CMSC250, Spring 2004 Homework 10

Due Wednesday, April 14 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

For these problems, you should reduce your answer to a form that only includes only addition, subtraction, multiplication, division, and factorials.

Also, showing your work and writing simple explanations will help in awarding partial credit.

1. "Musical chairs" is a children's game often played at parties. If there are n children at the party, then the game starts with $n-1$ chairs placed in a row. The children walk around the line of chairs while some music is being played. When the music stops, all the children must immediately sit down in any one of the chairs — only one person to a chair. Obviously, there will be one person who doesn't get a chair. That person is now out of the game, one chair is removed from the row, and the music and walking begin again. This continues, with one person being eliminated and one chair removed each round, until there is only one person left, who is declared the winner.

   (a) If there are 5 children at the party, how many ways can they be seated in the chairs when the music stops for the first time? (Assume the person who didn't get a chair has already walked away and is not considered.)

   (b) If there are n children at the party, how many ways can they be seated in the chairs when the music stops for the first time? (Assume the person who didn't get a chair has already walked away and is not considered.)

   (c) If there are 6 children at the party, one of whom is named Max, in how many different orders can they be eliminated during the game if you know that Max will be eliminated first? (All the children, including the winner, must be in the order.)

   (d) If there are n children at the party, one of whom is named Mindy, in how many different orders can they be eliminated during the game if you know that Mindy will be the winner? (All the children, including the winner, must be in the order.)

   (e) Suppose there are four children playing: John, Kate, Lisa, and Mike. You know that John is a better at this game than Mike, and Lisa is better than Kate. Suppose you also know that in playing musical chairs, if player X is better than player Y, then Y will be eliminated before X.

   Using this knowledge, draw a possibility tree showing the possible orders in which the children can be eliminated (so the winners will be at the leaves of the tree).

   How many different orders are there?

   (f) Suppose there are 6 children playing: three boys and three girls.

i. What is the probability that all the boys are eliminated before any girls are eliminated?

ii. What is the probability that the children are eliminated in alphabetical order by their last names? (Assume they all have different last names.)

iii. Consider the first round of the game (when there are 6 children and 5 chairs). Suppose a rule is added to the game that when the children sit in the chairs, no two children of the same gender may be seated next to each other. How many ways can they sit down in the first round?

2. All ranges in this problem are inclusive.

   (a) How many even integers are there from 10 through 99?

   (b) How many integers from 10 through 99 have distinct digits?

   (c) How many even integers from 10 through 99 have distinct digits?

   (d) What is the probability that a randomly chosen two-digit integer has distinct digits?

   (e) What is the probability that a randomly chosen two-digit integer has distinct digits and is even?

3. All ranges in this problem are inclusive.

   (a) How many integers from 20 through 1000 are multiples of 4 or multiples of 9?

   (b) Suppose an integer from 20 through 1000 is randomly chosen. What is the probability that this integer is divisible by 4 or divisible by 9?

   (c) How many integers from 20 through 1000 are neither multiples of 4 nor multiples of 9?

4. Suppose a group of six students attend a concert together.

   (a) How many different ways can they be seated in a row?

   (b) Suppose one of the six has to leave the concert early to finish a CMSC214 project. How many ways can the students be seated in a row of seats if exactly one of the seats is on the aisle and the hard-working CS student must be in the aisle seat?

   (c) Suppose the six students consist of three boyfriend-girlfriend couples and each couple wants to sit together so that the boy is on the right. How many ways can the six be seated?

   (d) Suppose the six students consist of three math majors and three CS majors. Each group wants to sit in three consecutive seats so that they can discuss their current homework problems between sets at the concert. How many ways can they be seated in a row so that the students of the same major are all seated consecutively?