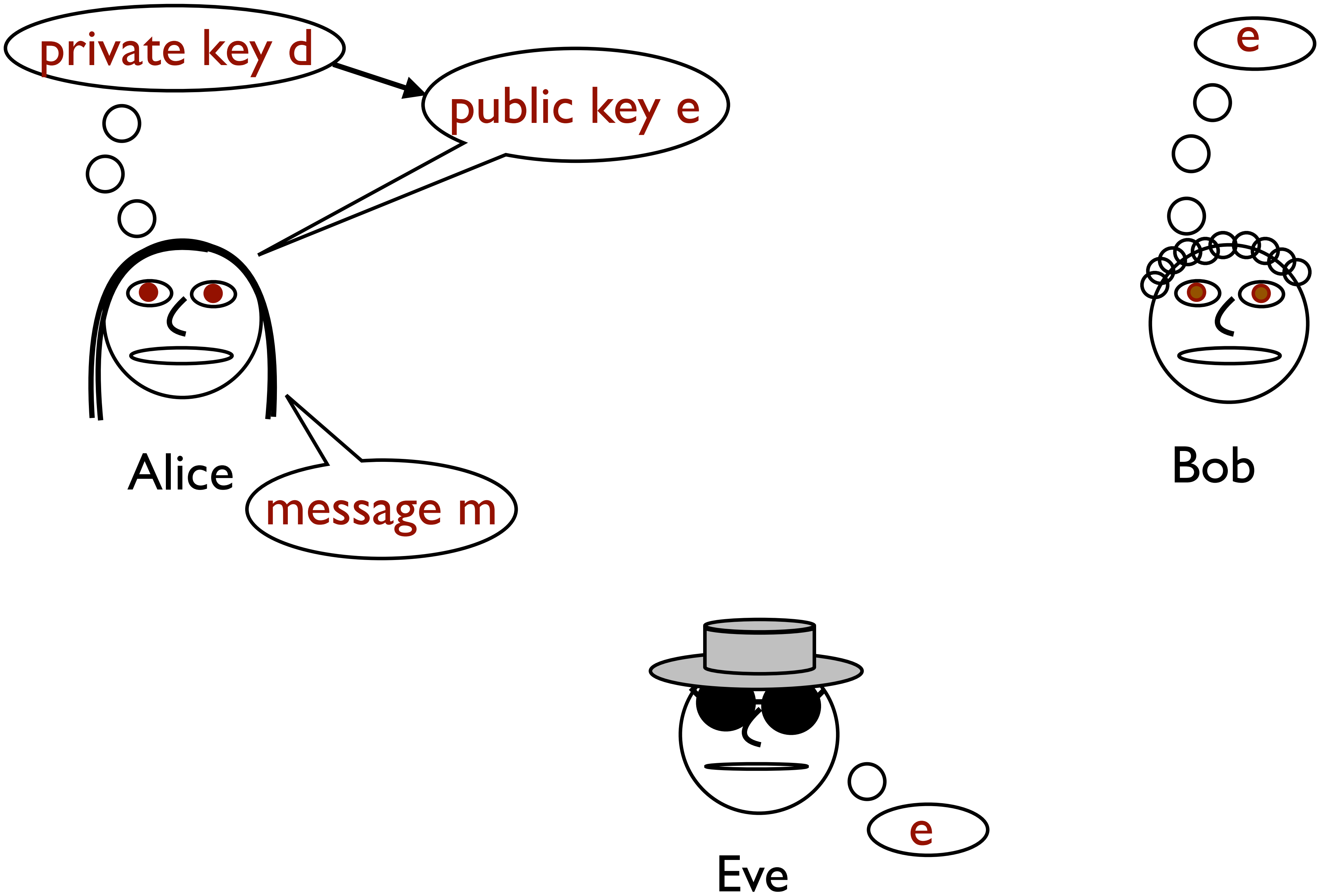

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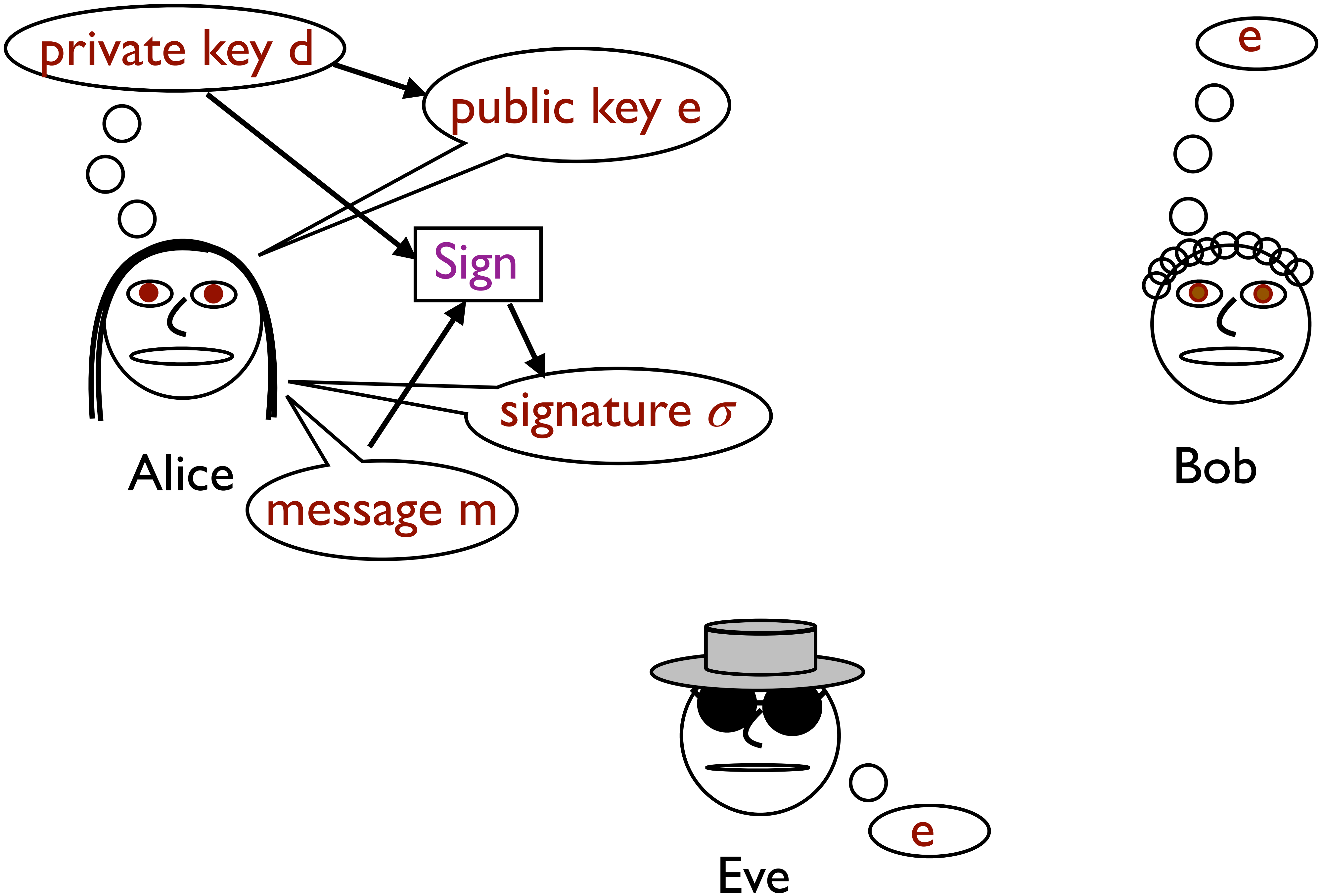

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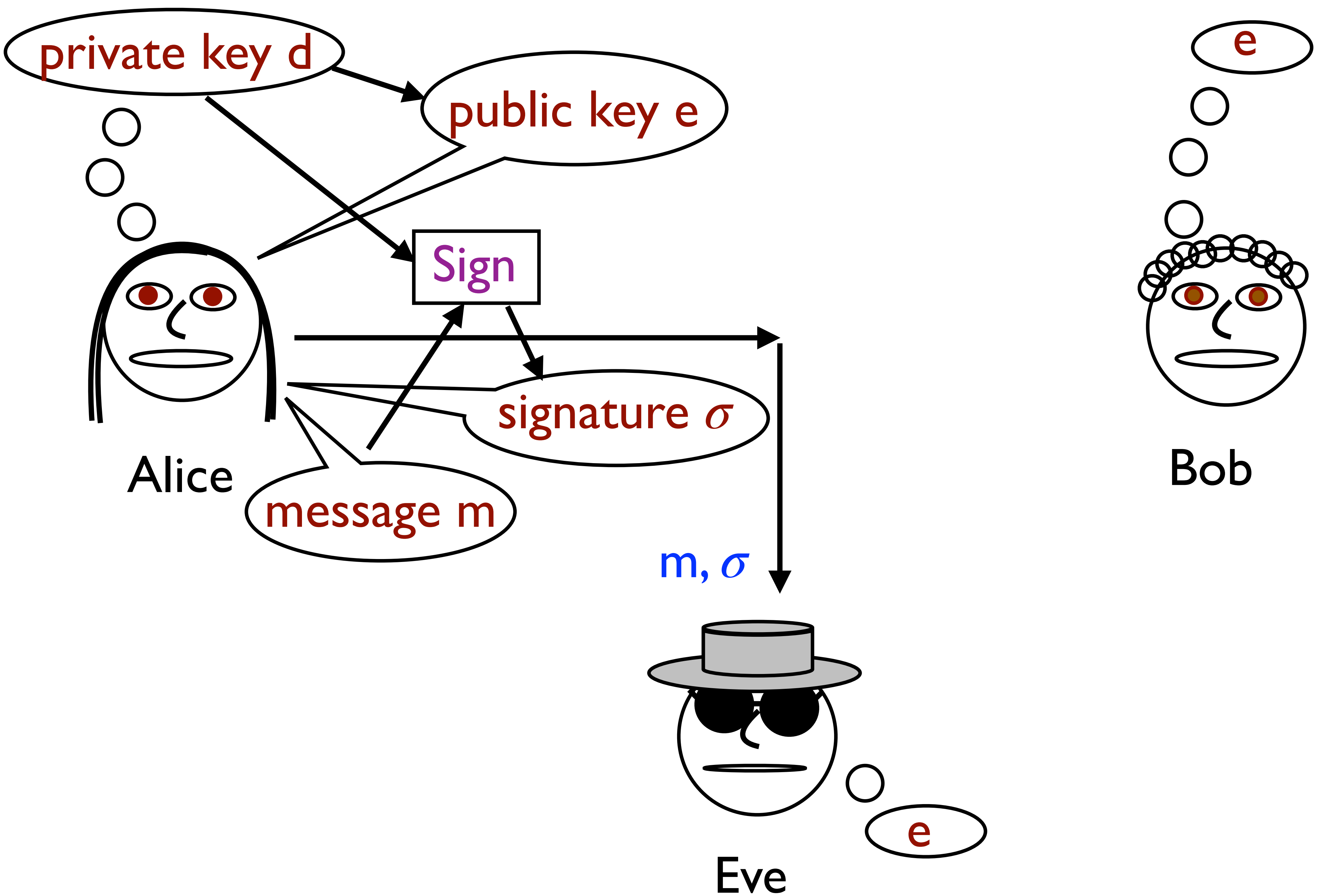

