CMSC 451 11/04

Network flow (IV): Applications.

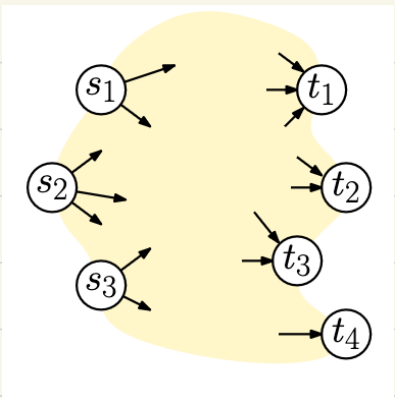
Solving / Reducing a Problem to Maxflow

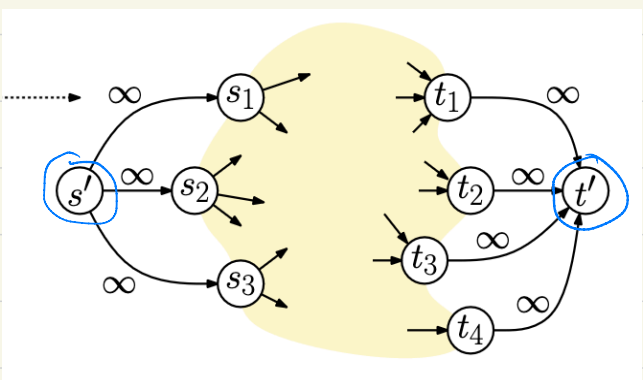
1. construct a flow network G
2. prove that max flow on G
   $\Leftrightarrow$ a solution of the problem
3. time complexity
   what's n & m of G.

Reducing A to B:

1. given an instance of A, construct an instance of B.
2. prove: B has a solution $\Leftrightarrow$
   A has a solution.
3. size of B instance is related to size of A instance.

Multi-source / Multi-sink.





Need to prove: flow in new graph
$\Leftrightarrow$ flow in old graph.

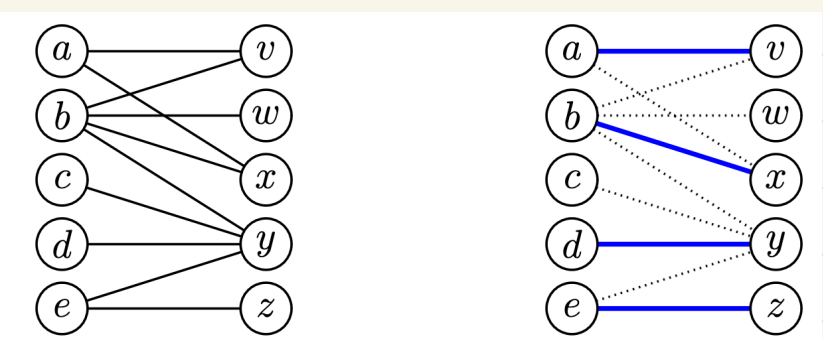
Bipartite Matching

Bipartite graph: An **undirected** graph $G=(V, E)$, s.t.

V can be **partitioned** into X and Y, and
all edges in E are **across** X and Y.

Matching of a bipartite graph:

Subset $M \subseteq E$, s.t. for each $v \in V$,
at most **one edge** in M **incident** to v.



Maximum Matching Problem: maximize $|M|$

Construct a flow network $G' = (V', E')$

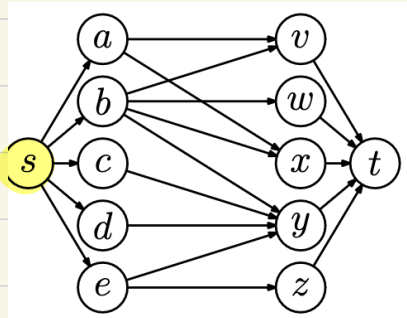
(1) add s & t.

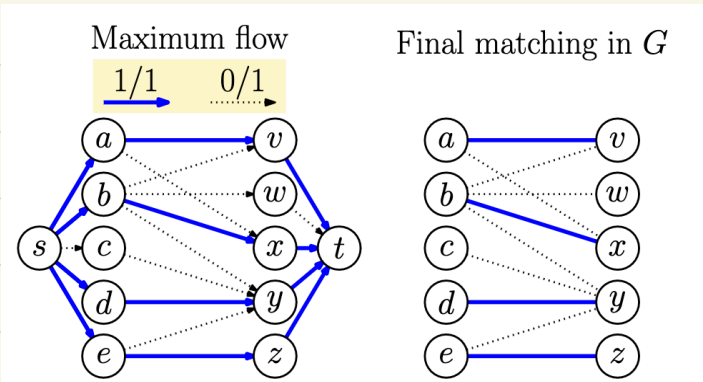
(2) $\forall u \in X$, add $s \to u$ w/ $c = 1$.

