Number Bases

- Number bases represent the number of unique digits in a system.
  - ex: Base-7 has 7 unique digits (0,1,2,3,4,5,6)
- Binary (base 2), octal (base 8), and hexadecimal (base 16) are the most common for computer science.
- Today we'll go over two basic things:
  - Converting base-`x` numbers to base-10 (Easier)
  - Convering base-10 numbers to base-`x` (Harder, a little)

What even is Base-10?

- Consider the number: 843
  - 843 = 8 * 100 + 4 * 10 + 3 * 1
  - We can also represent it slightly differently

What even is Base-10?

- Consider the number: 843
  - $843 = 8 * 100 + 4 * 10 + 3 * 1$
  - $843 = 8 * 10^2 + 4 * 10^1 + 3 * 10^0$
- Notice that in base 10, we use powers of 10!
  - We also only have digits 0-9 (10 digits)

Translating To Base-10

- If we have a number in base 6, how many digits would we have?
    - Correct, we'll have 6!
        - 0, 1, 2, 3, 4, 5
- Now, in the same way we represented 843 in base 10, how would we represent a number like 253_6?
  -Hint: Remember $843 = 8 * 10^2 + 4 * 10^1 + 3 * 10^0$
- $253_6 : 2 * 6^2 + 5 * 6^1 + 3 * 6^0 = 105$
  -Congrats, you just converted your first number to base-10

Base-10 Exercises

- Try some for yourself now:
    i. 136_7 = __
    ii. 423_5 = __

Base-10 Exercises

- Try some for yourself now:
  i. $136_7 = 76 = 1 * 7^2 + 3 * 7^1 + 6 * 7^0$
  ii. $423_5 = 113 = 4 * 5^2 + 2 * 5^1 + 3 * 5^0$

Translating from Base-10 to Base-`a`

- Less intuitive, but there is a process you follow to convert numbers and you'll be ok
- The rationale for why it works is because it relies on the fundamental concept of place value and the way different bases represent numeric values using powers of the base.
    - Basically, a person centuries ago decided this when numbers were invented

Translating from Base-10 to Base-`a`

- The process in question:

1. **Divide and Record**: Divide the number by the base and record the remainder.
2. **Repeat**: Repeat step 1 with the quotient until it becomes zero.
    - i.e: Something like 2 / 5 because 5 cannot go into 2 without talking about decimals
3. **Read in Reverse**: Read the remainders in reverse order to get the converted number.

- An example will make this (a lot) easier to swallow

Example - Base 2 (Binary)

- Let's convert 25 (base-10) to binary (base 2).
- Step 1: 25 / 2 = 12 with a remainder of 1.
  - Record the 1.
- Step 2: 12 / 2 = 6 with a remainder of 0.
  - Record the 0.
- Step 3: 6 / 2 = 3 with a remainder of 0.
  - Record the 0.
- Step 4: 3 / 2 = 1 with a remainder of 1.
  - Record the 1.
- Step 5: 1 / 2 = 0 with a remainder of 1.
  - Record the 1.
- Reading in reverse: 11001_2
  -Therefore, 25 in base 10 is 11001_2 in binary.

Example - Base 5

- Let's convert 52 (base-10) to base 5.
- Step 1: 52 / 5 = 10 with a remainder of 2.
  - Record the 2.
- Step 2: 10 / 5 = 2 with a remainder of 0.
  - Record the 0.
- Step 3: 2 / 5 = 0 with a remainder of 2.
  - Record the 2.
- Reading in reverse: 202. Therefore, 52 in base 10 is 202_5

Example - Base 16 (Hexadecimal)

- Wait, base 16? But we only have 9 numbers for digits
  - We use letters to represent bases greater than 10
- So for something like base 16, we need 6 additional letters to represent digits past 9.
  - Luckily, they just go in alphabetical order:
    - A = 10
    - B = 11
    - C = 12
    - D = 13
    - E = 14
    - F = 15
  - Remember there's no digit for `a` in base-`a`
    - We only go from digits 0-9 for base **10**

Example - Base 16 (Hexadecimal)

- Now, we just do the same process! But the remainders >= 10, we sub out for the letters.
    - Note that, this also applies from converting to from base 16 to base 10, we just use powers of 16 as our base and follow the same process
- Let's try this now:

  Convert 189 (base 10) to base 16 (hexadecimal)

Example - Base 16 (Hexadecimal)

- Let's convert 189 (base 10) to base 16 (hexadecimal)
- Step 1: 189 / 16 = 11 with a remainder of 13.
  - Record D (representing 13 in hexadecimal)
- Step 2: 11 / 16 = 0 with a remainder of 11.
  - Record B (representing 11 in hexadecimal)
- Reading in reverse: BD. Therefore, 189 in base 10 is BD in base 16 (hexadecimal).

Practice

1. Convert 0001 1010 1010 1110 from binary to base 16
2. Convert 41 from octal to base 2
3. Convert 1111 1000 from base 2 to decimal
4. Convert 0011 from base 2 to decimal
5. Convert 0110 0000 from base 2 to decimal
6. Convert 0100 from base 2 to base 10
7. Convert 76 from octal to base 2
8. Convert 4563 from hexadecimal to base 2
9. Convert 1111 1011 0111 1110 from base 2 to base 16
10. Convert 5 from octal to base 2

Answers: Online helper: https://www.rapidtables.com/convert/number/base-converter.html

Correct answer: 1aae

Correct answer: 100 001

Correct answer: 248

Correct answer: 3

Correct answer: 96

Correct answer: 4

Correct answer: 111 110

Correct answer: 0100 0101 0110 0011

Correct answer: fb7e

Correct answer: 101