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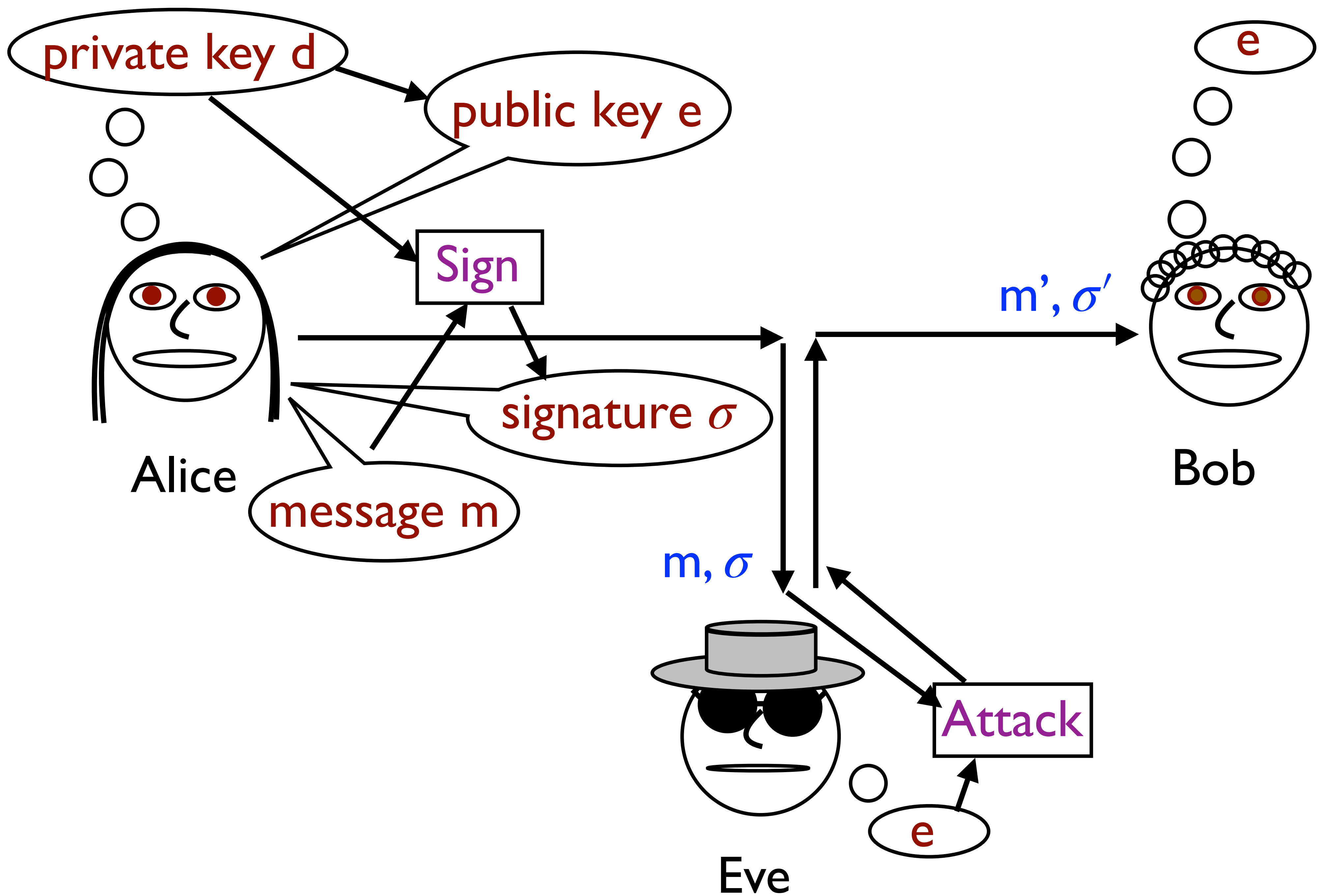

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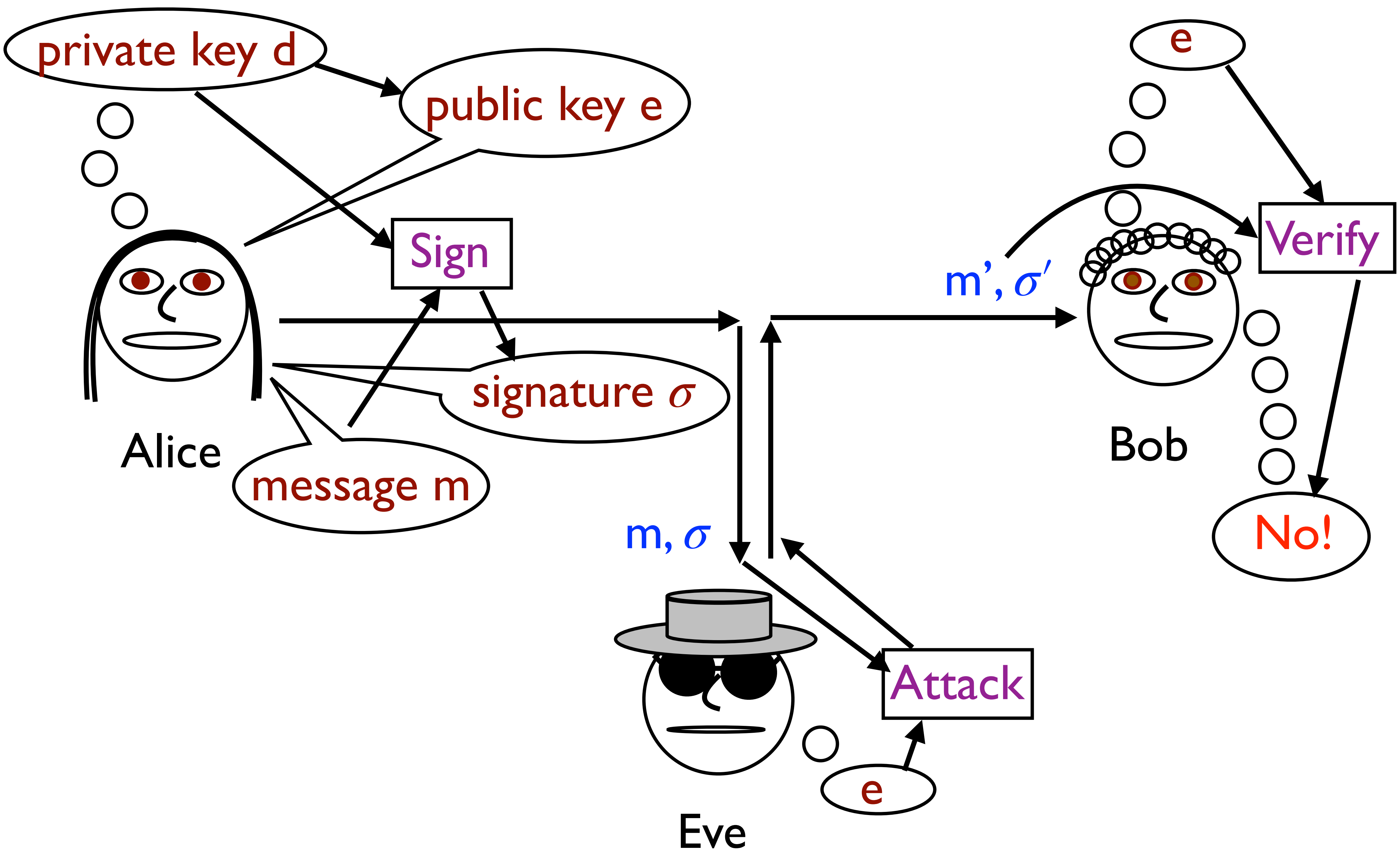

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Digital Signature Definition

Definition: A digital signature is a set of three probabilistic polynomial-time algorithms (Gen, Sign, Vrfy):

Gen is the key generation algorithm. It takes as input s, the security parameter, and outputs a public key, private key pair $(e, d) \in \{0,1\}^* \times \{0,1\}^*$.

Sign is the signing algorithm. It takes as input the private key d and a message $m \in \{0,1\}^*$ and outputs a signature $\sigma \in \{0,1\}^*$.

Vrfy is the verification algorithm. It takes as input the public key e and (m, σ) and outputs "valid" or "invalid."

The digital signature scheme is correct if

$$\mathrm{Vrfy}(e, m, \mathrm{Sign}(d, m)) = \mathrm{valid}$$

Note: Unlike a MAC, Vrfy cannot just generate a new signature, since that would require the private key.

