CMSC250, Spring 2004 — Homework 7

Due Wednesday, March 17 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Simplify this expression: $\ln \left[\prod_{i=1}^{n} e^{f(i)} \right]$.

2. Prove $\forall n \in \mathbf{Z}^+ \ \sum_{i=1}^{n} (2i)(2i-1) = \frac{n(n+1)(4n-1)}{3}$.

3. Prove $\forall n \in \mathbf{Z}^+ \ \sum_{i=1}^{n} (4i-3) = n(2n-1)$.

   Note: Do not use Theorem 4.2.2 to solve this problem.

4. Prove $\forall n \in \mathbf{Z}^+ \ \prod_{i=1}^{n} 2^i = 2^{\left(\frac{n}{2}+\frac{n^2}{2}\right)}$.

5. Recall the recursive definition of the *Fibonacci sequence*:

$$\begin{aligned} F_1 &= 1 \\ F_2 &= 1 \\ F_k &= F_{k-1} + F_{k-2} \quad \text{for } k > 2. \end{aligned}$$

   Prove these interesting facts about this sequence:

   (a) $\forall n \in \mathbf{Z}^+ \ \sum_{k=1}^{n} F_k = F_{n+2} - 1$

   (b) $\forall n \in \mathbf{Z}^+ \ \sum_{k=1}^{n} F_k^2 = F_n \cdot F_{n+1}$.

   Hint: You can perform a change of variable on the third line in the Fibonacci sequence formula to obtain equivalent formulas that may prove useful.