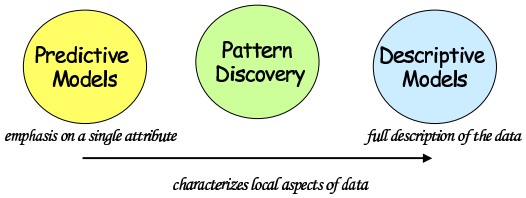
CMSC828G **Principles of Data Mining** Lecture #22

- Today's Reading:
  - HMS 6.7, ch. 13
- Today's Lecture:
  - Finding Patterns and Rules
    - Association Rules
- Upcoming Due Dates:
  - H3 due today
  - Project Writeup due 4/30

Big Picture



Finding Patterns and Rules

- Examples:
  - supermarket transaction database,
    10% of the customers buy wine and cheese
  - telecommunications alarms database
    if alarms A and B occur within 30 seconds of each other, then alarm C occurs within 60 seconds with probability 0.5
  - web log dataset
    if a person visits the CNN Web site, there is 60% chance the person will visit the ABC News Web site in the same month

Mining Rules

- Problems?
  - number of rules grows exponentially with the number of attributes
  - among the large number of rules generated, how do we determine interesting rules?

What Is Association Mining?

- Association rule mining:
  - Finding frequent patterns, associations, correlations, or causal structures among sets of items or objects in transaction databases, relational databases, and other information repositories.
- Applications:
  - Basket data analysis, cross-marketing, catalog design, loss-leader analysis, clustering, classification, etc.
- Examples.
  - Rule form: "Body → Head [support, confidence]".
  - buys(x, "diapers") → buys(x, "beers") [0.5%, 60%]
  - major(x, "CS") ^ takes(x, "DB") → grade(x, "A") [1%, 75%]

Association Rule Flavors

- Boolean vs. quantitative associations (Based on the types of values handled)
  - buys(x, "SQLServer") ^ buys(x, "DMBook") → buys(x, "DBMiner") [0.2%, 60%]
  - age(x, "30..39") ^ income(x, "42..48K") → buys(x, "PC") [1%, 75%]
- Single dimension vs. multiple dimensional associations
- Single level vs. multiple-level analysis
  - What brands of beers are associated with what brands of diapers?
- Various extensions
  - Correlation, causality analysis
    - Association does not necessarily imply correlation or causality
  - Maxpatterns and closed itemsets
  - Constraints enforced
    - E.g., small sales (sum < 100) trigger big buys (sum > 1,000)?

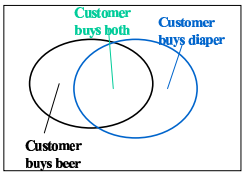
Market Basket Analysis

- Given some set of *items*
  1. database of transactions,
  2. each transaction is a list of items (purchased by a customer in a visit), called a *basket*
- Find: all rules that correlate the presence of one set of items with that of another set of items in a basket
  - E.g., *98% of people who purchase tires and auto accessories also get automotive services done*
- Other problems with this structure:
  - baskets = documents; items = words
  - baskets = web pages; items = links

Market Basket Analysis, cont

- Typically transaction database too large to fit in main memory
  - common sizes
    number of transactions: $10^5 - 10^8$
    number of items: $10^2 - 10^6$
  - typically sparse, 0.1% chance of random purchase
  - commonly stored
    - in a relational data base, Baskets(BID,item)
    - flat file (BID,$item_1$,$item_2$,...,$item_n$)
- Evaluating running time:
  - count the number of passes through the data
  - principle cost time to read data from disk

Rule Measures: Support and Confidence



Find all the rules $X \& Y \Rightarrow Z$ with minimum confidence and support

support, *s*, proportion of transactions containing{X, Y, Z}

confidence, *c*, proportion of transactions which have {X, Y} and also contain *Z*

Example: Support and Confidence

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

Let minimum support 50%, and minimum confidence 50%, we have:

