CMSC250, Spring 2004 — Homework 5

Due Wednesday, March 3 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

Prove each of the following statements true or false. Remember, an example/counter-example may only be used to prove that a universal statement is false or that an existential statement is true. All examples must include specific values for the variables and enough algebra/justification to show that they are truly examples.

1. For all integers a, b, and c, if $a + b = c$ and $a \mid b$, then $a \mid c$.
2. $\forall a, b, c \in \mathbf{Z}\ [(a \mid c) \wedge (b \mid c)] \rightarrow [(a \mid b) \vee (b \mid a)]$
3. $\forall a, b, c \in \mathbf{Z}\ [(a \mid b) \wedge (a \mid c)] \rightarrow [(b \mid c) \vee (c \mid b)]$
4. If $a \mid b$ and $b \mid c$, then $a \mid c$, for any integers a, b, and c.
5. If x is an odd integer, then $x^2 - 1$ is divisible by 4.
6. If a prime greater than 2 can be represented as $3k + 2$ for some integer k, then that prime can be represented as $6m + 5$ for some integer m.
7. For any integer n, $n^3 \not\equiv_4 2$.
8. $\forall x \in \mathbf{Z}^{\text{even}}\ (3 \nmid x) \rightarrow (4 \mid x^2)$

Extra credit problem:

9. There exists an integer $m > 1$ such that $m^4 - 1$ is prime.