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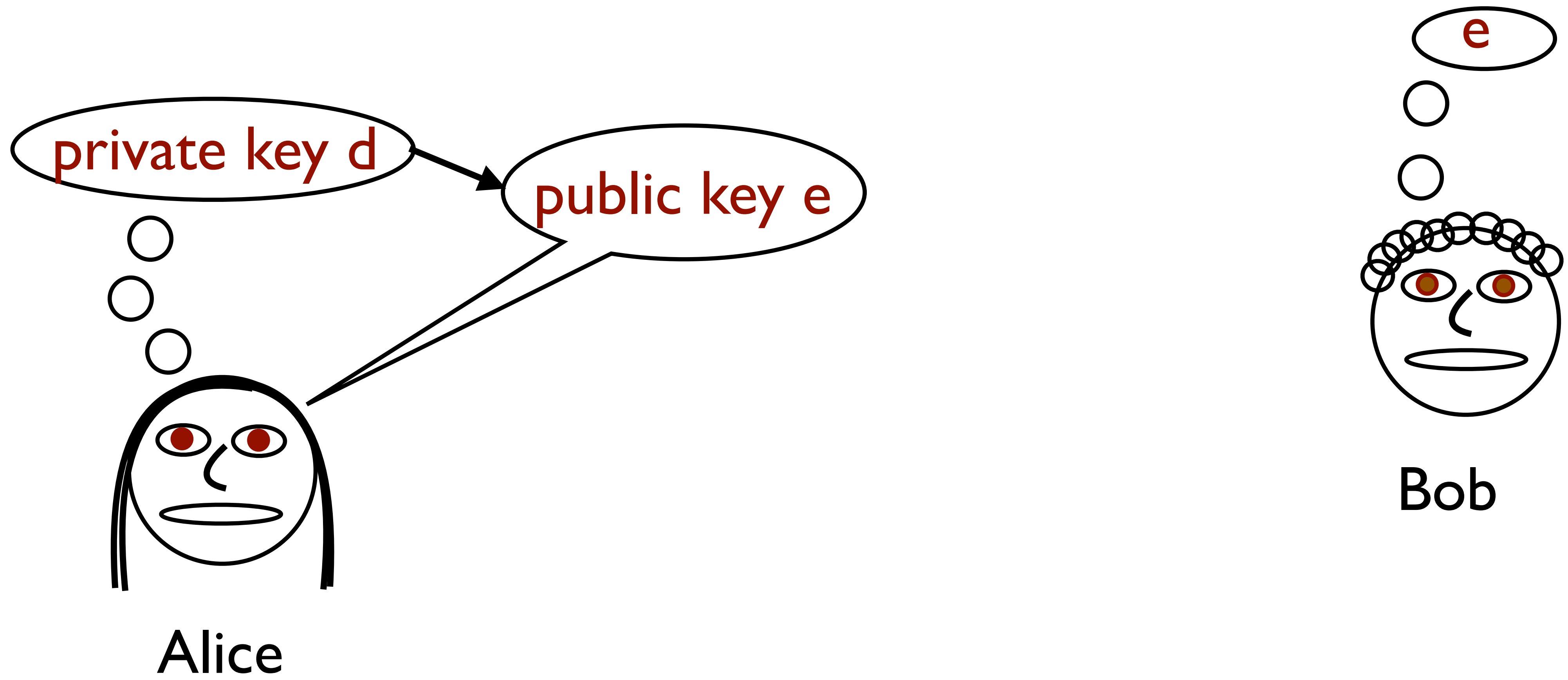
Transferability and Non-Repudiation



Because signatures use a public key, they are transferable between recipients. And Alice cannot repudiate the message, claiming she did not send it, since it can only be created using her private key.

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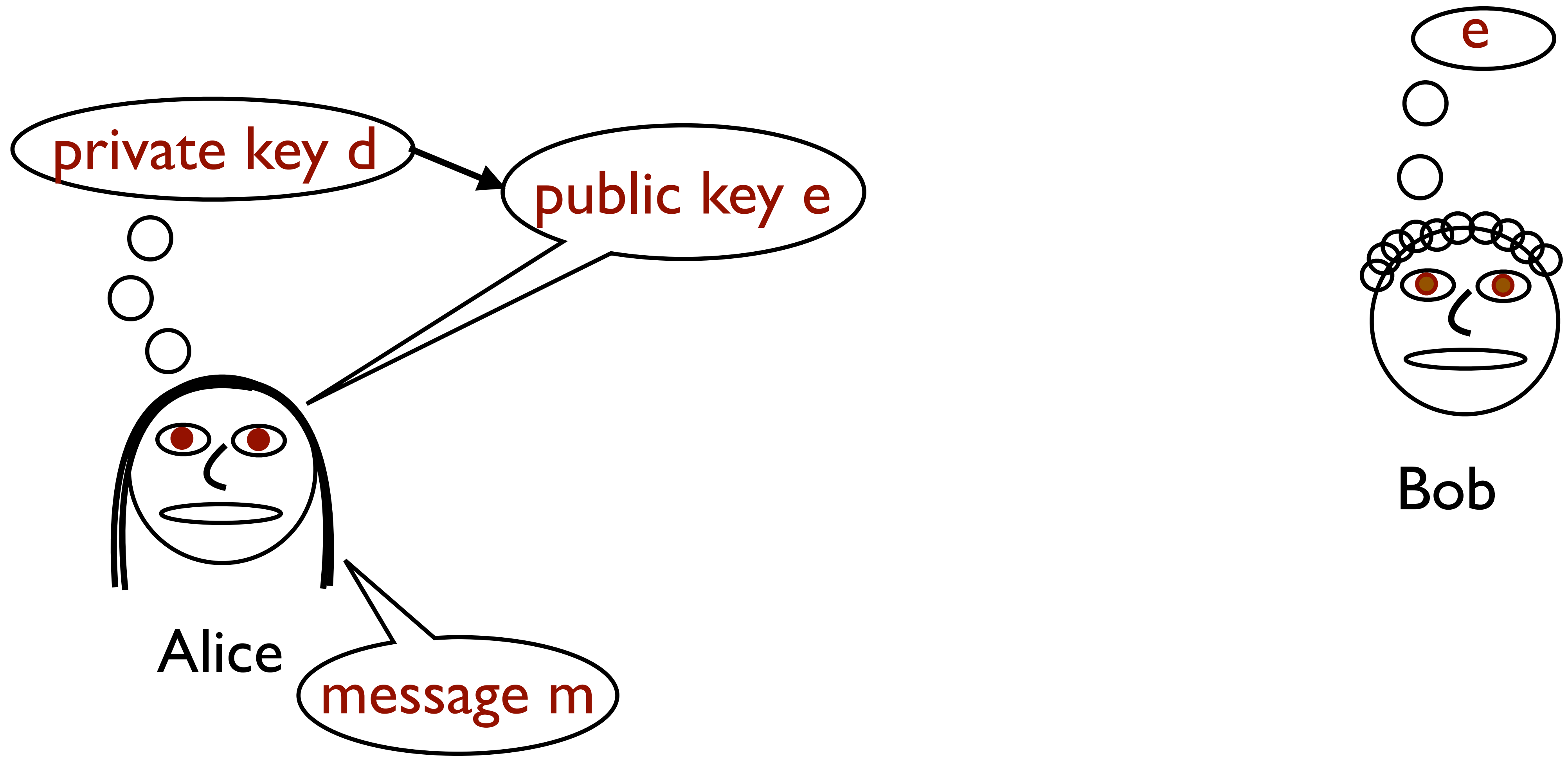

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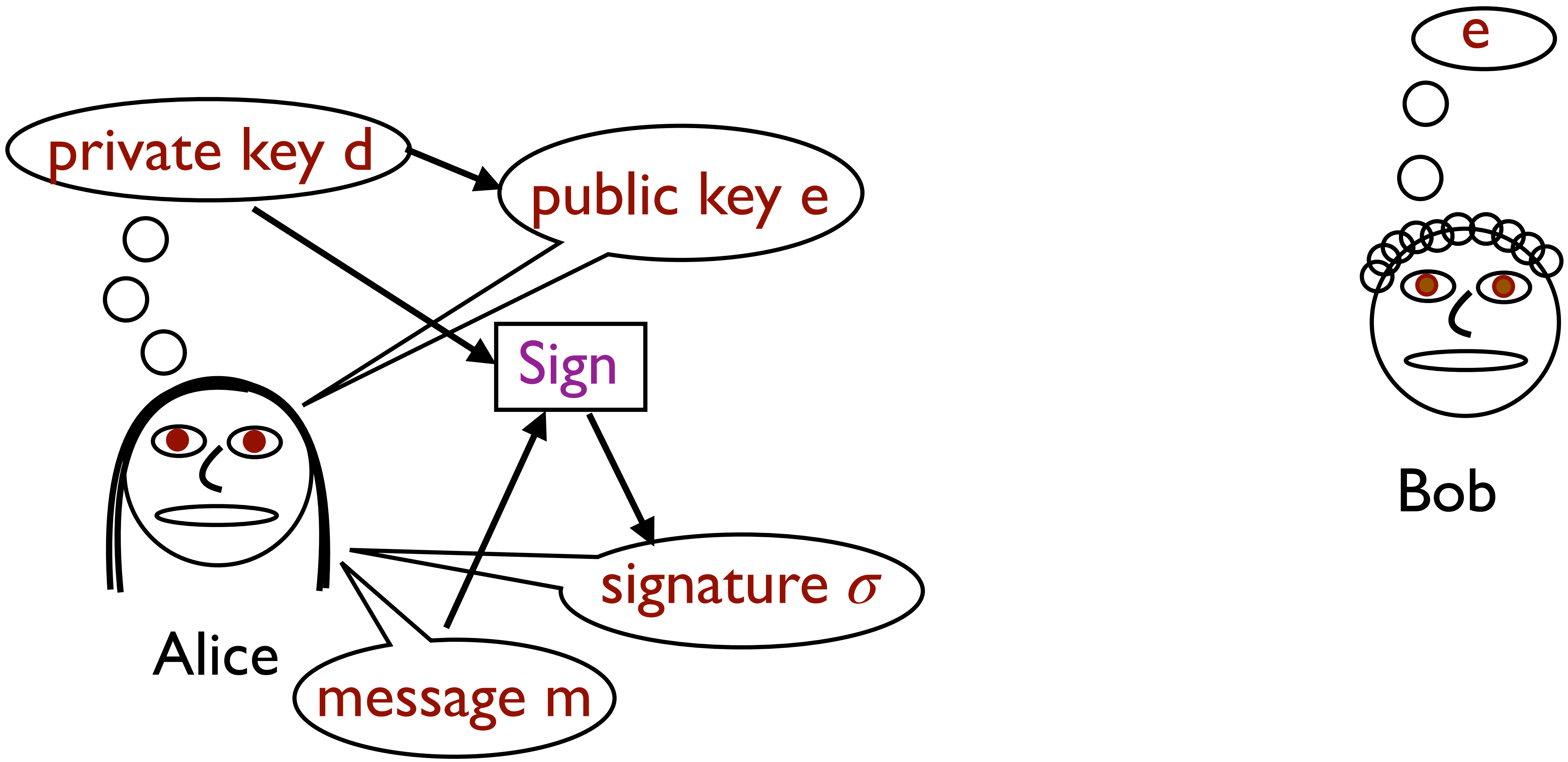

