Problem Set #2

CMSC 858L
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Due on Gradescope Oct. 8, 2026, 11:59 PM

General instructions: No use of LLMs. If you collaborate or use any outside resources, remember to cite them in your solutions.

Problem #1. Semi-local simulation of the Clifford group (50 pts.)

A stabilizer state is any state that can be created by applying a Clifford group circuit to the all 0s state $|00\ldots0\rangle$. Imagine we have a stabilizer state where the qubits are in spatially separated locations, possibly with multiple qubits in each location. A *local* Clifford circuit is one for which each layer is a tensor product of Clifford group gates on the different locations, and in particular, there are no gates that span multiple locations. At the end of the circuit, all qubits are measured in the standard basis.

A *local* classical simulation is one where after an initial set-up phase that depends on the state (but not the circuit) and may involve communication, all classical computations (which now depend on the circuit) are themselves local, involving only data and computation at each location separately. To be clear, here is the structure of a local classical simulation:

1. We are given a description of a stabilizer state $|\psi\rangle$ (provided by giving its stabilizer) with a specified split between multiple sites. Using classical communication and storage, we set up variables localized to each site.

2. We are given the description of a local Clifford circuit. Each site is only informed of the part of the Clifford circuit at its own location.

3. We now must perform local classical computations at each site to perform the simulation.

4. Each site outputs one or more bits.

In all cases in this problem, the goal is to perform an exact *weak* simulation, i.e. to output bit strings with exactly the same probability distribution as the corresponding quantum circuit.

a) (10 pts.) Suppose the initial state $|\psi\rangle$ is an n-qubit stabilizer state which is a tensor product between 2 sites. Show that there is an efficient local classical simulation of any local Clifford circuit acting on $|\psi\rangle$ followed by a standard basis measurement.

b) (10 pts.) Suppose the initial state is $|\psi\rangle = |00\rangle + |11\rangle$, which has stabilizer generated by $M_1 = X \otimes X$ and $M_2 = Z \otimes Z$, with the two qubits split between separate sites. Consider local Clifford circuits consisting of a tensor product of two single-qubit Clifford group gates, followed by standard basis measurement. Find a local classical simulation of this family of circuits.

c) (10 pts.) Now suppose the initial state is the three-qubit state $|000\rangle + |111\rangle$ (stabilizer generated by $M_1 = X \otimes X \otimes X$, $M_2 = Z \otimes Z \otimes I$, $M_3 = I \otimes Z \otimes Z$) split between 3 sites. Show that if each site *does* learn the full Clifford circuit, including gates applied to other sites, that there is an otherwise local simulation of the circuit.

d) (10 pts.) Again consider the state $|000\rangle + |111\rangle$ split between 3 sites. This state displays non-locality under local Clifford circuits and measurements, and thus there is no fully local classical simulation (efficient or inefficient) where the sites do not know the Clifford group gates on the other sites. Find a classical simulation in which one site is allowed to broadcast one or two classical bits of information to the other sites, but the simulation is in other respects local. (It is possible to do this with 1 bit, in fact, but 2 is easier.)

e) (10 pts.) Suppose you are given an arbitrary stabilizer state split between 2 sites A and B each with n qubits and the local Clifford circuits on sites A and B use T_A and T_B Clifford group gates, respectively. Find a classical simulation with $O(\text{poly}(n, T_A, T_B))$ gates which is local except for a transmission of $O(n^2)$ classical bits from A to B, independent of T_A and T_B. (Note that no communication from B to A is allowed.)

Problem #3. Matchgates with a $|+\rangle$ state (50 pts.)

a) (10 pts.) Show that X_i and H_i are not valid matchgates for any i. Here, X_i is the bit flip on the ith qubit and H_i is the Hadamard on the ith qubit.

b) (10 pts.) Show that, given the state $|+\rangle \otimes |\psi\rangle$, you can create the state $|+\rangle \otimes X_i|\psi\rangle$ for any i. Here, $|+\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$.

c) (10 pts.) Show that, given the state $|+\rangle \otimes |\psi\rangle$, you can create the state $|+\rangle \otimes H_1|\psi\rangle$. (The state $|+\rangle$ is qubit 0 and you need to implement Hadamard on the adjacent qubit, the first qubit of $|\psi\rangle$.)

d) (10 pts.) Show that, given the state $|+\rangle \otimes |00\ldots0\rangle$, you can use matchgates to prepare any state of the form $|+\rangle \otimes |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_{n-1}\rangle$, i.e., a tensor product of one $|+\rangle$ state with arbitrary single-qubit states.

Hint: You may wish to use the fact that Hadamard plus single-qubit diagonal gates generate all single-qubit unitaries.

e) (10 pts.) Suppose we have a computer which starts with the state $|+\rangle \otimes |00\ldots0\rangle$ and can perform matchgates and SWAP gates interchanging two adjacent qubits, ending with measurement of every qubit in the standard basis. Show that this computer can give an efficient exact weak simulation of any quantum circuit with arbitrary gates starting with the $|00\ldots0\rangle$ state and ending with measurement of every qubit in the standard basis.