$A \Rightarrow C$ S = 50% C = 66.6%

$C \Rightarrow A$ S = 50% C = 100%

Mining Association Rules—An Example

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

Min. support 50%
Min. confidence 50%

Frequent Itemset	Support
{A}	75%
{B}	50%
{C}	50%
{A,C}	50%

For rule $A \Rightarrow C$:
support = support({*A*,*C*}) = 50%
confidence = support({*A*,*C*})/support({*A*}) = 66.6%

The Apriori principle:
Any subset of a frequent itemset must be frequent

Finding Frequent Itemsets

- Find the *frequent itemsets*: the sets of items that have minimum support
  - **Monotonicity principle**: A subset of a frequent itemset must also be a frequent itemset
    - i.e., if {*AB*} is a frequent itemset, both {*A*} and {*B*} should be a frequent itemset
  - Iteratively find frequent itemsets with cardinality from 1 to *k* (*k*-itemset)
- Use the frequent itemsets to generate association rules.

Finding frequent itemsets, cont.

- Two approaches
  - Proceed levelwise, first find frequent items (sets of size 1), then frequent pairs, next frequent triples
    - most time consuming step is often finding pairs
    - one pass over the data is made for each level
  - Find all maximal frequent itemsets (I.e., sets S such that no proper superset of S is frequent)
    - one or several passes over the data

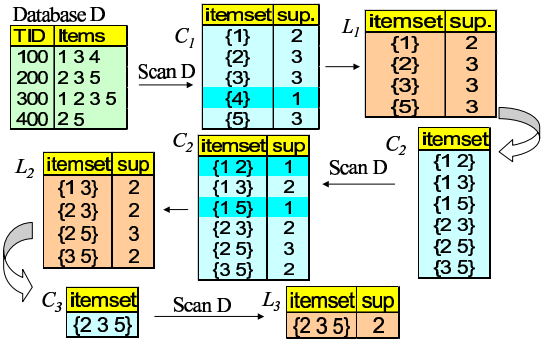
The Apriori Algorithm

- Join Step: C_k is generated by joining L_{k-1} with itself
- Prune Step: Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset

C_k: Candidate itemset of size k
L_k: frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
end
return $\cup_k L_k$;

The Apriori Algorithm — Example



How to Generate Candidates?

- Suppose the items in L_{k-1} are ordered
- Step 1: self-joining L_{k-1}

  insert into C_k
  select $p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}$
  from L_{k-1} p, L_{k-1} q
  where $p.item_1=q.item_1, \ldots, p.item_{k-2}=q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$
- Step 2: pruning

  forall ***itemsets c in*** C_k do
    forall ***(k-1)-subsets s of c*** do
      if (*s is not in* L_{k-1}) **then delete** *c* **from** C_k

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

Example of Generating Candidates

- L_3={*abc, abd, acd, ace, bcd*}
- Self-joining: L_3*L_3
  - *abcd* from *abc* and *abd*
  - *acde* from *acd* and *ace*
- Pruning:
  - *acde* is removed because *ade* is not in L_3
- C_4={*abcd*}

Methods to Improve Apriori's Efficiency

- Hash-based itemset counting: A k-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- Transaction reduction: A transaction that does not contain any frequent k-itemset is useless in subsequent scans
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
- Sampling: mining on a subset of given data, lower support threshold + a method to determine the completeness
- Dynamic itemset counting: add new candidate itemsets only when all of their subsets are estimated to be frequent

Is Apriori Fast Enough? — Performance Bottlenecks

- The core of the Apriori algorithm:
  - Use frequent $(k-1)$-itemsets to generate candidate frequent k-itemsets
  - Use database scan and pattern matching to collect counts for the candidate itemsets
- The bottleneck of *Apriori*: candidate generation
  - Huge candidate sets:
    - 10^4 frequent 1-itemset will generate 10^7 candidate 2-itemsets
    - To discover a frequent pattern of size 100, e.g., $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ candidates.
  - Multiple scans of database:
    - Needs $(n+1)$ scans, n is the length of the longest pattern

