CMSC250, Spring 2004 Homework 11 Answers

Due Wednesday, April 21 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

For these problems, you should reduce your answer to a form that only includes only addition, subtraction, multiplication, division, and factorials.

Also, showing your work and writing simple explanations will help in awarding partial credit.

1. The Baltimore Orioles are in the last few days of Spring Training in Fort Lauderdale, Florida. They currently have 40 baseball players in camp. The Orioles need to pick 25 players to send to Baltimore when the regular season starts. How many ways can they pick the 25? (Ignoring what position each person plays.)

   Solution: $\binom{40}{25} = \frac{40!}{25!15!}$

2. Assume the 40 players in camp consist of 18 pitchers, 5 catchers, 10 infielders, and 7 outfielders. The 25 players to be sent to Baltimore must consist of 11 pitchers, 2 catchers, 7 infielders, and 5 outfielders. Now how many different sets of players can they send?

   Solution: $\binom{18}{11} \cdot \binom{5}{2} \cdot \binom{10}{7} \cdot \binom{7}{5} = \frac{18!}{11!7!} \cdot \frac{5!}{2!3!} \cdot \frac{10!}{7!3!} \cdot \frac{7!}{5!2!}$

3. Now that the Orioles are in Baltimore with their 25 chosen players, it's time to start playing baseball! Assume the 14 players who are available to bat in the lineup (the 11 pitchers don't bat) consist of 4 "rookies" (young players), and 10 "veterans" (more experienced players). The Orioles need to pick 9 players from these 14 to put in the lineup. To make it more exciting, they decide to choose at random.

   (a) What is the probability that the lineup consists of exactly 2 rookies?

   Solution: $\frac{\binom{4}{2}\binom{10}{7}}{\binom{14}{9}} = \frac{\frac{4!}{2!2!}\frac{10!}{7!3!}}{\frac{14!}{9!5!}} \approx .360$

   (b) What is the probability that the lineup consists of 2 or fewer rookies?

   Solution: $\frac{\binom{4}{2}\binom{10}{7} + \binom{4}{1}\binom{10}{8} + \binom{4}{0}\binom{10}{9}}{\binom{14}{9}} = \frac{\frac{4!}{2!2!}\frac{10!}{7!3!} + \frac{4!}{1!3!}\frac{10!}{8!2!} + \frac{4!}{0!4!}\frac{10!}{9!1!}}{\frac{14!}{9!5!}} \approx .455$

   (c) What is the probability that the lineup consists of all rookies?

   Solution: 0. (Since there aren't enough rookies to make a lineup consisting of all rookies.)

4. The Orioles have now chosen the nine players who will bat in the game.

(a) In how many orders can those nine chosen players be put in?

Solution: $9!$

(b) Assume that four of the nine chosen players are Miguel Tejada, Rafael Palmeiro, Javy Lopez, and Jay Gibbons. Being the best hitters, these four players must all bat in the four spots in the lineup which are positions 3–6. Within those spots, the four may hit in any order. The other 5 players (of the nine originally chosen) may go in any of the other 5 spots in the lineup. With this restriction, how many lineups are possible?

Solution: $4! \cdot 5!$

(c) Assume that the four players mentioned above now must be in consecutive spots in the order, but not necessarily positions 3–6; they may, for example appear in spots 1–4 or spots 6–9. Again, within the four spots, they may hit in any order. With this less strict restriction, how many lineups are possible now?

Solution: $6 \cdot 4! \cdot 5!$

5. The Orioles' eleven pitchers consist of 6 right-handed pitchers and 5 left-handed pitchers. They need to pick five of these pitchers to start the ballgames. Again, to have some fun, they decide to pick at random.

(a) What is the probability that they pick 3 right-handers and 2 left-handers?

Solution: $\frac{\binom{6}{3}\binom{5}{2}}{\binom{11}{5}} = \frac{\frac{6!}{3!3!} \cdot \frac{5!}{2!3!}}{\frac{11!}{5!6!}} \approx .433$

(b) What is the probability that they pick 3 or more right-handers?

Solution: $\frac{\binom{6}{3}\binom{5}{2} + \binom{6}{4}\binom{5}{1} + \binom{6}{5}\binom{5}{0}}{\binom{11}{5}} = \frac{\frac{6!}{3!3!} \cdot \frac{5!}{2!3!} + \frac{6!}{4!2!} \cdot \frac{5!}{1!4!} + \frac{6!}{5!1!} \cdot \frac{5!}{0!5!}}{\frac{11!}{5!6!}} \approx .608$

(c) Considering only handed-ness (not actually *who* the pitchers are), how many different combinations of right-handers and left-handers are there? (order doesn't matter)

Solution: $\binom{2+5-1}{5} = \binom{6}{5} = 6.$

(d) Again, considering only handed-ness, how many different combinations of right-handers and left-handers are there if each combination must have at least one right-hander and at least one left-hander? (order doesn't matter)

Solution: $\binom{2+3-1}{3} = \binom{4}{3} = 4.$

6. The Orioles' catcher, Javy Lopez, is on a hot streak and is getting tons of hits. Assume Javy has ESP and knows he will get four hits in the game tonight — consisting of some combination of singles, doubles, triples, and home runs.

(a) How many different combinations of four hits (for example, 3 singles and a double) can Javy put together, assuming he doesn't care in what order he hits them?

Solution: $\binom{4+4-1}{4} = \binom{7}{4} = \frac{7!}{4!3!} = 35.$

(b) How many different combinations of four hits can Javy put together, assuming he will hit exactly one home run? (Again, he doesn't care in what order.)

Solution: $\binom{3+3-1}{3} = \binom{5}{3} = \frac{5!}{3!2!} = 10.$

(c) Assuming each kind of hit is equally likely each of the four times, what is the probability that Javy hits exactly one home run?

Note: Just because each *kind* of hit is equally likely each time doesn't mean each *combination* of hits is equally likely. For example, it's more likely for Javy to hit 3 singles and a double than 4 singles.

Solution: $\frac{4 \cdot 3^3 \cdot 1}{4^4} \approx .422$

Explanation: Event space — the home run can be in any of the four slots, so that's 4. Then there are 3 ways to hit a non-homerun, which he has to do 3 times. And there's one way to hit a homerun, that's the 1. And the sample space is any type of hit (4) in any of the 4 slots.

(d) Assuming each kind of hit is equally likely each of the four times, what is the probability that Javy hits two singles and two doubles?

Solution: $\frac{\binom{4}{2}\binom{2}{2}}{4^4} = \frac{\frac{4!}{2!2!}\frac{2!}{2!0!}}{4^4} \approx .023$

Explanation: Event space is permutations of 2 doubles and 2 singles; sample space is all permutations of 4 hits.

(e) Again, assuming each kind of hit is equally likely, what is the probability that Javy hits for the cycle? (Meaning he hits exactly one of each kind of hit.)

Solution: $\frac{P(4,4)}{4^4} = \frac{4!}{4^4} \approx .094$