

Chapter 4

Mathematical Induction

- Used to verify a property of a sequence
- $2, 4, 6, 8, \dots$ for $i \geq 1$ $a_i = 2i$
 - infinite sequence with infinite distinct values
- for $i \geq 1$ $b_i = (-1)^i$
 - infinite sequence with finite distinct values
- for $1 \leq i \leq 6$ $c_i = i + 5$
 - finite sequence (with finite distinct values)

Finding the Explicit Formula

- Figure the formula of this sequence

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \dots$$

- Different sequences with same initial values
 $k \geq 0$

$$a_k = 2k + 1$$

$$b_k = (k - 1)^3 + k + 2$$

Summation & Product Notation

- Sum of Items Specified

$$\sum_{k=1}^6 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

- Product of Items Specified

$$\prod_{k=1}^5 2k = 2(1) * 2(2) * 2(3) * 2(4) * 2(5)$$

Variable ending point

- n as the index of the final term

$$\sum_{k=0}^n \frac{k+1}{n+k}$$

- for $n = 2$
- for $n = 3$

Nesting of Sum/Product Notation

- Variations (same or different??):

$$\sum_{j=1}^J \sum_{i=1}^{n_j} Y_{ij}^2 \quad \sum_{j=1}^J \left(\sum_{i=1}^{n_j} Y_{ij} \right)^2 \quad \left(\sum_{j=1}^J \sum_{i=1}^{n_j} Y_{ij} \right)^2$$

Telescoping Series

$$\sum_{k=1}^n \left(\frac{k}{k+1} - \frac{k+1}{k+2} \right)$$

$$\prod_{i=1}^n \left(\frac{i}{i+1} \right)$$

Properties

- Merging and Splitting

$$\sum_{k=m}^n a_k + \sum_{k=m}^n b_k = \sum_{k=m}^n (a_k + b_k)$$

$$\sum_{k=m}^n a_k = \sum_{k=m}^i a_k + \sum_{k=i+1}^n a_k$$

$$\prod_{k=m}^n a_k * \prod_{k=m}^n b_k = \prod_{k=m}^n (a_k * b_k)$$

$$\prod_{k=m}^n a_k = \prod_{k=m}^i a_k * \prod_{k=i+1}^n a_k$$

- Distribution

$$c * \sum_{k=m}^n a_k = \sum_{k=m}^n (c * a_k)$$

Factorial

- $n! = n * (n-1) * (n-2) * \dots * 2 * 1$

- Definition

$$0! = 1$$

$$n! = n * (n-1)!$$