Mining Frequent Patterns Without Candidate Generation

- Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only

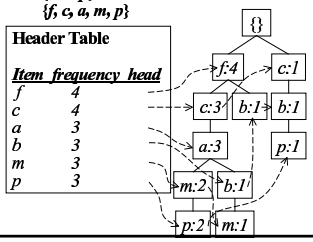
Construct FP-tree from a Transaction DB

TID	Items bought	(ordered) frequent items
100	{f, a, c, d, g, i, m, p}	{f, c, a, m, p}
200	{a, b, c, f, l, m, o}	{f, c, a, b, m}
300	{b, f, h, j, o}	{f, b}
400	{b, c, k, s, p}	{c, b, p}
500	{a, f, c, e, l, p, m, n}	{f, c, a, m, p}

min_support = 0.5

Steps:

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Order frequent items in frequency descending order
3. Scan DB again, construct FP-tree



Benefits of the FP-tree Structure

- Completeness:
  - never breaks a long pattern of any transaction
  - preserves complete information for frequent pattern mining
- Compactness
  - reduce irrelevant information—infrequent items are gone
  - frequency descending ordering: more frequent items are more likely to be shared
  - never be larger than the original database
  - Example: For Connect-4 DB, compression ratio could be over 100

Mining Frequent Patterns Using FP-tree

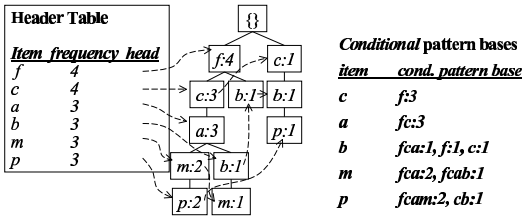
- General idea (divide-and-conquer)
  - Recursively grow frequent pattern path using the FP-tree
- Method
  - For each item, construct its conditional pattern-base, and then its conditional FP-tree
  - Repeat the process on each newly created conditional FP-tree
  - Until the resulting FP-tree is empty, or it contains only one path (single path will generate all the combinations of its sub-paths, each of which is a frequent pattern)

Major Steps to Mine FP-tree

1) Construct conditional pattern base for each node in the FP-tree
2) Construct conditional FP-tree from each conditional pattern-base
3) Recursively mine conditional FP-trees and grow frequent patterns obtained so far
   - If the conditional FP-tree contains a single path, simply enumerate all the patterns

Step 1: From FP-tree to Conditional Pattern Base

- Starting at the frequent header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item
- Accumulate all of transformed prefix paths of that item to form a conditional pattern base

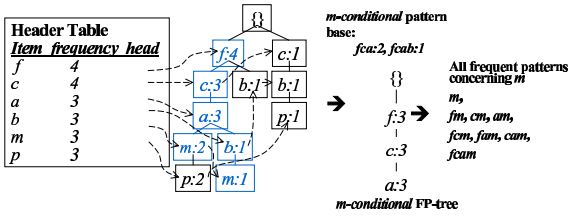


Properties of FP-tree for Conditional Pattern Base Construction

- Node-link property
  - For any frequent item a_i, all the possible frequent patterns that contain a_i can be obtained by following a_i's node-links, starting from a_i's head in the FP-tree header
- Prefix path property
  - To calculate the frequent patterns for a node a_i in a path P, only the prefix sub-path of a_i in P need to be accumulated, and its frequency count should carry the same count as node a_i.

Step 2: Construct Conditional FP-tree

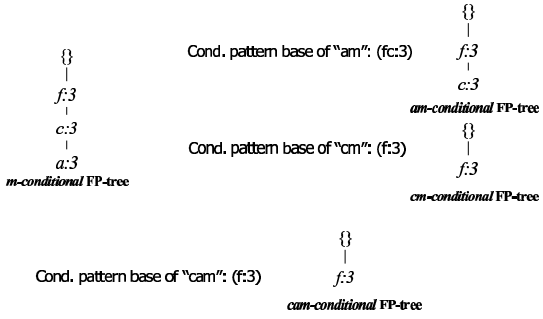
- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base



