CMSC250, Spring 2004 — Homework 9

Due Wednesday, April 7 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

Note: $\mathcal{P}(A)$ denotes the power set of A.

1. For each of the following, give a proof of the statement if it is true, or a counterexample if the statement is false. Remember, counterexamples must include specific values and enough work shown to demonstrate that they are actual counterexamples.

   (a) For all sets A, B, and C, if $A \cup C = B \cup C$, then $A = B$.

   (b) For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \times B \subseteq B \times C$.

   (c) For all sets A, B, and C, $(A - B) - (B - C) = A - B$.

   (d) For all sets A and B, if $A \cap B = \emptyset$ then $A \times B = \emptyset$.

   (e) For all sets A, B, and C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.

   (f) For all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

2. Let $A_1, A_2, \ldots$ be sets. Prove the generalized DeMorgan's law:

$$\forall n \in \mathbf{Z}^+ \ (A_1 \cup A_2 \cup \cdots \cup A_n)' = A_1' \cap A_2' \cap \cdots \cap A_n'$$

   Hint: Use induction, and the facts that

   - $A_1 \cup A_2 \cup \cdots \cup A_n = A_1 \cup A_2 \cup \cdots \cup A_{n-1} \cup A_n$.
   - $A_1 \cap A_2 \cap \cdots \cap A_n = A_1 \cap A_2 \cap \cdots \cap A_{n-1} \cap A_n$.

3. Let $B_1, B_2, \ldots$ be sets.

   Let $B_1 = \{0, 1\}$, $B_2 = \{1, 2\}$, and $\forall i \in \mathbf{Z}^{>2}\ B_i = (B_{i-1} \cup \{i\}) - B_{i-2}$.

   Prove $\forall n \in \mathbf{Z}^+\ B_n = \{n - 1, n\}$.