CMSC 250 Quiz #8 ANSWERS Wed., Mar. 17, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [12 pnts.] Write each of the following using summation or product notation:

   a. $3 + 12 + 27 + 48 + 75$
      answer $= \sum_{i=1}^{5}(3 * i^2)$
   b. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$
      answer $= \sum_{i=2}^{n}(\frac{1}{i})$
   c. $\frac{2}{n} \times \frac{3}{n} \times \cdots \times \frac{n}{n}$
      answer $= \prod_{i=2}^{n}(\frac{i}{n})$

2. [8 pnts.] Expand the following:

   a. $\sum_{i=3}^{6} \frac{i}{i+1}$
      answer $= \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$
   b. $\prod_{i=2}^{5}(i^2)$
      answer $= 2^2 \times 3^2 \times 4^2 \times 5^2$

3. [10 pnts.] Use induction to prove the following. Make sure you clearly specify each step in the appropriate area. The statement to be proved is that

$$\sum_{i=1}^{n}(2 * 3^{i-1}) = 3^n - 1$$

Base Case: $(n = 1)$

$\sum_{i=1}^{1}(2 * 3^{i-1})$ by substituting a 1 for every n on the left side
$= 2 * 3^{1-1}$ by expanding the summation its one term
$= 2 * 3^0$
$= 2 * 1$
$= 2$

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$3^1 - 1$ by sustituting a 1 for every n on the right side
$= 3 - 1$
$= 2$

Inductive Hypothesis: $(n = k)$

$\sum_{i=1}^{k}(2 * 3^{i-1}) = 3^k - 1$

Inductive Step: $(n = k + 1)$

Show:

$\sum_{i=1}^{k+1}(2 * 3^{i-1}) = 3^{k+1} - 1$

Proof:

$\sum_{i=1}^{k+1}(2 * 3^{i-1})$ the left hand side of the equality we want to prove
$= \sum_{i=1}^{k}(2 * 3^{i-1}) + \sum_{i=k+1}^{k+1}(2 * 3^{i-1})$ by splitting the sum
$= \sum_{i=1}^{k}(2 * 3^{i-1}) + (2 * 3^{(k+1)-1})$ by expanding the sum on the right its one term
$= 3^k - 1 + (2 * 3^k)$ by the induction hypothesis
$= 3 * 3^k - 1$ by combining like terms
$= 3^{k+1} - 1$ by adding exponents on the 3's

QED