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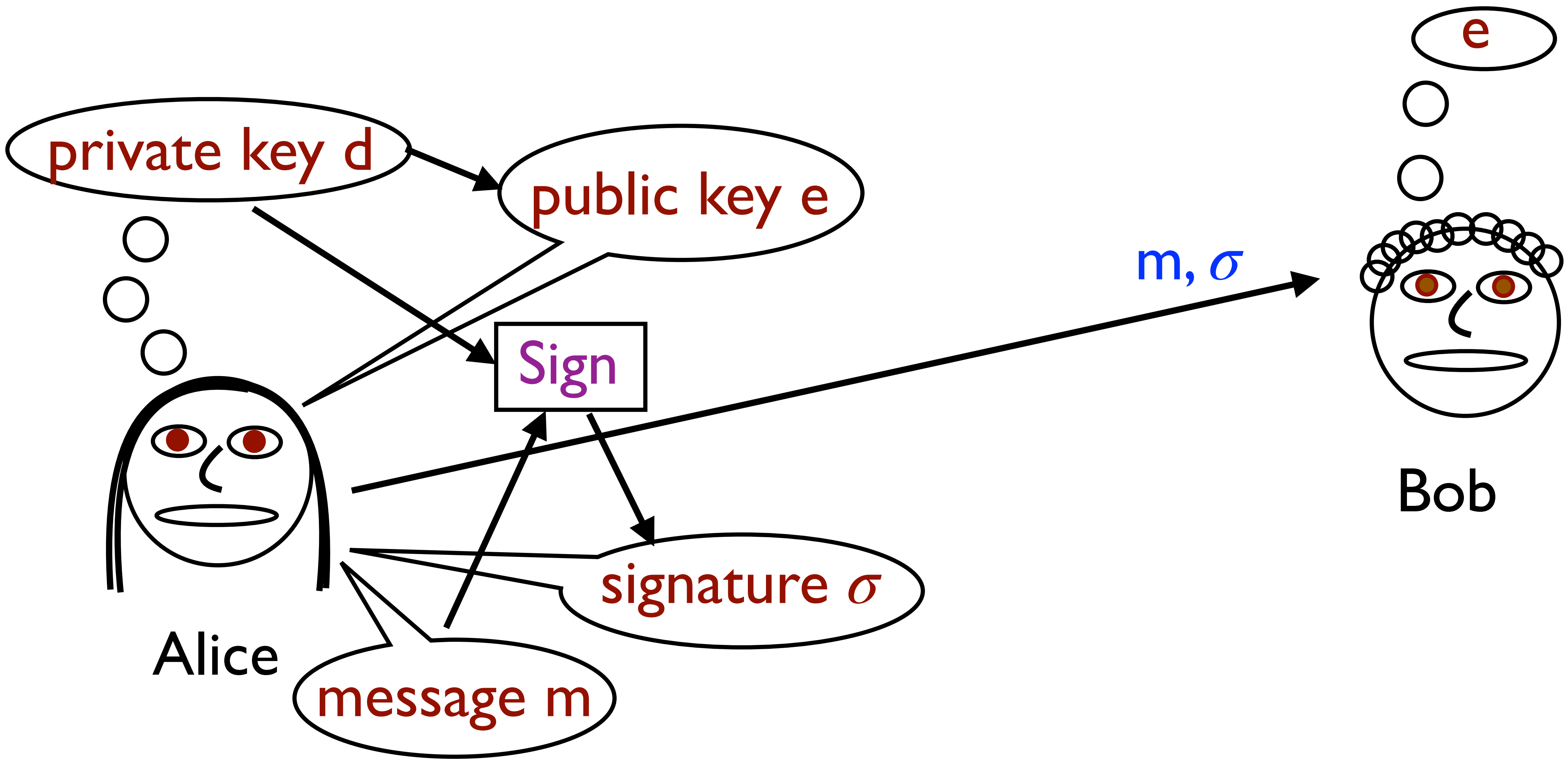

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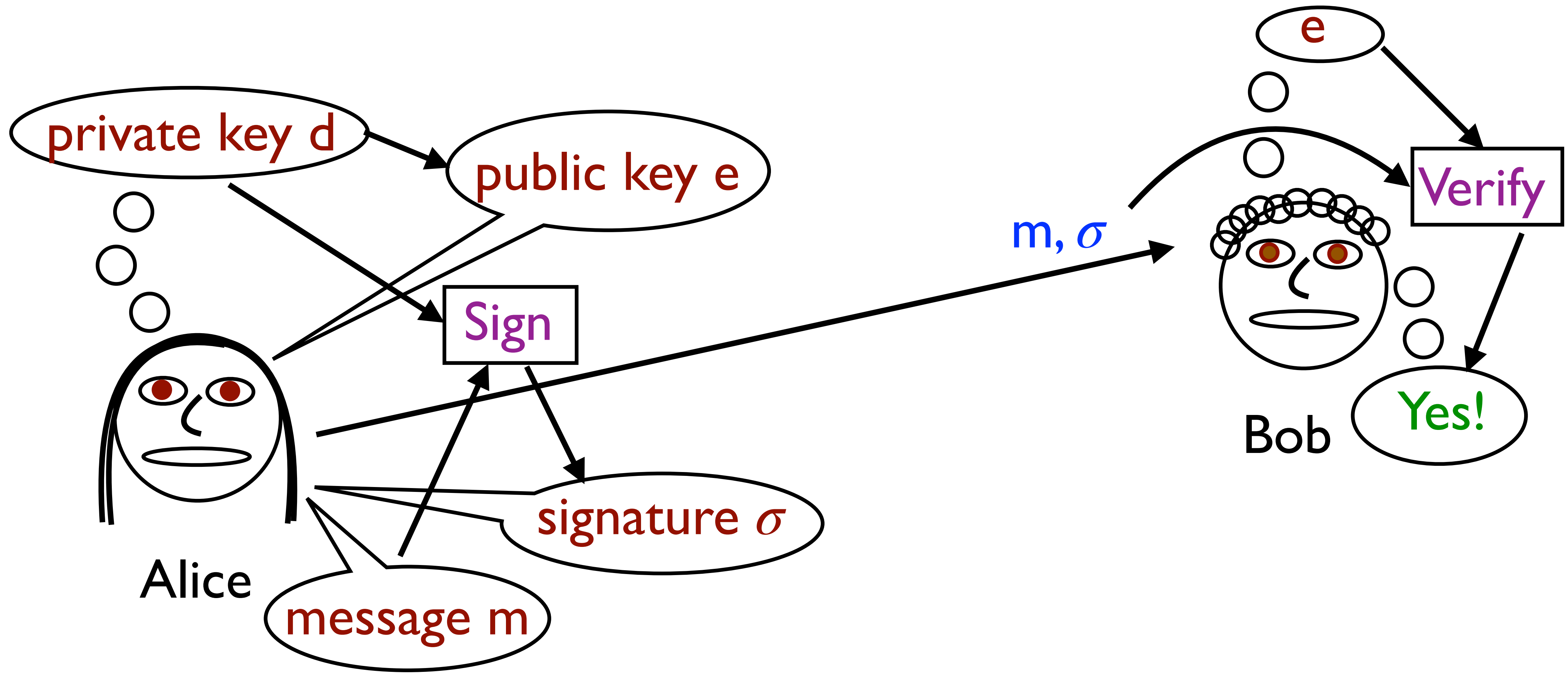

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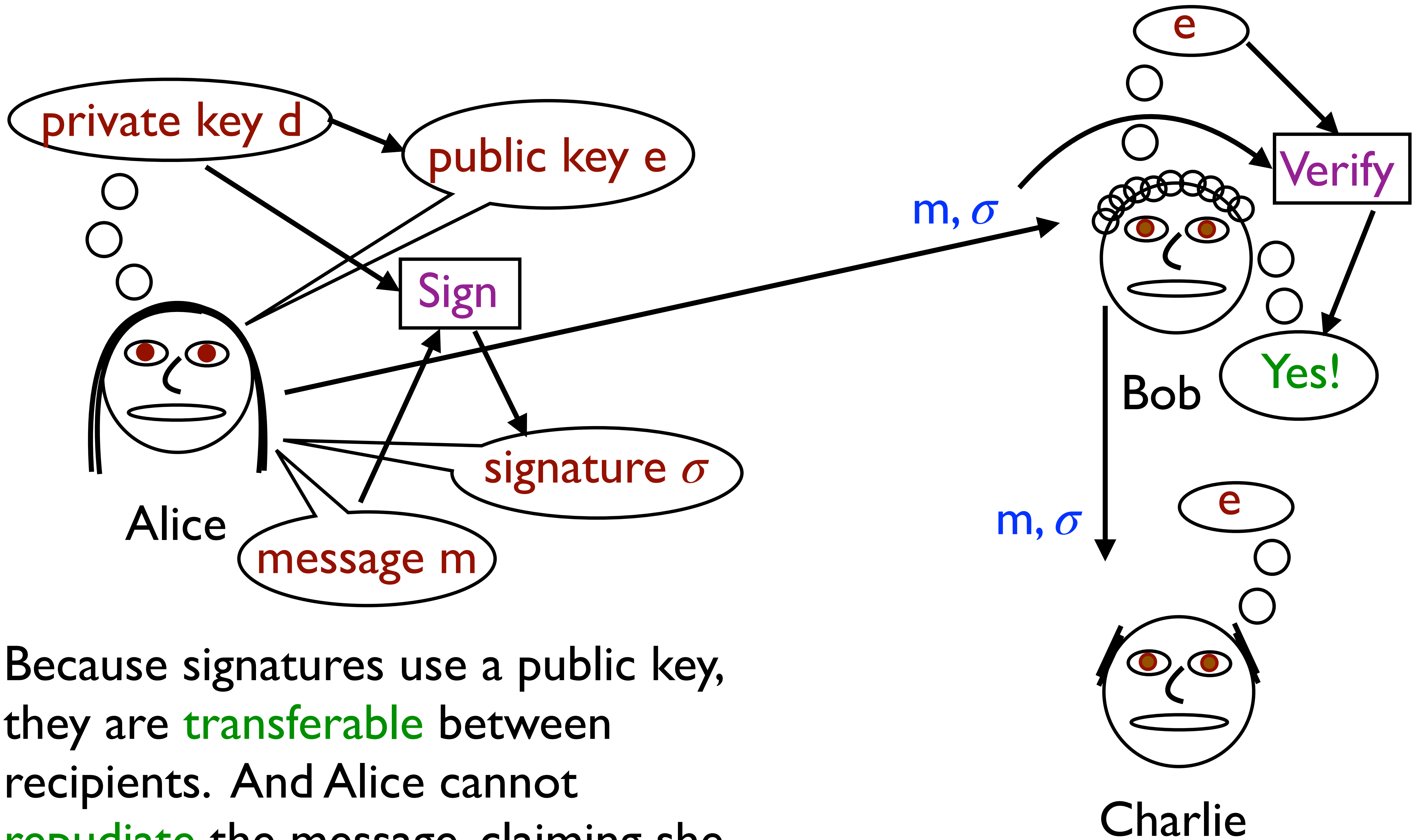