Mining Frequent Patterns by Creating Conditional Pattern-Bases

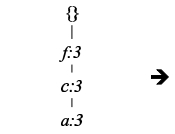
Item	Conditional pattern-base	Conditional FP-tree
p	{(fcam:2), (cb:1)}	{(c:3)}|p
m	{(fca:2), (fcab:1)}	{(f:3, c:3, a:3)}|m
b	{(fca:1), (f:1), (c:1)}	Empty
a	{(fc:3)}	{(f:3, c:3)}|a
c	{(f:3)}	{(f:3)}|c
f	Empty	Empty

Step 3: Recursively mine the conditional FP-tree



Single FP-tree Path Generation

- Suppose an FP-tree T has a single path P
- The complete set of frequent pattern of T can be generated by enumeration of all the combinations of the sub-paths of P



→ **All frequent patterns concerning *m***
***m*,**
***fm, cm, am*,**
***fcm, fam, cam*,**
fcam

***m-conditional* FP-tree**

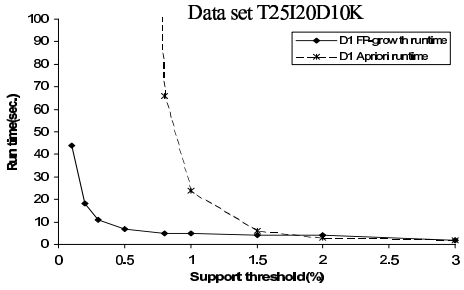
Principles of Frequent Pattern Growth

- Pattern growth property
  - Let α be a frequent itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then $\alpha \cup \beta$ is a frequent itemset in DB iff β is frequent in B.
- "*abcdef*" is a frequent pattern, if and only if
  - "*abcde*" is a frequent pattern, and
  - "*f*" is frequent in the set of transactions containing "*abcde*"

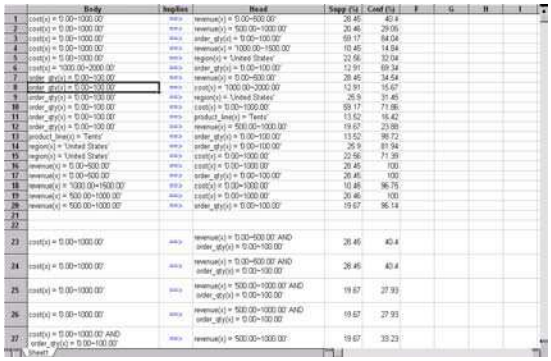
Why Is Frequent Pattern Growth Fast?

- Performance study shows
  - in some cases FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection
- Reasoning
  - No candidate generation, no candidate test
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building

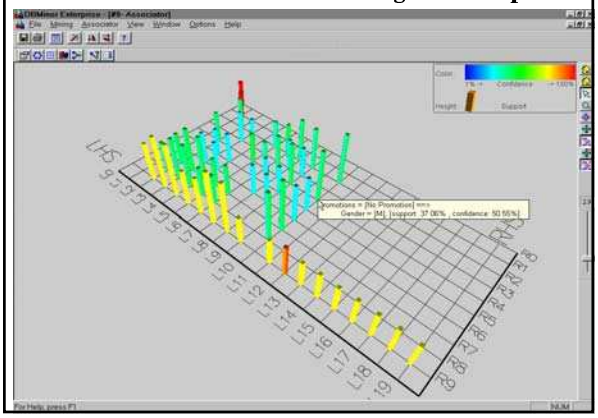
FP-growth vs. Apriori: Scalability With the Support Threshold



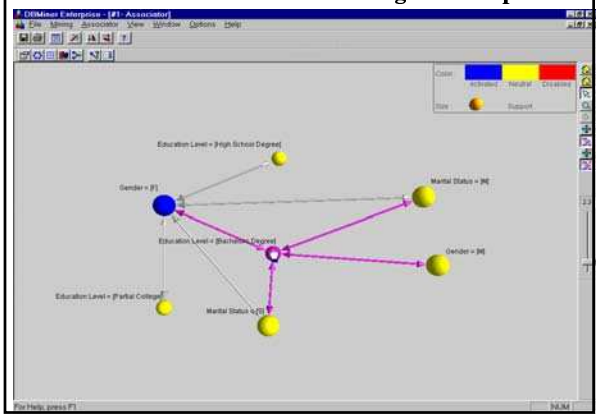
Presentation of Association Rules (Table Form)



