CMSC250, Spring 2004 — Homework 14

Never due. Use this for practice.

1. How many binary relations are there from a set with m elements to a set with n elements?

2. Determine whether each of the following relations are reflexive, symmetric, transitive, antisymmetric or none of these. Justify each answer with a proof or counterexample.

   (a) D is the "divides" relation on $\mathbf{Z}$: for all integers m and n, $m\ D\ n \leftrightarrow m \mid n$.

   (b) Let $X = \{a, b, c\}$ and $P(X)$ be the power set of X. A binary relation # is defined on $P(X)$ as follows: for all $A, B \in P(X)$, $A \# B \leftrightarrow n(A) = n(B)$.

   (c) Let $X = \{a, b, c\}$ and $P(X)$ be the power set of X. A binary relation R is defined on $P(X)$ as follows: for all $A, B \in P(X)$, $A\ R\ B \leftrightarrow n(A) < n(B)$.

   (d) Let $X = \{a, b, c\}$ and $P(X)$ be the power set of X. A binary relation T is defined on $P(X)$ as follows: for all $A, B \in P(X)$, $A\ T\ B \leftrightarrow n(A) \neq n(B)$.

   (e) Let C be the set of all boolean formulas in three variables p, q and r. Define I to be the "implies" relation on C: for all boolean statements a and b in C, $(a\ I\ b) \leftrightarrow (a \rightarrow b$ is true).

3. Let $A = \{1, 2, 3, 4, 5\}$. For each part, define a binary relation R on set A so that R is

   (a) reflexive and symmetric, but not transitive.

   (b) symmetric and transitive, but not reflexive.

   (c) reflexive, symmetric, transitive, and antisymmetric.

4. Let Q be the relation on $\mathbf{R}$ defined as follows:

$$\text{for all } x, y \in \mathbf{R}, \quad x\ Q\ y \leftrightarrow x - y \in \mathbf{Z}.$$

   (a) Prove Q is an equivalence relation.

   (b) Describe the equivalence classes of Q.

5. Let $C = \{106, 114, 214, 250, 311, 330, 351\}$, and let the binary relation P on C be defined as follows: $P = \{(106, 114), (114, 214), (250, 311), (250, 330), (250, 351), (214, 311), (214, 330), (214, 351)\}$.

   (a) Can you figure out what relation P represents?

   (b) Write P in matrix form.

   (c) Draw the directed graph for P.

   (d) Write the transitive and reflexive closure of P in matrix form, call this new relation P'. Check that P' is now a partial order relation.

   (e) Draw the Hasse diagram for P'.

   (f) Find the least, minimal, greatest, and maximal element(s) for P' if they exist.