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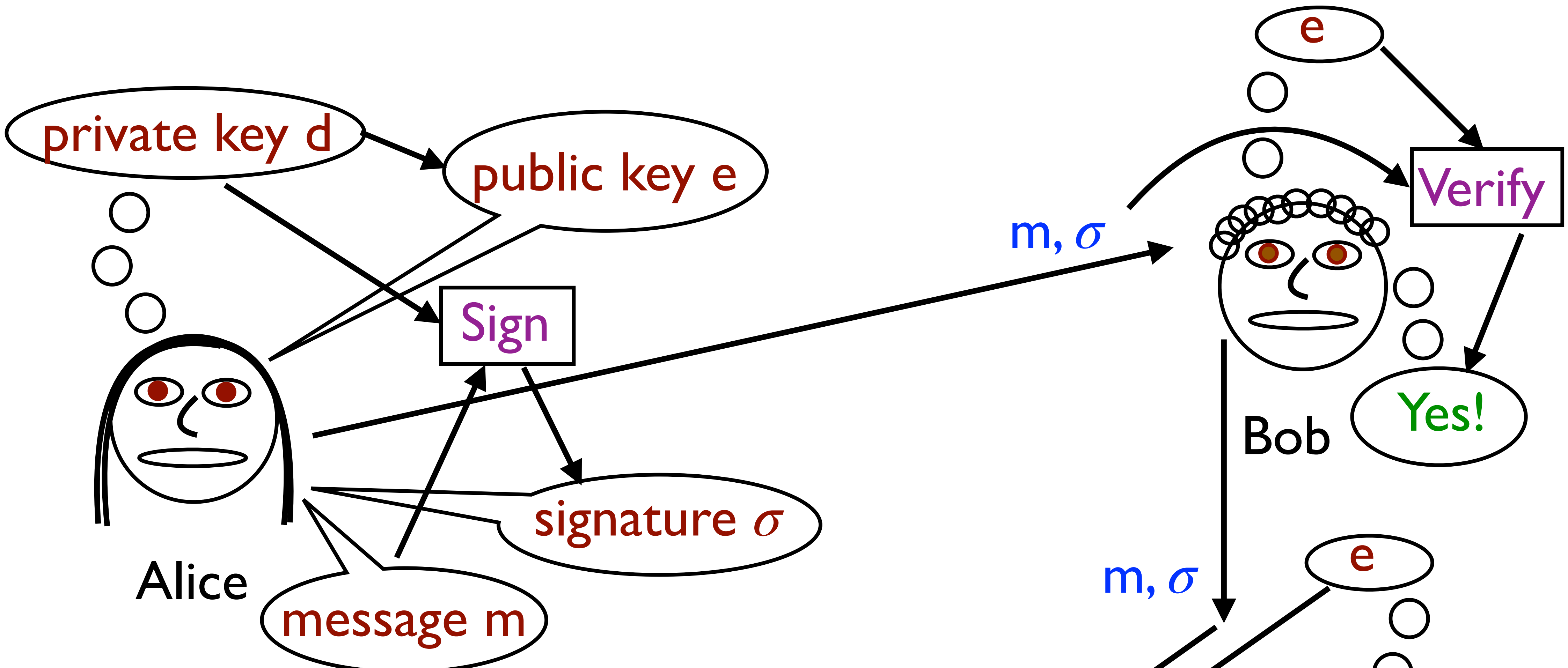

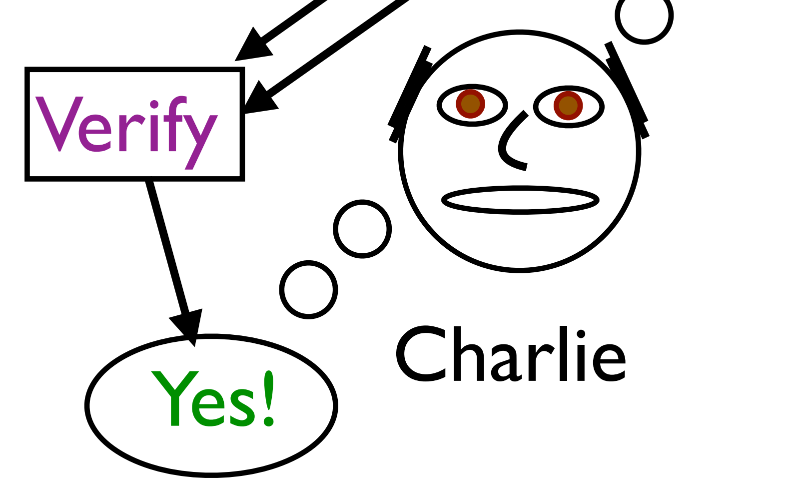

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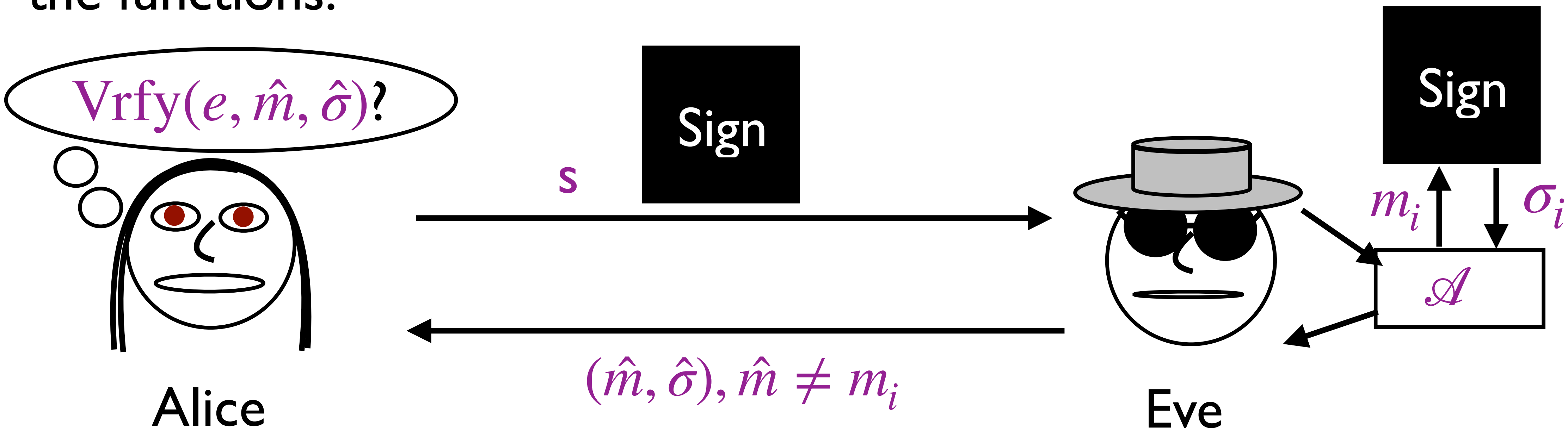
Digital Signature Security Definition

Definition: A digital signature (Gen, Sign, Vrfy) with security parameter s is secure (against an adaptive chosen-message attack) if, for any polynomial-time attack $\mathscr{A}$ with the public key e and oracle access to $\mathrm{Sign}(d, m)$, where $\mathscr{A}$ outputs $(\hat{m}, \hat{\sigma})$ such that $\mathscr{A}$ never queried the oracle for $m = \hat{m}$,

$$\Pr(\mathrm{Vrfy}(e, \hat{m}, \hat{\sigma}) = \mathrm{valid}) \leq \epsilon(s)$$

where $\epsilon(s)$ is a negligible function and the probability is averaged over (e,d) generated by Gen and the randomness used in any of the functions.



RSA Digital Signatures

Gen: Generate two random primes p and q which are s bits long. Let N = pq. Choose $e, d \in \mathbb{Z}_N^*$ such that $ed = 1 \bmod \varphi(N)$. The public key is (N, e) and the private key is (N, d).

Sign: Given message m and private key (N, d). The signature is $\sigma = m^d \bmod N$.

Vrfy: Given (message, signature) pair (m, σ) and public key (N, e). The message is accepted as valid if $m = \sigma^e \bmod N$.

Vote: Is this secure (yes/no/unknown)?

(If RSA assumption is true.)

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Answer: No.

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Attack on Plain RSA Signatures

What if we come up with the signature σ *first*?

We then need to find a message m such that $\sigma = m^d \bmod N$.

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m decrypts to σ, so σ encrypts to m.

In this case, Eve picks a random σ and therefore gets a random m. This is maybe of limited practical use, but it is definitely a violation of the security definition.

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And notice that this attack doesn't even require Eve to see any valid signatures.

Forging a Specific Message

Suppose we want to forge a specific message m.

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Notice: Given signatures for two messages m_1 and m_2,

$$\sigma_1 = m_1^d \bmod N$$
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Therefore, to forge the message $m_1 m_2$, all we need are the signatures of the two messages m_1 and m_2.

Revised RSA Signatures

New Idea: Put m through a hash function H first.

Gen: Generate two random primes p and q which are s bits long. Let N = pq. Choose $e, d \in \mathbb{Z}_N^*$ such that $ed = 1 \bmod \varphi(N)$. The public key is (N, e) and the private key is (N, d).

Sign: Given message m and private key (N, d). The signature is $\sigma = H(m)^d \bmod N$.

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Answer: Yes, if H is a random oracle. The standards pad the output of a standard hash function instead, in which case the answer is unknown.

Security Discussion

With the revised definition, the first attack fails because finding m given σ requires inverting H(m).

But it is hard to invert a random oracle.

Since $H(m_1)$, $H(m_2)$, and $H(m_1 m_2)$ are random, they have no relationship to each other and in particular, $H(m_1 m_2) \neq H(m_1)H(m_2)$.
This foils the second attack.

However, we do need to be careful that we can't find multiplicative relations among the outputs of the $H(m_i)$.

Padding H helps to achieve this in practice, but there is no proof that the particular schemes which are widely used actually work — but no attacks are known despite having been used for many years.