Visualization of Association Rule Using Plane Graph



Visualization of Association Rule Using Rule Graph



Iceberg Queries

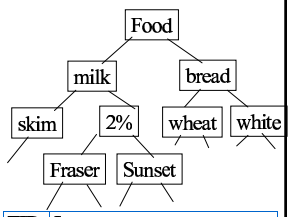
- Icerberg query: Compute aggregates over one or a set of attributes only for those whose aggregate values is above certain threshold
- Example:

```
select P.custID, P.itemID, sum(P.qty)
from purchase P
group by P.custID, P.itemID
having sum(P.qty) >= 10
```

- Compute iceberg queries efficiently by Apriori:
  - First compute lower dimensions
  - Then compute higher dimensions only when all the lower ones are above the threshold

Multiple-Level Association Rules

- Items often form hierarchy.
- Items at the lower level are expected to have lower support.
- Rules regarding itemsets at appropriate levels could be quite useful.
- Transaction database can be encoded based on dimensions and levels
- We can explore shared multi-level mining



TID	Items
T1	{111, 121, 211, 221}
T2	{111, 211, 222, 323}
T3	{112, 122, 221, 411}
T4	{111, 121}
T5	{111, 122, 211, 221, 413}

Mining Multi-Level Associations

- A top_down, progressive deepening approach:
  - First find high-level strong rules:
    milk → bread [20%, 60%].
  - Then find their lower-level "weaker" rules:
    2% milk → wheat bread [6%, 50%].
- Variations at mining multiple-level association rules.
  - Level-crossed association rules:
    2% *milk* → *Wonder wheat bread*
  - Association rules with multiple, alternative hierarchies:
    2% *milk* → *Wonder bread*

Multi-level Association: Uniform Support vs. Reduced Support

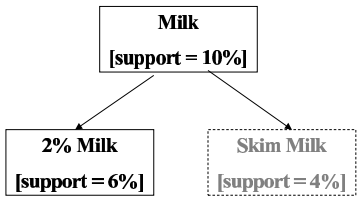
- Uniform Support: the same minimum support for all levels
  - + One minimum support threshold. No need to examine itemsets containing any item whose ancestors do not have minimum support.
  - – Lower level items do not occur as frequently. If support threshold
    - too high ⇒ miss low level associations
    - too low ⇒ generate too many high level associations
- Reduced Support: reduced minimum support at lower levels
  - There are 4 search strategies:
    - Level-by-level independent
    - Level-cross filtering by k-itemset
    - Level-cross filtering by single item
    - Controlled level-cross filtering by single item

Uniform Support

Multi-level mining with uniform support

Level 1
min_sup = 5%

Level 2
min_sup = 5%



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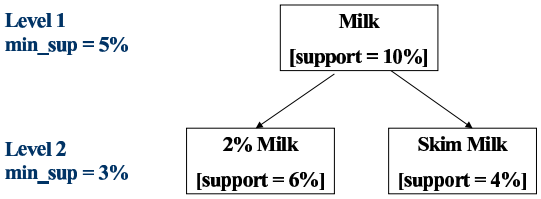
Reduced Support

Multi-level mining with reduced support



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Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to "ancestor" relationships between items.
- Example
  - milk ⇒ wheat bread [support = 8%, confidence = 70%]
  - 2% milk ⇒ wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule.
- A rule is redundant if its support is close to the "expected" value, based on the rule's ancestor.

Multi-Level Mining: Progressive Deepening

- A top-down, progressive deepening approach:
  - First mine high-level frequent items:
    milk (15%), bread (10%)
  - Then mine their lower-level "weaker" frequent itemsets:
    2% milk (5%), wheat bread (4%)
- Different min_support threshold across multi-levels lead to different algorithms:
  - If adopting the same *min_support* across multi-levels
    then toss *t* if any of *t*'s ancestors is infrequent.
  - If adopting reduced *min_support* at lower levels
    then examine only those descendents whose ancestor's support is frequent/non-negligible.

