Name (PRINTED): _____

Student ID #: _____

Section # (or TA's: name and time) _____

CMSC 250 Quiz #12 ANSWERS Wed., Apr. 21, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

Note: you only need to take the answer as far as a mathematical expression including addition, subtraction, multiplication, division, exponents and factorials.

1. [15 pnts.] Assume you have fruit to sell in your fruitstand. You have 7 apples, 4 oranges and 6 bananas. Fruit of the same type are indistinguishable from each other.

   You make a fruit basket to deliver to a friend at the hospital. It has 5 pieces of fruit in it answer the following questions about that fruit basket assuming you filled the fruit basket at random (not thinking about what the friend likes or what would look good in the basket). Assume the order you put the fruit into the basket does not make the basket different.

   a. How many different baskets can be created with the criteria above?

   $\frac{(5+(3-1))!}{5!(3-1!} - 1 = \frac{7!}{5!2!} - 1$

   EXPLANATION: We need to distribute the 5 indistinguishable x's into the three categories known as "apples", "oranges" and "bananas". The x's are indistinguishable from each other and the bars separating the categories are indistinguishable from each other. Once we have the number of ways to distribute the five x's among the three categories, we must subtract the one that is not possible because we can't have a basket that is just oranges since the basket has 5 fruits and there are only 4 oranges.

   b. What is the probability that all of the fruit in the basket are apples?

   $\frac{\binom{7}{5}}{\binom{17}{5}} = \frac{\frac{7!}{5!2!}}{\frac{17!}{5!12!}} = \frac{7!12!}{2!17!}$

   EXPLANATION: We can not use the number from part "a" since all of those baskets do not have equal probability. So we need to calculate as our event space "how many ways there are to select the 5 apples from the 7 available" and for our sample space "how many ways to select 5 fruits from the 17 available". The probability is then the size of the event space over the size of the sample space.

   c. Assuming you must have at least one of each type of fruit in the basket, how many different baskets could be formed?

   $\frac{(2+(3-1))!}{2!(3-1)!} = \frac{4!}{2!2!} = 6$

   EXPLANATION: We put the 3 pieces of fruit (one of each type into the basket first - then they are not considered when calculating the number of baskets. Now we only have the other two pieces of fruit to decide upon. So we are distributing 2 X's among the three categories and use the formula shown above. Or you can just partition these possibilities - partition into those where the two fruits are of the same type and those where the two fruits are of different types. To calculate where the two fruits are the same type: there are $\binom{3}{1} = 3$ ways to select that one

fruit. To calculate where the two fruits are of different types: there are $\binom{3}{2} = 3$ ways to select the two fruits. Since this was a partition you then add the two answers and get $3 + 3 = 6$.

d. Assume your friend received 2 baskets where the first basket has has 3 apples and 2 oranges and the second basket has 2 apples and 3 bananas. Your friend lines up the fruit in a strait line to decide in what order to eat them. How many different linear arrangements can he make assuming the fruit of the same type is indistinguishable?

$\binom{10}{5}\binom{5}{3}\binom{2}{2} = \frac{10!}{5!3!2!} = 2520$

EXPLANATION: There are 10 things to put in a line, but there are subgroups of indistinguishable things where the subgroups are of size 5, 3 and 2. Or, to look at it a different way: We need to select the 5 places where the apples will be placed in the line that is 10 long, then we need to select the 3 places where the bananas will be placed from the 5 remianing slots, then we need to select the 2 places from the 2 remaining slots for the oranges.

2. [15 pnts.] Assume you have 27 indistinguishable chocolate candy bars and 5 distinguishable friends. Your friends want you to share the candy bars with them. Each of the questions is separate from the others - any conditions mentioned in one question do not carry to any of the others.

   Note: you only need to take the answer as far as a mathematical expression including addition, subtraction, multiplication, division, exponents and factorials.

   a. How many ways can you distribute the candy bars among yourself and your 5 frineds?

      $\frac{(27+(6-1))!}{27!(6-1)!} = \frac{32}{27!5!} = \binom{32}{27}$
      EXPLANATION: we need to position the 27 indistinguishable candy bars into the 6 categories (the people receiving the candy bars).

   b. You don't want any of your friends (or yourself) to be left out of the candy bar eating so you recalculate - how many ways are there to distribute the candy bars among yourself and your five friends assuming you want to be sure that each person gets at least two candy bars?

      $\frac{(15+(6-1))!}{15!(6-1)!} = \frac{20!}{15!5!}$
      EXPLANATION: Since $27 - 12 = 15$ and there are 12 of the candy bars already distributed, we need to position the 15 remaining candy bars in the 6 categories.

   c. You decide these candy bars are small so instead of each person receiving at least two as mentioned in the previous question, you want to be sure you and each of your friends each receive at least 5 candy bars. Now, how many ways to distribute the candy bars?

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      EXPLANATION: Since there are only 27 candy bars, you can not give 5 to each of the 6 people because that would be 30 candy bars (more than you actually have.

   d. Bill one of the friends is allergic to chocolate, so you need to make sure he does not receive any of the candy bars. Now how many ways are there to distribute them? (remember ignore restrictions in previous questions)

      $\frac{(27+(5-1))!}{27!(5-1)!} = \frac{31!}{27!4!}$
      EXPLANATION: Since Bill is no longer one of the categories that can receive candy bars, you only have 5 categories rather than 6.

   e. Since Bill is allergic to chocolate and can't eat the candy bars, you will add 12 peppermint candycanes to the candy you are distributing. How many ways can you distribute the candy assuming Bill can not eat the chocolate candy bars, but he can eat the peppermint candycanes?

      $\frac{(27+(5-1))!}{27!(5-1)!} * \frac{(12+(6-1))!}{12!(6-1)!} = \frac{31!}{27!4!} * \frac{17!}{12!5!}$
      EXPLANATION: You must distribute the candy bars to the 5 people who can receive them and then distribute the candycanes to the 6 people who can receive them. Since this is a two step process where both steps must be done in order to distribute the candy, you multiply the two steps to get all of the combinations.