CMSC250, Spring 2004 — Homework 6

Due Wednesday, March 10 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Let $p_1, p_2, \ldots, p_n$ be distinct prime numbers such that $p_1 = 2$ and $n > 1$, and let $a = p_1 p_2 \cdots p_n$. Prove that $a \equiv_4 2$.

   Hint: Use the quotient-remainder theorem, and eliminate the other possibilities.

2. An integer is called *squarefree* if it is not divisible by the square of any prime number. What can you deduce about the standard factored form of a squarefree number?

   Hint: Examine the exponents.

3. Prove that any integer $a > 1$ can be written as either a perfect square, a squarefree integer, or the product of a squarefree integer and a perfect square.

   Hint: Break this into the three cases. Two of them should be easy. For the third case, write a as $(p_1^{e_1} p_2^{e_2} \cdots p_n^{e_n})(q_1^{d_1} q_2^{d_2} \cdots q_m^{d_m})$ where all the e_i's are even and all the d_j's are odd.

4. Prove that for all real numbers x, $x - 1 < \lfloor x \rfloor \leq x$.

5. Write the first four terms of each of the following sequences:

   (a) $\forall j > 1,\ s_j = j(j-1)$

   (b) $\forall n \geq 0,\ r_n = \dfrac{n}{n!}$

6. Reduce each of the following expressions to a single numeric value:

   (a) $\displaystyle\sum_{i=1}^{4} \frac{-1}{i^2 + i}$

   (b) $\displaystyle\prod_{h=0}^{10} \frac{h}{h! + h + 1}$

   (c) $\displaystyle\sum_{m=1}^{3} \left(\prod_{n=1}^{m} \frac{m}{n} \right)$

7. Write each of the following using sum and/or product notation:

   (a) $n + 0 - n - 1 + n + 2 - n - 3 + n + 4$

   (b) $\sqrt{5}(1) + 2(1+2) + \sqrt{3}(1+2+3) + \sqrt{2}(1+2+3+4) + 1(1+2+3+4+5)$