CMSC250, Spring 2004 — Homework 4 Answers

Due Wednesday, February 25 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Complete the following proofs using the method described in class (line numbers, rules, etc).

(a)

P1	$\forall x \in D\ P(x) \to (T(x) \vee Q(x))$
P2	$\forall y \in D\ Q(y) \vee (R(y) \wedge P(y))$
P3	$\forall z \in D\ (T(z) \wedge R(z)) \to S(z)$
$\therefore$	$\forall w \in D\ \sim Q(w) \to S(w)$

Answer:

Line	Statement	Rule	Lines Used
1	$Q(a) \vee (R(a) \wedge P(a))$	$\forall$ instantiation	P2
2	$\sim Q(a)$	Assume	—
3	$R(a) \wedge P(a)$	Disjunctive syllogism	1, 2
4	$P(a)$	Conjunctive simplification	3
5	$T(a) \vee Q(a)$	$\forall$ modus ponens	4, P1
6	$R(a)$	Conjunctive simplification	3
7	$T(a)$	Disjunctive syllogism	2, 5
8	$T(a) \wedge R(a)$	Conjunctive addition	6, 7
9	$S(a)$	$\forall$ modus ponens	P3, 8
10	$\sim Q(a) \to S(a)$	CCW w/out contra	2–9
11	$\forall w \in D\ \sim Q(w) \to S(w)$	$\forall$ generalization	10

Another way:

Line	Statement	Rule	Lines Used
1	$Q(a) \vee (R(a) \wedge P(a))$	$\forall$ instantiation	P2
2	$\sim (\sim Q(a) \to S(a))$	Assume	—
3	$\sim Q(a) \wedge \sim S(a)$	Definition of $\to$	2
4	$\sim Q(a)$	Conjunctive simplification	3
5	$\sim S(a)$	Conjunctive simplification	3
6	$\sim (T(a) \wedge R(a))$	$\forall$ modus tollens	5, P3
7	$\sim T(a) \vee \sim R(a)$	DeMorgan's law	6
8	$R(a) \wedge P(a)$	Disjunctive syllogism	1, 4
9	$R(a)$	Conjunctive simplification	8
10	$P(a)$	Conjunctive simplification	8
11	$\sim T(a)$	Disjunctive syllogism	7, 9
12	$\sim T(a) \wedge \sim Q(a)$	Conjunctive addition	4, 11
13	$\sim (T(a) \vee Q(a))$	DeMorgan's law	12
14	$\sim P(a)$	$\forall$ modus tollens	P1, 13
15	$\sim P(a) \wedge P(a)$	Conjunctive addition	14, 10
16	$\sim Q(a) \to S(a)$	CCW w/ contra	2–15
17	$\forall w \in D\ \sim Q(w) \to S(w)$	$\forall$ generalization	16

(b)

P1	$\forall t \in D\ (A(t) \to B(t)) \to (C(t) \vee D(t))$
P2	$\exists u \in D\ \sim A(u) \wedge (D(u) \to E(u))$
P3	$\forall v \in D\ \sim E(v) \to (C(v) \to A(v))$
$\therefore$	$\exists h \in D\ \sim B(h) \vee E(h)$

Answer:

Line	Statement	Rule	Lines Used
1	$\sim A(a) \wedge (D(a) \to E(a))$	$\exists$ instantiation	P2
2	$\sim A(a)$	Conjunctive simplification	1
3	$\sim A(a) \vee B(a)$	Disjunctive addition	2
4	$A(a) \to B(a)$	Definition of $\to$	3
5	$C(a) \vee D(a)$	$\forall$ modus ponens	P1, 4
6	$D(a) \to E(a)$	Conjunctive simplification	1
7	$\sim (\sim B(a) \vee E(a))$	Assume	—
8	$B(a) \wedge \sim E(a)$	Double negation, DeMorgan's law	7
9	$\sim E(a)$	Conjunctive simplification	8
10	$\sim D(a)$	Modus tollens	6, 9
11	$C(a)$	Disjunctive syllogism	5, 10
12	$C(a) \wedge \sim A(a)$	Conjunctive addition	2, 11
13	$\sim (\sim C(a) \vee A(a))$	Double negation, DeMorgan's law	12
14	$\sim (C(a) \to A(a))$	Definition of $\to$	13
15	$E(a)$	$\forall$ modus tollens, Double negation	P3, 14
16	$E(a) \wedge \sim E(a)$	Conjunctive addition	15, 9
17	$\sim B(a) \vee E(a)$	Closing cond world w/ contra	7–16
18	$\exists h \in D\ \sim B(h) \vee E(h)$	$\exists$ generalization	17

Another way:

Line	Statement	Rule	Lines Used
1	$\sim A(a) \wedge (D(a) \to E(a))$	$\exists$ instantiation	P2
2	$\sim A(a)$	Conjunctive simplification	1
3	$D(a) \to E(a)$	Conjunctive simplification	1
4	$\sim A(a) \vee B(a)$	Disjunctive addition	2
5	$A(a) \to B(a)$	Definition of $\to$	4
6	$C(a) \vee D(a)$	$\forall$ modus ponens	P1, 5
7	$C(a)$	Assume	—
8	$C(a) \wedge \sim A(a)$	Conjunctive addition	7, 2
9	$\sim (C(a) \to A(a))$	Definition of $\to$	8
10	$E(a)$	$\forall$ modus tollens	9, P3
11	$C(a) \to E(a)$	Closing cond world w/out contra	7–10
12	$E(a)$	Dilemma	6, 3, 11
13	$\sim B(a) \vee E(a)$	Disjunctive addition	12
14	$\exists h \in D\ \sim B(h) \vee E(h)$	$\exists$ generalization	13

(c)

P1	$\forall w \in D\ J(w) \to M(w)$
P2	$\forall x \in D\ M(x) \to ((N(x) \to \sim J(x)) \wedge \sim K(x))$
P3	$\forall y \in D\ N(y) \vee \sim L(y)$
P4	$\forall z \in D\ (J(z) \wedge K(z)) \vee (L(z) \wedge M(z))$
$\therefore$	$\forall q \in D\ \sim J(q)$

Answer:

Line	Statement	Rule	Lines Used
1	$N(a) \vee \sim L(a)$	$\forall$ instantiation	P3
2	$(J(a) \wedge K(a)) \vee (L(a) \wedge M(a))$	$\forall$ instantiation	P4
3	$J(a)$	Assume	—
4	$M(a)$	$\forall$ modus ponens	P1, 3
5	$(N(a) \rightarrow \sim J(a)) \wedge \sim K(a)$	$\forall$ modus ponens	P2, 4
6	$N(a) \rightarrow \sim J(a)$	Conjunctive simplification	5
7	$\sim N(a)$	Modus tollens, double neg	6, 3
8	$\sim L(a)$	Disjunctive syllogism	1, 7
9	$\sim K(a)$	Conjunctive simplification	5
10	$\sim J(a) \vee \sim K(a)$	Disjunctive addition	9
11	$\sim (J(a) \wedge K(a))$	double neg, DeMorgan's law	10
12	$L(a) \wedge M(a)$	Disjunctive syllogism	2, 11
13	$L(a)$	Conjunctive simplification	12
14	$L(a) \wedge \sim L(a)$	Conjunctive addition	8, 13
15	$\sim J(a)$	CCW w/ contra	3–14
16	$\forall q \in D \ \ \sim J(q)$	$\forall$ generalization	15

Another way:

Line	Statement	Rule	Lines
1	$M(a) \rightarrow ((N(a) \rightarrow \sim J(a)) \wedge \sim K(a))$	$\forall$ instantiation	P2
2	$\sim M(a) \vee ((N(a) \rightarrow \sim J(a)) \wedge \sim K(a))$	Definition of $\rightarrow$	1
3	$\sim M(a)$	Assume	—
4	$\sim J(a)$	$\forall$ modus tollens	P1, 3
5	$\sim M(a) \rightarrow \sim J(a)$	CCW w/out contra	3–4
6	$(N(a) \rightarrow \sim J(a)) \wedge \sim K(a)$	Assume	—
7	$N(a) \rightarrow \sim J(a)$	Conjunctive simp	6
8	$\sim K(a)$	Conjunctive simp	6
9	$\sim J(a) \vee \sim K(a)$	Disjunctive addition	8
10	$\sim (J(a) \wedge K(a))$	DeMorgan's law	9
11	$(J(a) \wedge K(a)) \vee (L(a) \wedge M(a))$	$\forall$ instantiation	P4
12	$L(a) \wedge M(a)$	Disjunctive syllogism	10, 11
13	$L(a)$	Conjunctive simp	12
14	$N(a) \vee \sim L(a)$	$\forall$ instantiation	P3
15	$N(a)$	Disjunctive syllogism	13, 14
16	$\sim J(a)$	Modus ponens	15, 7
17	$[(N(a) \rightarrow \sim J(a)) \wedge \sim K(a)] \rightarrow \sim J(a)$	CCW w/out contra	6–16
18	$\sim J(a)$	Dilemma	2, 5, 17
19	$\forall q \in D \ \ \sim J(q)$	$\forall$ generalization	18

2. Translate each of the following into formal language using the sets and predicates given.

   (a) Exactly two people completely understand quantum physics. ($U = \{\text{universal set}\}$, $P(m) = m$ is a person, $Q(n) = n$ completely understands quantum physics.)

   Answer:
   $\exists a, b \in U \ P(a) \wedge Q(a) \wedge P(b) \wedge Q(b) \wedge a \neq b \wedge \forall x \in U \ (P(x) \wedge Q(x)) \rightarrow (a = x \vee b = x)$

(b) I own at least three cats. ($C = \{\text{all cats}\}$, $N(x) =$ I own x.)

