CMSC250, Spring 2004 Homework 10 Answers

Due Wednesday, April 14 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

For these problems, you should reduce your answer to a form that only includes only addition, subtraction, multiplication, division, and factorials.

Also, showing your work and writing simple explanations will help in awarding partial credit.

1. "Musical chairs" is a children's game often played at parties. If there are n children at the party, then the game starts with $n-1$ chairs placed in a row. The children walk around the line of chairs while some music is being played. When the music stops, all the children must immediately sit down in any one of the chairs — only one person to a chair. Obviously, there will be one person who doesn't get a chair. That person is now out of the game, one chair is removed from the row, and the music and walking begin again. This continues, with one person being eliminated and one chair removed each round, until there is only one person left, who is declared the winner.

   (a) If there are 5 children at the party, how many ways can they be seated in the chairs when the music stops for the first time? (Assume the person who didn't get a chair has already walked away and is not considered.)

   Solution: $P(5,4) = 5!/1! = 5!$.

   You may have interpreted this question to mean that the person who walks away never had a chance to sit in a chair in the first place. In that case, the answer is $P(4,4) = 4!$.

   (b) If there are n children at the party, how many ways can they be seated in the chairs when the music stops for the first time? (Assume the person who didn't get a chair has already walked away and is not considered.)

   Solution: $P(n, n-1) = n!/1! = n!$.

   You may have interpreted this question to mean that the person who walks away never had a chance to sit in a chair in the first place. In that case, the answer is $P(n-1, n-1) = (n-1)!$.

   (c) If there are 6 children at the party, one of whom is named Max, in how many different orders can they be eliminated during the game if you know that Max will be eliminated first? (All the children, including the winner, must be in the order.)

   Solution: Pretend the six children stand in a line in the order they're eliminated. Max is always at the front, so there are 5 spots left over for 5 children. Answer is 5!.

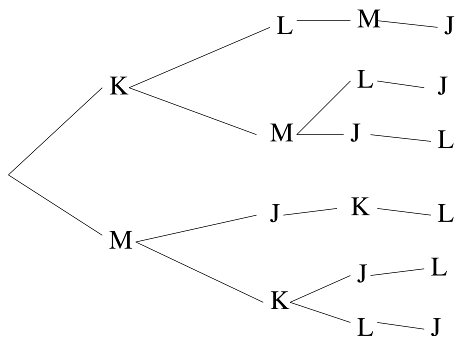
   (d) If there are n children at the party, one of whom is named Mindy, in how many different orders can they be eliminated during the game if you know that Mindy will be the winner? (All the children, including the winner, must be in the order.)

   Solution: Pretend the n children stand in a line in the order they're eliminated. Mindy is always at the end, so there are $n-1$ spots left over for $n-1$ children. Answer is $(n-1)!$.

(e) Suppose there are four children playing: John, Kate, Lisa, and Mike. You know that John is a better at this game than Mike, and Lisa is better than Kate. Suppose you also know that in playing musical chairs, if player X is better than player Y, then Y will be eliminated before X.

Using this knowledge, draw a possibility tree showing the possible orders in which the children can be eliminated (so the winners will be at the leaves of the tree).

How many different orders are there?



Solution:

There are 6 orders.

(f) Suppose there are 6 children playing: three boys and three girls.

i. What is the probability that all the boys are eliminated before any girls are eliminated?

Solution: The order must be BBBGGG. So the size of the event space is $P(3,3)P(3,3) = 36$. The size of the sample space is $P(6,6) = 6!$. Answer is $36/6! = 1/20$.

ii. What is the probability that the children are eliminated in alphabetical order by their last names? (Assume they all have different last names.)

Solution: If you have six people with different last names, then there's only one way to sort them alphabetically. Answer is $1/6! = 1/720$.

iii. Consider the first round of the game (when there are 6 children and 5 chairs). Suppose a rule is added to the game that when the children sit in the chairs, no two children of the same gender may be seated next to each other. How many ways can they sit down in the first round?

Solution: There are five chairs, and the children must be arranged as BGBGB or GBGBG. Consider the first case. Since the locations for each of the sexes are set in advance, all we have to do is permute the 3 boys: $P(3,3) = 3! = 6$ and permute 2 out of the 3 girls: $P(3,2) = 6$. So there are $6 * 6 = 36$ ways to seat them BGBGB. The case for GBGBG also yields 36 possibilities. The answer is $36 + 36 = 72$.

2. All ranges in this problem are inclusive.

(a) How many even integers are there from 10 through 99?

Solution: 45. The first integer is 10, the last is 98. Divide each by two, the first becomes 5, the last is 49. $49 - 5 + 1 = 45$.

(b) How many integers from 10 through 99 have distinct digits?

Solution: There are $99 - 10 + 1 = 90$ numbers in the range. We have to remove all that don't have distinct digits — that is, whose two digits match (11, 22, etc). There are nine of these, so $90 - 9 = 81$.

(c) How many even integers from 10 through 99 have distinct digits?

Solution: We already know there are 45 even integers in the range. Again, we need to remove all the ones that have duplicate digits. But now there are only four of these: 22, 44, 66, 88. $45 - 4 = 41$.

(d) What is the probability that a randomly chosen two-digit integer has distinct digits?

Solution: 81/90.

(e) What is the probability that a randomly chosen two-digit integer has distinct digits and is even?

Solution: 41/90.

3. All ranges in this problem are inclusive.

(a) How many integers from 20 through 1000 are multiples of 4 or multiples of 9?

Solution: Number of integers from 20–1000 that are multiples of 4: The first integer in the range is 20, the last is 1000. Divide each by 4, we get 5 and 250. $250-5+1 = 246$.
Number of integers from 20–1000 that are multiples of 9: The first integer in the range is 27, the last is 999. Divide each by 9, we get 3 and 111. $111 - 3 + 1 = 109$.
Now we need to figure out the number that are divisible by both 4 AND 9 — which means divisible by 36. Number of integers from 20-1000 that are multiples of 36: The first integer in the range is 36, the last is 972. Divide each by 36, we get 1 and 27. $27 - 1 + 1 = 27$.
Now use the inclusion/exclusion rule: $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 246 + 109 - 27 = 328$.

(b) Suppose an integer from 20 through 1000 is randomly chosen. What is the probability that this integer is divisible by 4 or divisible by 9?

Solution: 328/981.

(c) How many integers from 20 through 1000 are neither multiples of 4 nor multiples of 9?

Solution: $981 - 328 = 653$.

4. Suppose a group of six students attend a concert together.

(a) How many different ways can they be seated in a row?

Solution: 6!.

(b) Suppose one of the six has to leave the concert early to finish a CMSC214 project. How many ways can the students be seated in a row of seats if exactly one of the seats is on the aisle and the hard-working CS student must be in the aisle seat?

Solution: 5!.

(c) Suppose the six students consist of three boyfriend-girlfriend couples and each couple wants to sit together so that the boy is on the right. How many ways can the six be seated?

Solution: 3!.

(d) Suppose the six students consist of three math majors and three CS majors. Each group wants to sit in three consecutive seats so that they can discuss their current homework problems between sets at the concert. How many ways can they be seated in a row so that the students of the same major are all seated consecutively?

Solution: $2 * 3! * 3! = 72$.