$\forall v \in Y$, add $v \to t$ w/ $c = 1$.

(3) $\forall (u, v) \in E$, $u \in X$ & $v \in Y$,

add $u \to v$ w/ $c = 1$.





Claim: Max-flow of value k in G'
$\Leftrightarrow$ Maximum matching of size k in G.

Proof.

(1) matching of size k in G $\Rightarrow$
flow of size k in G'.

construct a flow:

$(u,v) \in M \Rightarrow f(u,v) = 1$

$(u,v) \notin M \Rightarrow f(u,v) = 0$

(2) max. flow of value k in G' $\Rightarrow$
matching of size k in G

Integer flow theorem $\Rightarrow$ f is integer flow.

$f(u,v) = 0$ or 1.

$(u,v) \in M$ if $f(u,v) = 1$

... $\notin$ ... $= 0$

Circulations.

Given a digraph $G = (V, E)$,

$(u, v) \in E$ has a capacity $c(u, v)$

$u \in V$ has a demand value $d(u)$

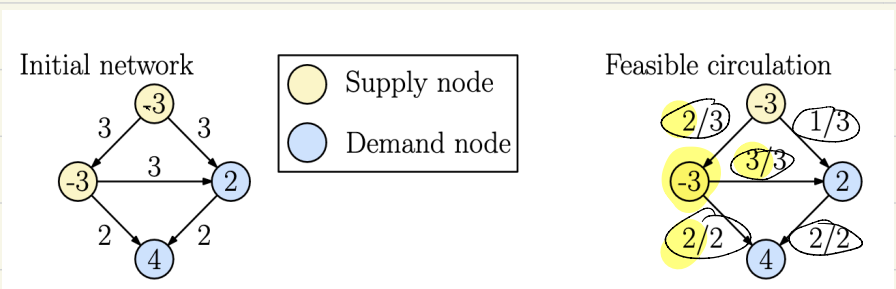
supply when $d(u) < 0$.

Find flow value $f(u, v)$, s.t.

Capacity constraints: $0 \leq f(u, v) \leq c(u, v)$

Supply / Demand constraints:

$$f^{in}(u) - f^{out}(u) = d(u)$$



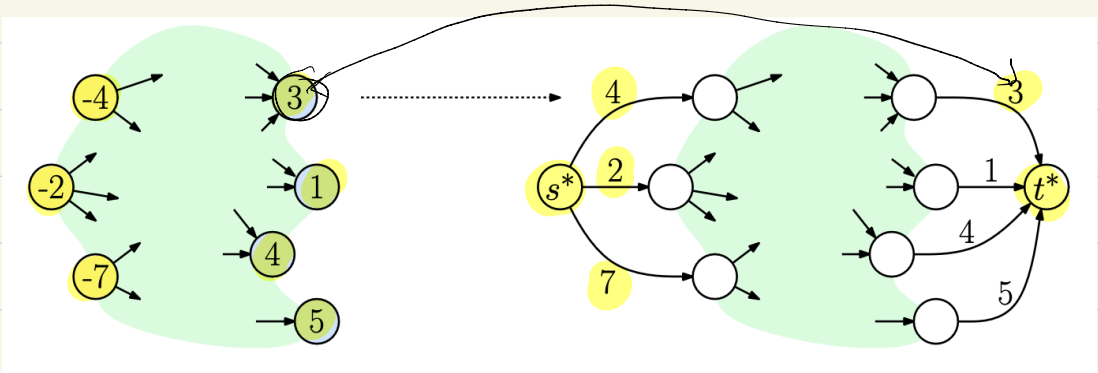
- A decision problem (Yes / No)

Max-flow → Circulation : (Circ. satisfied ⇔ Max flow ≥ D)

Threshold D, let $d(s) = -D$, $d(t) = D$

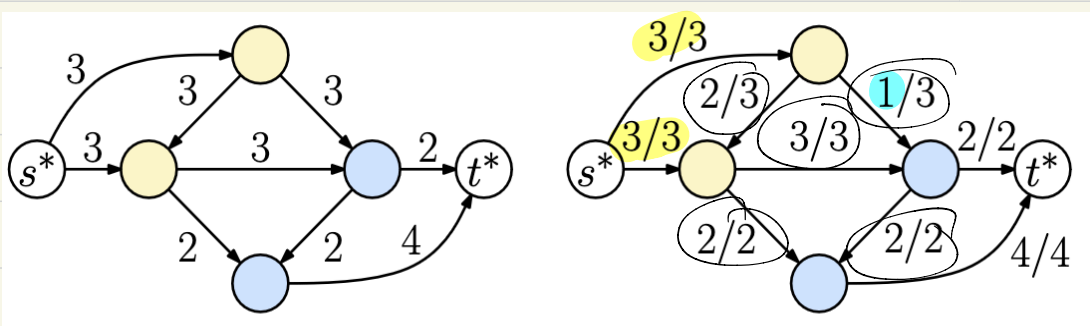
$d(u) = 0$

Reducing Circulation to Max Flow



Max-flow on G' = $\sum$ demands.

