

Name (PRINTED): _____

Student ID #: _____

Section # (or TA's: _____
name and time)

CMSC 250

Quiz #4 ANSWERS

Wednesday, Feb. 18, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [16 pnts.] For each of the following English Sentences, translate the meaning into formal notation using the symbols ($\exists$, $\forall$, $\wedge$, $\vee$, $\sim$, and $\rightarrow$). You may also use the following relational operators: $=$ and $\neq$ as well as the algebraic operators: $+$, $-$, $*$ (multiplication) and $/$ (division). On the next line write the negation of the original statement using formal notation. A negation symbol may only appear immediately before an individual predicate.

There is a tree taller than any building.

Domains: B = “all buildings” and D = “all trees”

Predicate: $T(x,y)$ = x is taller than y

statement: $\exists t \in D, \forall b \in B, T(t,b)$

negation: $\forall t \in D, \exists b \in B, \sim T(t,b)$

The square of any even integer is an even integer.

Domain: Z = “all integers”

Predicates: $E(x)$ = x is even

statement: $\forall x \in Z, E(x) \rightarrow E(x * x)$

negation: $\exists x \in Z, E(x) \wedge \sim E(x * x)$

No pigs have wings.

Domain: P = “all pigs”

Predicate: $W(x)$ = x has wings

statement: $\forall x \in P, \sim W(x)$

negation: $\exists x \in P, W(x)$

There are at least two people here.

Domain: P = “all people”

Predicate: $H(x)$ = x is here

statement: $\exists a, b \in P, H(a) \wedge H(b) \wedge (a \neq b)$

negation: $\forall a, b \in P, \sim H(a) \vee \sim H(b) \vee (a = b)$

↓ TURN OVER ↓

2. [14 pnts.] Given the following truth-table, do each of the following tasks.

p	q	r	output
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	0

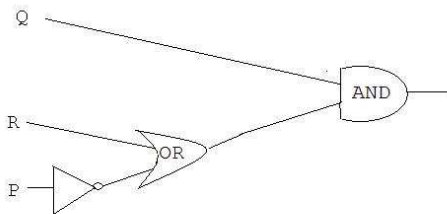
- a. Give the three situations (values for p,q, and r) that will give a 1 as the output in a single (long) logic statement of the form $(situation1) \vee (situation2) \vee (situation3)$.

ANSWER: $(p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r)$

- b. Show the reduction of that line to an equivalent statement that has the minimum number of logical operators. (This reduction is in the standard form of a proof.)

	Equivalent Statement	Rule
1	$(q \wedge (p \wedge r)) \vee (q \wedge (\sim p \wedge r)) \vee (q \wedge (\sim p \wedge \sim r))$	Assoc and Comm
2	$q \wedge ((p \wedge r) \vee ((\sim p \wedge r) \vee (\sim p \wedge \sim r)))$	Distrib and Assoc
3	$q \wedge ((p \wedge r) \vee (\sim p \wedge (r \vee \sim r)))$	Distrib
4	$q \wedge ((p \wedge r) \vee (\sim p \wedge t))$	Negation
5	$q \wedge ((p \wedge r) \vee \sim p)$	identity
6	$q \wedge ((p \vee \sim p) \wedge (r \vee \sim p))$	distribution
7	$q \wedge (t \wedge (r \vee \sim p))$	Negation
8	$q \wedge (r \vee \sim p)$	identity

- c. Draw the circuit represented by this truth table using as few gates as possible.



- d. Give the equivalent logic statement without using any **and** gates.

ANSWER: $\sim (\sim q \vee \sim (r \vee \sim p))$