CMSC250, Spring 2004 — Homework 12

Due Wednesday, April 28 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Prove or disprove the following:
   (a) $f : (0, \infty) \to \mathbf{R}$ defined by $f(x) = \ln(x)$ is 1-1.
   (b) $f : (0, \infty) \to \mathbf{R}$ defined by $f(x) = \ln(x)$ is onto.

2. Let A and B be sets, and $f : A \to B$ be the function $f(x) = x^2$. For each of the four following questions, define the sets A and B such that f is
   (a) 1-1 and onto.
   (b) 1-1 but not onto.
   (c) onto but not 1-1.
   (d) neither 1-1 nor onto.

   Note: You do not need to prove that each of your definitions of A and B has the desired properties; just give the sets.

3. Let f be an arbitrary bijective function from $\mathbf{R}$ to $\mathbf{R}$.
   (a) Define a function $g_1 : \mathbf{R} \to \mathbf{R}$ in terms of f so that g_1 is 1-1 but not onto. Prove that g_1 satisfies this criteria.
   (b) Define a function $g_2 : \mathbf{R} \to \mathbf{R}$ in terms of f so that g_2 is onto but not 1-1. Prove that g_2 satisfies this criteria.

4. Let D denote the set of odd integers.
   That is, $D = \{n \in \mathbf{Z} \mid n = 2k + 1 \text{ for some integer } k\}$.
   (a) Define a bijective function $h_1 : D \to \mathbf{Z}$. Prove that h_1 is a bijection.
   (b) Define a bijective function $h_2 : \mathbf{Z} \to D$. Prove that h_2 is a bijection.