$\Leftrightarrow$ Feasible circulation.



Circulations w/ lower bounds

Given a digraph $G = (V, E)$,

$(u,v) \in E$ has a capacity $c(u,v)$

a lower bound $\ell(u,v)$

$u \in E$ has a demand value $d(u)$

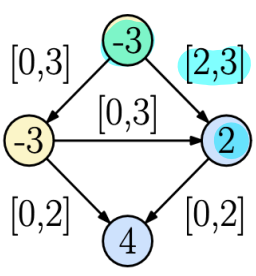
supply when $d(u) < 0$.

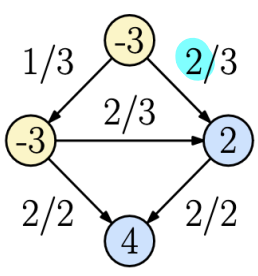
Find flow value $f(u,v)$, s.t.

Capacity constraints: $\ell(u,v) \leq f(u,v) \leq c(u,v)$

Supply / Demand constraints:

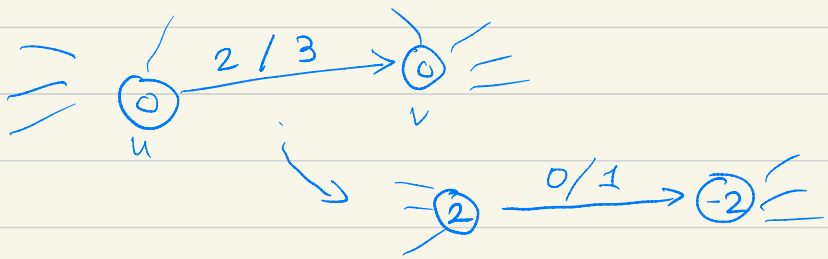
$$f^{in}(u) - f^{out}(u) = d(u)$$

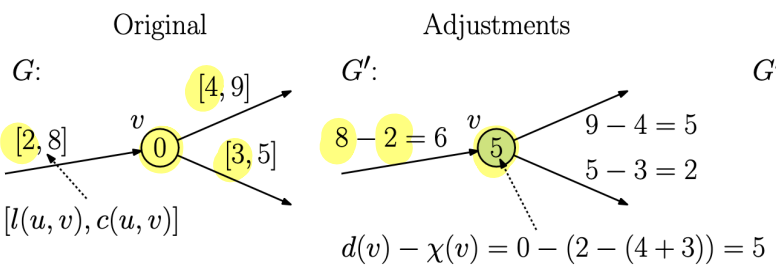




Reducing Circulation w/ lower bound to w/0 lower bound (then to max flow)

1. force all flow w/ lower bound.
   problem: it is not balanced.
2. adjust supply / demand to balance





Survey Design

- k different products
- n customers

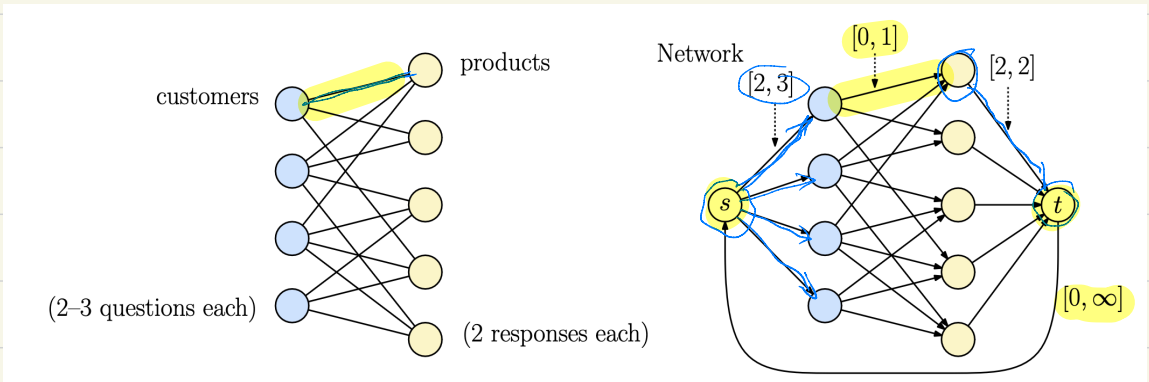
want to send questionnaire to customers s.t.

1. only ask about products the customer has bought
2. for customer #i, only ask $[c_i^-, c_i^+]$ products
3. for product #j, only ask $[p_j^-, p_j^+]$ customers

Q: is it feasible?

→ bipartite graph.

Plan: reducing to circulation w/ lower bounds.


customers
products
(2–3 questions each)
(2 responses each)
Network
[0, 1]
[2, 3]
[2, 2]
s
t
[0, ∞]