Same Key for Encryption & Signature

What if Bob decides to use the same N and the same (private key, public key) pair (e,d) for encryption and signatures?

Suppose Bob is using plain RSA encryption and signatures. They are already insecure, but this makes an even more powerful attack possible:

Alice sends Bob a ciphertext $c = m^e \bmod N$. Eve intercepts it and convinces Bob to sign the "message" c. Bob then produces the signature $\sigma = c^d = m^{ed} = m \bmod N$! Bob's signature is a decryption of Alice's message, revealing the message to Eve.

When correctly using the hash function with the signature, this particular attack doesn't work since c won't be correctly padded and it will be hard to find a message x to sign that gives H(x) = c, but this still seems like a vulnerability.

There are also good key management reasons not to do this.

Identification Schemes

Suppose Bob has Alice's public key. Bob is talking to someone who claims to be Alice and wants proof. Alice doesn't want to reveal information that would let Bob impersonate her.

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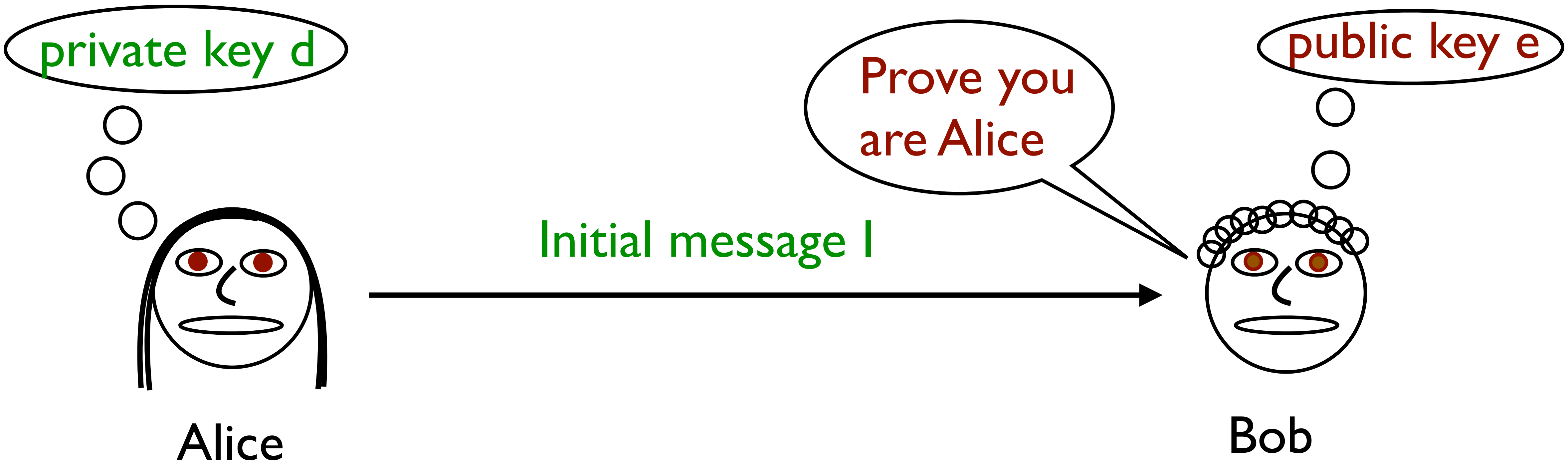

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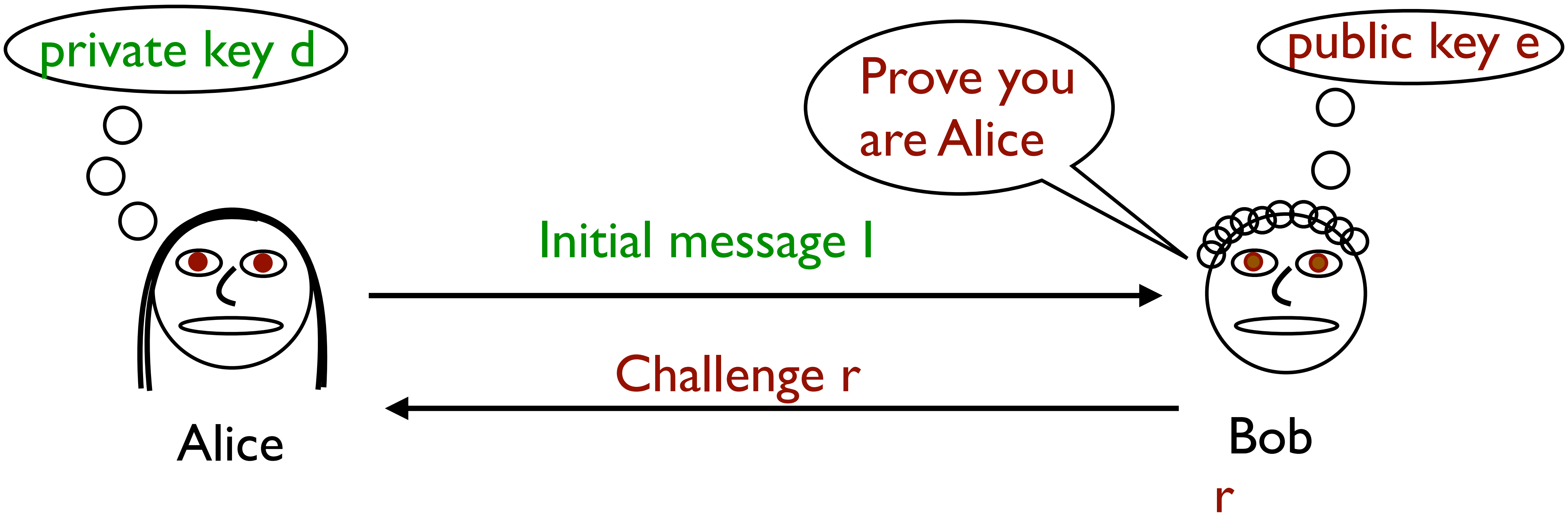

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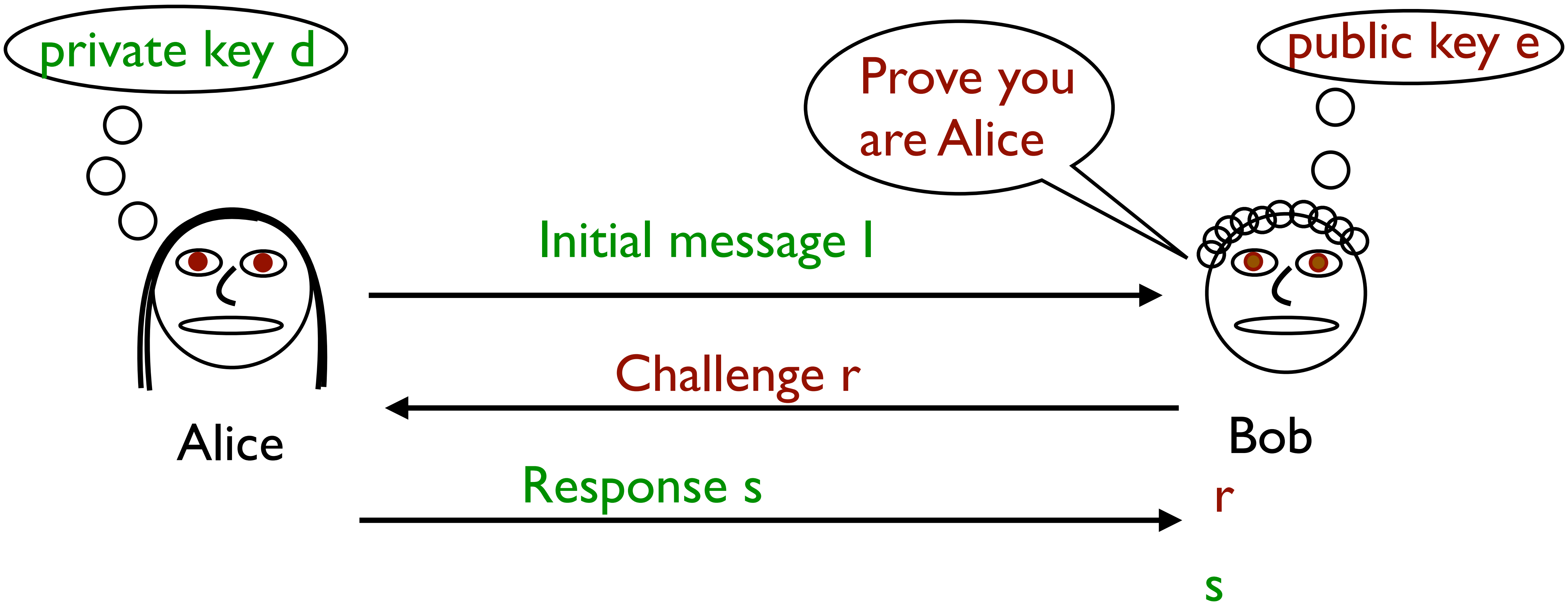

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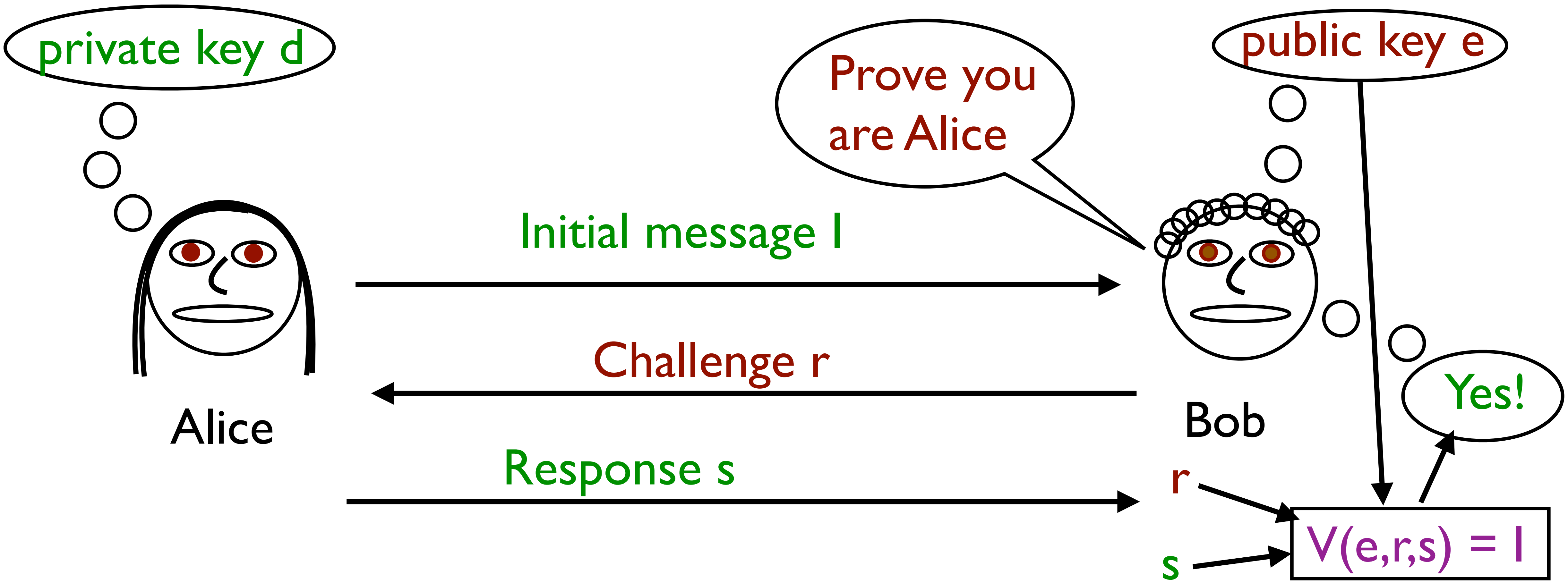