Progressive Refinement of Data Mining Quality

- Why progressive refinement?
  - Mining operator can be expensive or cheap, fine or rough
  - Trade speed with quality: step-by-step refinement.
- Superset coverage property:
  - Preserve all the positive answers—allow a positive false test but not a false negative test.
- Two- or multi-step mining:
  - First apply rough/cheap operator (superset coverage)
  - Then apply expensive algorithm on a substantially reduced candidate set (Koperski & Han, SSD'95).

Multi-Dimensional Association: Concepts

- Single-dimensional rules:
  buys(X, "milk") ⇒ buys(X, "bread")
- Multi-dimensional rules: ❍ 2 dimensions or predicates
  - Inter-dimension association rules (*no repeated predicates*)
    age(X,"19-25") ∧ occupation(X,"student") ⇒ buys(X,"coke")
  - hybrid-dimension association rules (*repeated predicates*)
    age(X,"19-25") ∧ buys(X, "popcorn") ⇒ buys(X, "coke")
- Categorical Attributes
  - finite number of possible values, no ordering among values
- Quantitative Attributes
  - numeric, implicit ordering among values

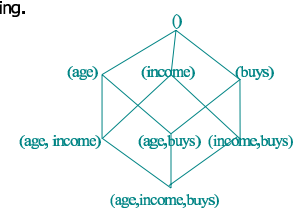
Techniques for Mining MD Associations

- Search for frequent *k*-predicate set:
  - Example: {age, occupation, buys} is a 3-predicate set.
  - Techniques can be categorized by how age are treated.

1. Using static discretization of quantitative attributes
   - Quantitative attributes are statically discretized by using predefined concept hierarchies.
2. Quantitative association rules
   - Quantitative attributes are dynamically discretized into "bins"based on the distribution of the data.
3. Distance-based association rules
   - This is a dynamic discretization process that considers the distance between data points.

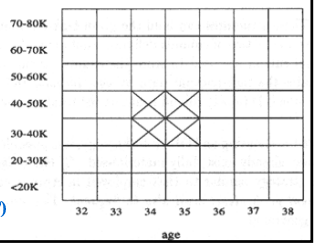
Static Discretization of Quantitative Attributes

- Discretized prior to mining using concept hierarchy.
- Numeric values are replaced by ranges.
- In relational database, finding all frequent k-predicate sets will require *k* or *k*+1 table scans.
- Data cube is well suited for mining.
- The cells of an n-dimensional cuboid correspond to the predicate sets.
- Mining from data cubes can be much faster.



Quantitative Association Rules

- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized.
- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Cluster "adjacent" association rules to form general rules using a 2-D grid.
- Example:



age(X,"30-34") ∧ income(X,"24K - 48K") ⇒ buys(X,"high resolution TV")

Clusters and Distance Measurements

- S[X] is a set of N tuples $t_1, t_2, \ldots, t_N$, projected on the attribute set X
- The diameter of S[X]:

$$d(S[X]) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} dist_X(t_i[X], t_j[X])}{N(N-1)}$$

  - $dist_X$: distance metric, e.g. Euclidean distance or Manhattan

Clusters and Distance Measurements(Cont.)

- The diameter, *d*, assesses the density of a cluster C_X, where

$$d(C_X) \le d_0^X$$

$$|C_X| \ge s_0$$

- Finding clusters and distance-based rules
  - the density threshold, d_0, replaces the notion of support
  - modified version of the BIRCH clustering algorithm

Interestingness Measurements

- Objective measures
  Two popular measurements:
  ☆ *support;* and
  ☆ *confidence*
- Subjective measures (Silberschatz & Tuzhilin, KDD95)
  A rule (pattern) is interesting if
  ☆ it is *unexpected* (surprising to the user); and/or
  ☆ *actionable* (the user can do something with it)

Criticism to Support and Confidence

- Example 1: (Aggarwal & Yu, PODS98)
  - Among 5000 students
    - 3000 play basketball
    - 3750 eat cereal
    - 2000 both play basket ball and eat cereal
  - *play basketball* ⇒ *eat cereal* [40%, 66.7%] is misleading because the overall percentage of students eating cereal is 75% which is higher than 66.7%.
  - *play basketball* ⇒ *not eat cereal* [20%, 33.3%] is far more accurate, although with lower support and confidence

	basketball	not basketball	sum(row)
cereal	2000	1750	3750
not cereal	1000	250	1250
sum(col.)	3000	2000	5000

Criticism to Support and Confidence (Cont.)

