CMSC250, Spring 2004 — Homework 8

Due Wednesday, March 31 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Prove $\forall n \in \mathbf{Z}^{+}\ \sum_{i=1}^{n} \left\lceil \frac{i}{2} \right\rceil > \frac{(n-1)(n+2)}{4}$.
2. Prove $\forall n \in \mathbf{Z}^{+}\ 8 \mid \left(2^{3n+1}\right)$.

   Hint: You do not need to use induction on this one.
3. Prove $\forall n \in \mathbf{Z}^{+}\ 8 \mid \left(3^{2n} + 7\right)$.
4. Prove $\forall n \in \mathbf{Z}^{+}\ 8 \mid \left(2^{3n+1} + 3^{2n} - 1\right)$.

   Hint: You may use the facts already proved true in problems 2 and/or 3 to prove the statement in problem 4.
5. Let $a_0 = 2$, $a_1 = 7$, and $\forall n \in \mathbf{Z}^{>1}\ a_n = 3a_{n-1} - 2a_{n-2}$.

   Prove $\forall n \in \mathbf{Z}^{\text{nonneg}}\ a_n \equiv_5 2$.
6. Let $c_0 = 0$, $c_1 = 1$, and $\forall n \in \mathbf{Z}^{>1}\ c_n = c_{\lfloor \frac{n}{2} \rfloor} + c_{\lfloor \frac{n}{3} \rfloor}$.

   Prove $\forall n \in \mathbf{Z}^{\geq 0}\ c_n \leq n$.
7. Prove $\forall n \in \mathbf{Z}^{\text{nonneg}}\ \sum_{i=0}^{n} \left((2n-i)\,2^{i}\right) = (n+1)\left(2^{n+1} - 2\right)$.