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Discrete-Log Identification

Suppose Alice uses a El Gamal-like public key: Prime p with large prime order q subgroup of $\mathbb{Z}_p^*$, base g (in the order q subgroup), and y, with $y = g^x \bmod p$. Here x is the private key.

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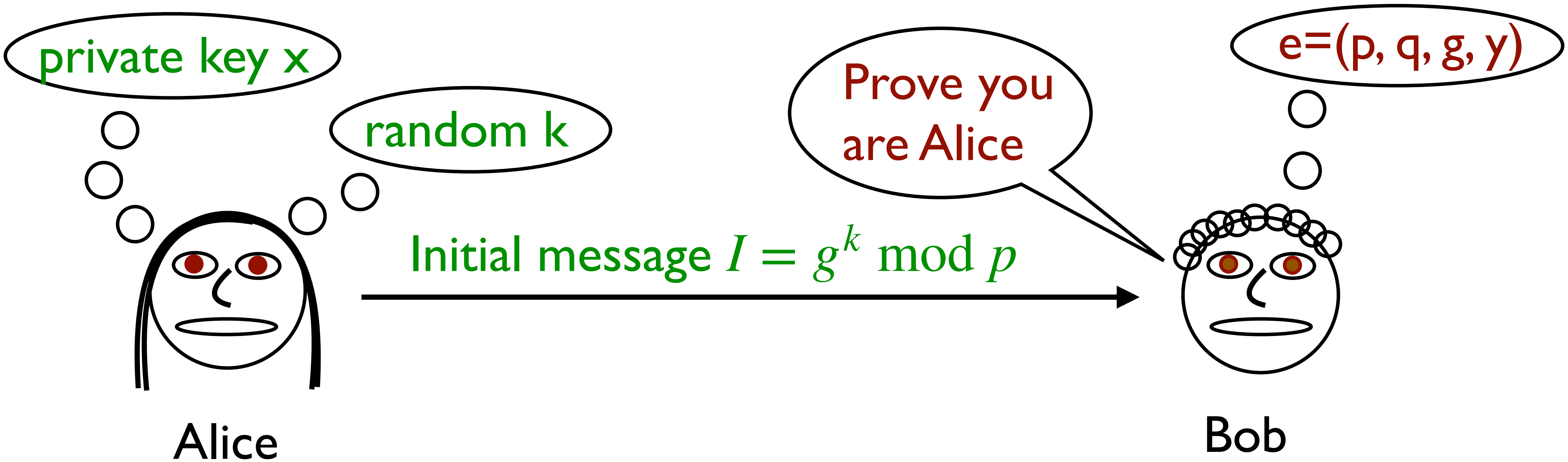

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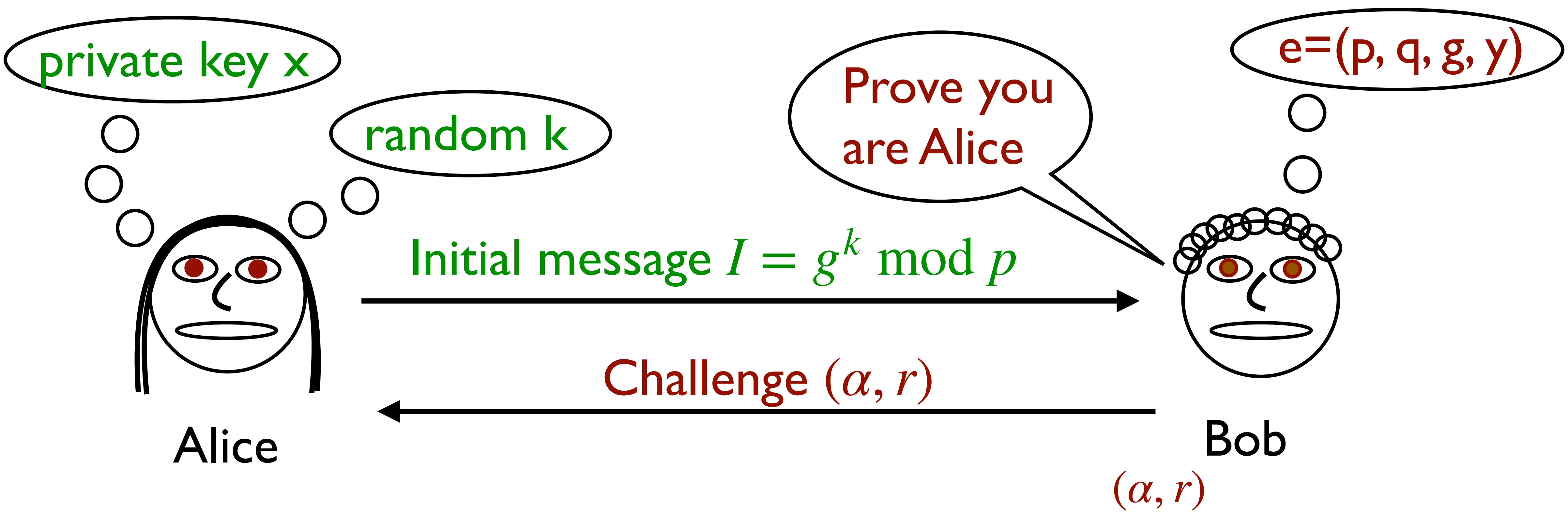

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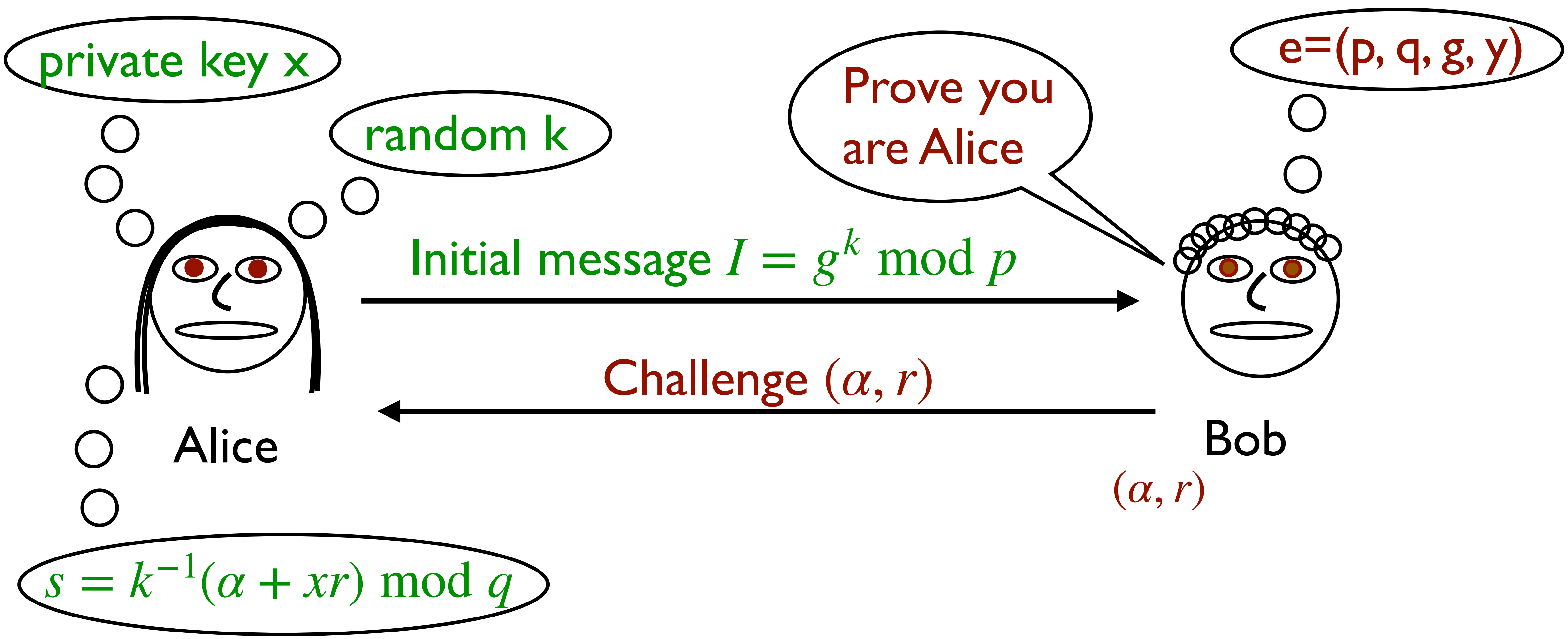