- Example 2:
  - X and Y: positively correlated,
  - X and Z, negatively related
  - support and confidence of X=>Z dominates
- We need a measure of dependent or correlated events

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

$$corr_{A,B} = \frac{P(A \cup B)}{P(A)P(B)}$$

Rule	Support	Confidence
X=>Y	25%	50%
X=>Z	37.50%	75%

- P(B|A)/P(B) is also called the lift of rule A => B

Other Interestingness Measures: Interest

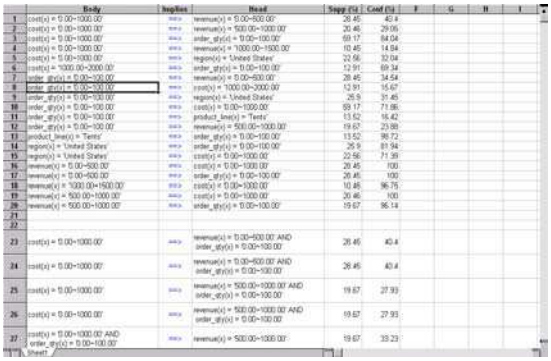
- Interest (correlation, lift) $\frac{P(A \wedge B)}{P(A)P(B)}$
  - taking both P(A) and P(B) in consideration
  - P(A^B)=P(B)*P(A), if A and B are independent events
  - A and B negatively correlated, if the value is less than 1; otherwise A and B positively correlated

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

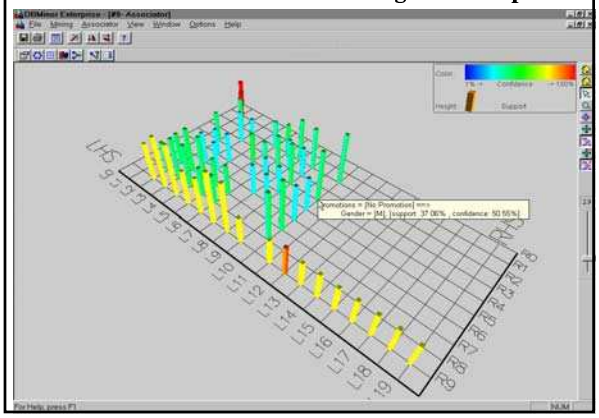
Itemset	Support	Interest
X,Y	25%	2
X,Z	37.50%	0.9
Y,Z	12.50%	0.57

Viewing Association Rules

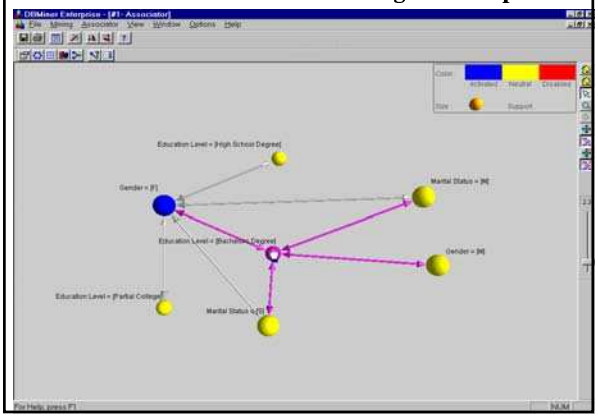
Presentation of Association Rules (Table Form)



Visualization of Association Rule Using Plane Graph



Visualization of Association Rule Using Rule Graph



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- slides from *Data Mining: Concepts and Techniques*, Jan and Kamber, Morgan Kaufman, 2001.