CMSC250, Spring 2004 Homework 4

Due Wednesday, February 25 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Complete the following proofs using the method described in class (line numbers, rules, etc).

   (a)

P1	$\forall x \in D\ P(x) \to (T(x) \vee Q(x))$
P2	$\forall y \in D\ Q(y) \vee (R(y) \wedge P(y))$
P3	$\forall z \in D\ (T(z) \wedge R(z)) \to S(z)$
$\therefore$	$\forall w \in D\ \sim Q(w) \to S(w)$

   (b)

P1	$\forall t \in D\ (A(t) \to B(t)) \to (C(t) \vee D(t))$
P2	$\exists u \in D\ \sim A(u) \wedge (D(u) \to E(u))$
P3	$\forall v \in D\ \sim E(v) \to (C(v) \to A(v))$
$\therefore$	$\exists h \in D\ \sim B(h) \vee E(h)$

   (c)

P1	$\forall w \in D\ J(w) \to M(w)$
P2	$\forall x \in D\ M(x) \to ((N(x) \to \sim J(x)) \wedge \sim K(x))$
P3	$\forall y \in D\ N(y) \vee \sim L(y)$
P4	$\forall z \in D\ (J(z) \wedge K(z)) \vee (L(z) \wedge M(z))$
$\therefore$	$\forall q \in D\ \sim J(q)$

2. Translate each of the following into formal language using the sets and predicates given.

   (a) Exactly two people completely understand quantum physics. ($U = \{$universal set$\}$, $P(m) = m$ is a person, $Q(n) = n$ completely understands quantum physics.)

   (b) I own at least three cats. ($C = \{$all cats$\}$, $N(x) =$ I own x.)

   (c) No more than two people own both a kangaroo and a polar bear. ($P = \{$all people$\}$, $K(p) = p$ owns a kangaroo, $B(p) = p$ owns a polar bear.)

3. For each of the following, decide if the argument is valid or invalid, and write "invalid" or "valid" as appropriate. If it is invalid, draw an Euler diagram to verify this fact. If it is valid, draw an Euler diagram that shows the premises and conclusion all to be true.

   (a)
   - All shortshops can steal bases.
   - Some shortstops can hit home runs.
   - Therefore, some shortstops can steal bases and hit home runs.

(b)
- All CS professors are intelligent.
- All CS professors like music.
- Therefore, all intelligent people like music.

(c)
- Some textbooks are cheap.
- Some textbooks are useful.
- Therefore, some textbooks are cheap and useful.

4. In this problem, you are given a number of situations in English. For each situation, you must determine which symbolic expression(s) from the given list are true in that situation.

   Let L be the set of people {Kate, Lisa, John}, let M be the set of musical instruments {piano, trumpet, accordian}, and let the predicate $P(x, y)$ mean "person x plays instrument y."

   Symbolic expressions to choose from:

   (1) $\forall x \in L\ \exists y \in M\ \ P(x, y)$

   (2) $\exists x \in L\ \forall y \in M\ \ P(x, y)$

   (3) $\forall y \in M\ \exists x \in L\ P(x, y)$

   (4) $\exists y \in M\ \forall x \in L\ P(x, y)$

   You may assume that in each situation, each person plays *only* the instruments listed for him or her, and no others. In other words, if its not listed, they don't play it!

   Remember, for each situation, write down the corresponding numbers of *all* the symbolic expressions that apply to that situation.

   Situations:

   (a) John plays piano, Kate plays trumpet, and Lisa plays accordian.

   (b) John plays piano, Kate plays piano and trumpet, and Lisa plays piano and accordian.

   (c) John plays trumpet, Kate plays piano, trumpet, and accordian, and Lisa doesn't play anything.

   (d) John plays trumpet, Kate plays piano and trumpet, and Lisa plays trumpet.

   (e) John plays trumpet, Kate doesn't play anything, and Lisa plays piano and accordian.

   (f) John plays accordian, Kate plays piano and accordian, and Lisa plays piano.

   (g) John plays piano, trumpet, and accordian, Kate plays trumpet and accordian, and Lisa plays accordian.

   (h) John plays piano and trumpet, Kate plays piano and accordian, and Lisa plays piano, trumpet, and accordian.