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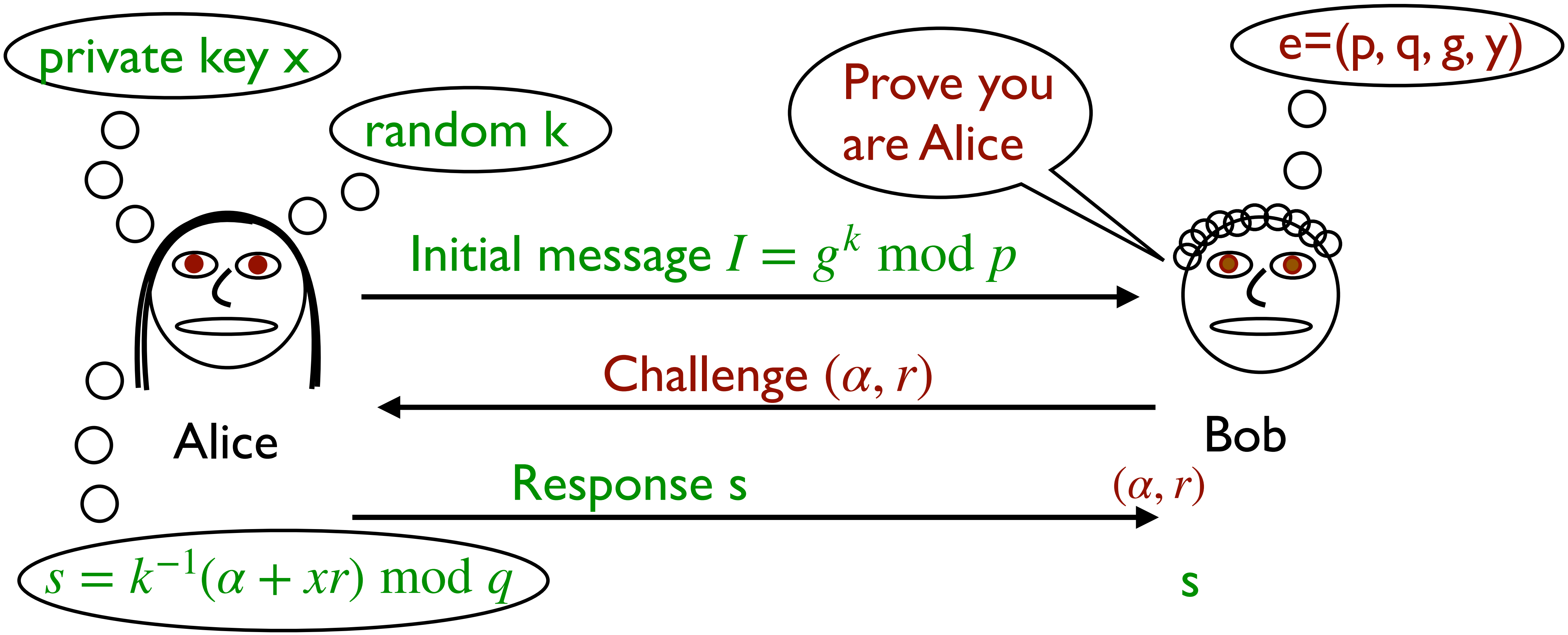

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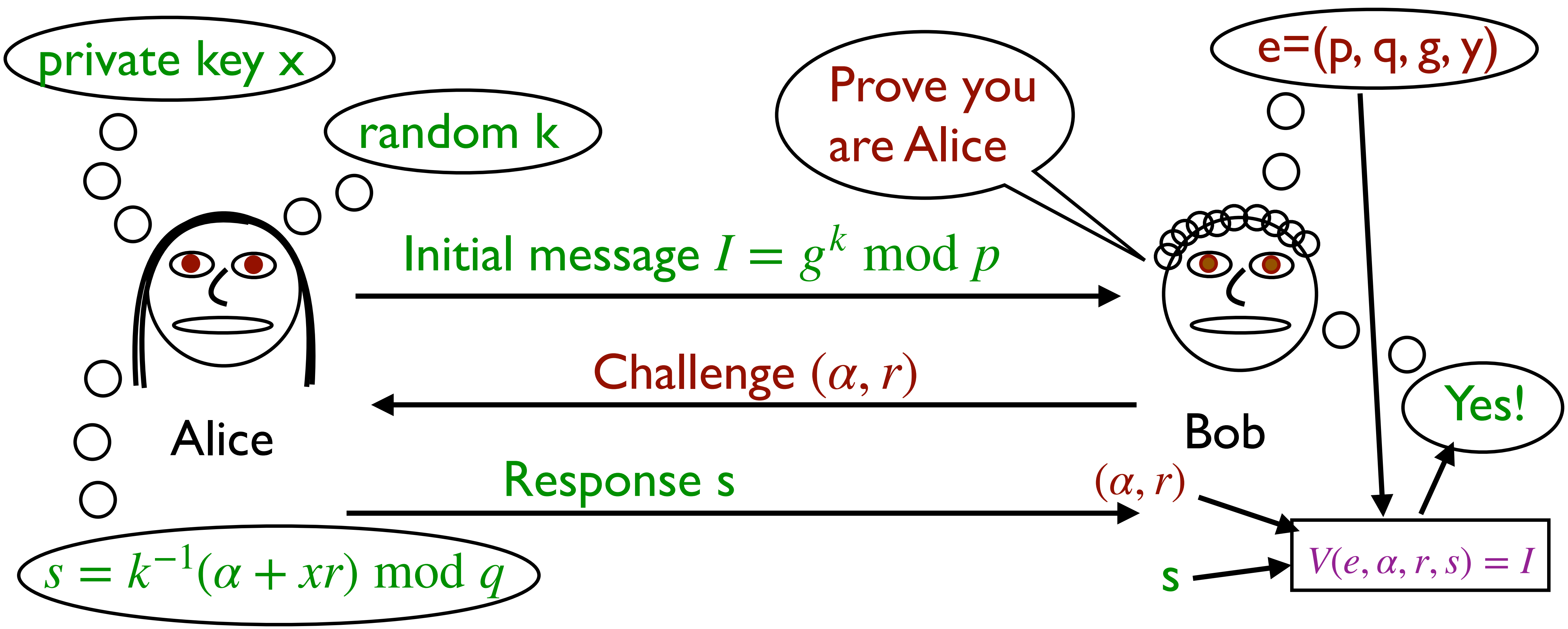

$$V(e, \alpha, r, s) = g^{\alpha s^{-1}} y^{r s^{-1}}$$

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3. Alice computes $s = k^{-1}(\alpha + xr) \bmod q$ using her two secrets k and x (along with public (α, r, q)) and sends s to Bob.

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3. Alice computes $s = k^{-1}(\alpha + xr) \bmod q$ using her two secrets k and x (along with public (α, r, q)) and sends s to Bob.
4. Bob computes $V(e, \alpha, r, s) = g^{\alpha s^{-1}} y^{rs^{-1}} \bmod p$ and accepts if $V(e, \alpha, r, s) = I$ and $s \neq 0$.

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Correctness of Identification

When Alice is actually present she can pass this test:

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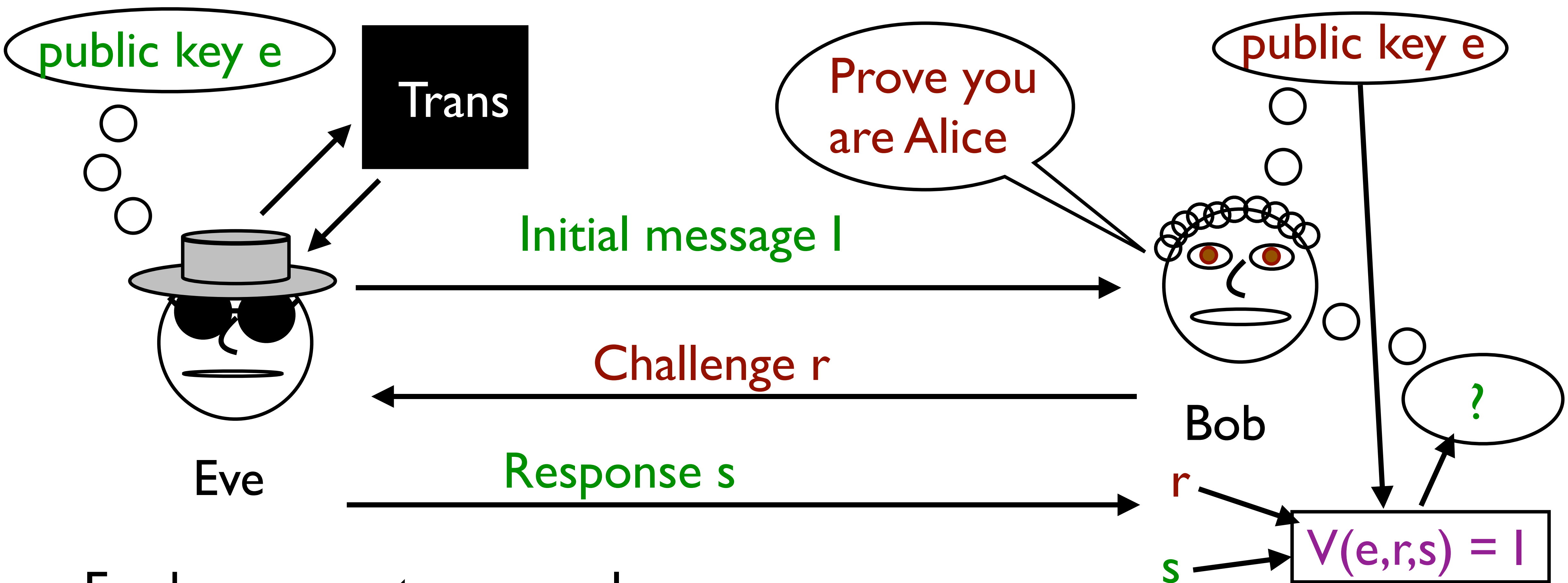
Bob checks for 1 more thing: that $s \neq 0$. But this happens only if $\alpha + xr = 0 \bmod q$, which happens with probability only 1/q.

Security of Identification

An identification protocol is secure if Eve can't masquerade as Alice even after seeing transcripts of previous identification sessions (all public information of the session).

Identification security game:



Eve has access to an oracle to generate transcripts.

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> Claim: Eve can solve discrete log if she can give correct responses to two pairs (α, r_1) and (α, r_2) for the same I.

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I will pass Bob's test:

$V(e, \alpha, r, s) = g^{\alpha s^{-1}} y^{r s^{-1}} \bmod p$ automatically equals I.

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Since g and y have order q, if we let

$$x = \alpha (s_1^{-1} - s_2^{-1})(r_2 s_2^{-1} - r_1 s_1^{-1})^{-1} \bmod q$$

then

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Security: Unproven, even when H is a random oracle.

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Repeated k

k must be new each time and kept hidden from Eve.

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$$x = r^{-1}(ks - H(m)) \bmod q$$

and Eve learns the private key x.