Answer: $\exists a, b, c \in C\ N(a) \wedge N(b) \wedge N(c) \wedge a \neq b \wedge b \neq c \wedge c \neq a$

(c) No more than two people own both a kangaroo and a polar bear. ($P = \{\text{all people}\}$, $K(p) = p$ owns a kangaroo, $B(p) = p$ owns a polar bear.)

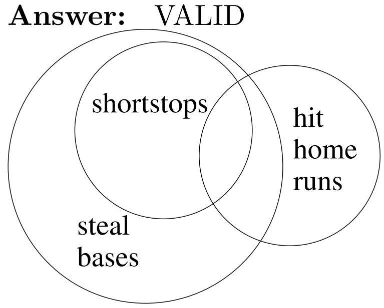
Answer:

$\forall p, q, r \in P\ (K(p) \wedge K(q) \wedge K(r) \wedge B(p) \wedge B(q) \wedge B(r)) \rightarrow (p = q \vee q = r \vee r = p)$

3. For each of the following, decide if the argument is valid or invalid, and write "invalid" or "valid" as appropriate. If it is invalid, draw an Euler diagram to verify this fact. If it is valid, draw an Euler diagram that shows the premises and conclusion all to be true.

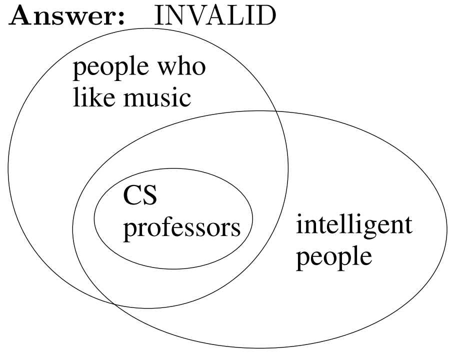
(a)
- All shortshops can steal bases.
- Some shortstops can hit home runs.
- Therefore, some shortstops can steal bases and hit home runs.

Answer: VALID



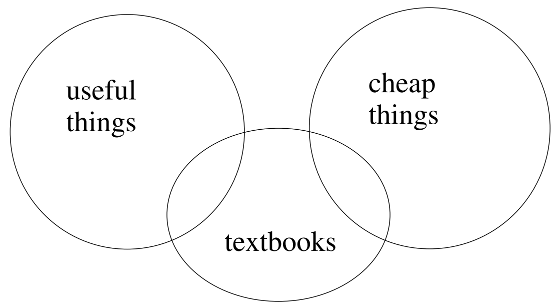
(b)
- All CS professors are intelligent.
- All CS professors like music.
- Therefore, all intelligent people like music.

Answer: INVALID



(c)
- Some textbooks are cheap.
- Some textbooks are useful.
- Therefore, some textbooks are cheap and useful.

Answer: INVALID



4. In this problem, you are given a number of situations in English. For each situation, you must determine which symbolic expression(s) from the given list are true in that situation.

   Let L be the set of people {Kate, Lisa, John}, let M be the set of musical instruments {piano, trumpet, accordian}, and let the predicate $P(x, y)$ mean "person x plays instrument y."

   Symbolic expressions to choose from:

   (1) $\forall x \in L \; \exists y \in M \;\; P(x, y)$
   (2) $\exists x \in L \; \forall y \in M \;\; P(x, y)$
   (3) $\forall y \in M \; \exists x \in L \;\; P(x, y)$
   (4) $\exists y \in M \; \forall x \in L \;\; P(x, y)$

   You may assume that in each situation, each person plays *only* the instruments listed for him or her, and no others. In other words, if its not listed, they don't play it!

   Remember, for each situation, write down the corresponding numbers of *all* the symbolic expressions that apply to that situation.

   Situations:

   (a) John plays piano, Kate plays trumpet, and Lisa plays accordian.
   Answer: 1 and 3.

   (b) John plays piano, Kate plays piano and trumpet, and Lisa plays piano and accordian.
   Answer: 1, 3, and 4.

   (c) John plays trumpet, Kate plays piano, trumpet, and accordian, and Lisa doesn't play anything.
   Answer: 2 and 3.

   (d) John plays trumpet, Kate plays piano and trumpet, and Lisa plays trumpet.
   Answer: 1 and 4.

   (e) John plays trumpet, Kate doesn't play anything, and Lisa plays piano and accordian.
   Answer: 3.

   (f) John plays accordian, Kate plays piano and accordian, and Lisa plays piano.
   Answer: 1.

   (g) John plays piano, trumpet, and accordian, Kate plays trumpet and accordian, and Lisa plays accordian.
   Answer: 1, 2, 3, and 4.

   (h) John plays piano and trumpet, Kate plays piano and accordian, and Lisa plays piano, trumpet, and accordian.
   Answer: 1, 2, 3, and 4.