CMSC250, Spring 2004 — Homework 1 Answers

Due Wednesday, February 4 at the beginning of your discussion section.

1. Use the following propositions for this problem.

   - b = "John is a basketball player."
   - h = "John is a hard worker."
   - c = "John is a computer science major."

   Write each of the following statements in symbolic form using the symbols $\sim$, $\wedge$, and $\vee$ and the letters defined above.

   (a) John is a basketball player, however he is not a computer science major.
   Answer: $b \wedge \sim c$

   (b) John is either a hard worker or a basketball player, but not both.
   Answer: $(h \wedge \sim b) \vee (\sim h \wedge b)$ OR $(h \vee b) \wedge \sim (h \wedge b)$

   (c) John is a hard worker, but he is neither a basketball player nor a computer science major.
   Answer: $h \wedge \sim b \wedge \sim c$

2. Use DeMorgan's Laws to write the negations of the following as English sentences.

   (a) Mary and Sam are both engineering majors.
   Answer: Mary is not an engineering major or Sam is not an engineering major.

   (b) Morgan is is neither a swimmer nor a runner.
   Answer: Morgan is either a swimmer or a runner (or both).

   (c) Either the weather is nice or the game is cancelled.
   Answer: The weather is not nice and the game is not cancelled.

3. Assuming x is a particular real number,

   (a) write the negation of this statement: $-1 < x \leq 4$.
   Answer: $(-1 \geq x) \vee (x > 4)$

   (b) Give one possible value that x could be if the statement "$-1 < x \leq 4$" is false.
   Answer: Many possible answers. Any number less than or equal to -1 or greater than 4 is fine.

4. State whether each of the following sentences are statements. If a sentence is a statement, state whether it is true or false.

   (a) "Why is the sky blue?"
   Answer: Not a statement — it's a question.

   (b) "My CMSC250 instructor is Jan Plane."
   Answer: True statement.

   (c) "This sentence is not a statement."

Answer: False statement.

(d) "This sentence is both true and false."

Answer: False statement.

5. Construct a complete a truth table for each of the following statements, and tell whether each is a tautology, contradiction, or neither.

(a) $(p \wedge \sim q) \wedge (\sim p \vee q)$

Answer:

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$ (X)	$\sim p \vee q$ (Y)	(X) $\wedge$ (Y)
1	1	0	0	0	1	0
1	0	0	1	1	0	0
0	1	1	0	0	1	0
0	0	1	1	0	1	0

This statement is a contradiction.

(b) $\sim p \wedge (q \vee \sim r)$

Answer:

p	q	r	$\sim p$	$\sim r$	$q \vee \sim r$	$\sim p \wedge (q \vee \sim r)$
1	1	1	0	0	1	0
1	1	0	0	1	1	0
1	0	1	0	0	0	0
1	0	0	0	1	1	0
0	1	1	1	0	1	1
0	1	0	1	1	1	1
0	0	1	1	0	0	0
0	0	0	1	1	1	1

This statement is a neither a tautology nor a contradiction.

(c) $(p \wedge \sim q) \vee (p \wedge q) \vee \sim p$

Answer:

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$ (X)	$p \wedge q$ (Y)	(X) $\vee$ (Y)	(X) $\vee$ (Y) $\vee \sim p$
1	1	0	0	0	1	1	1
1	0	0	1	1	0	1	1
0	1	1	0	0	0	0	1
0	0	1	1	0	0	0	1

This statement is a tautology.