Chapter 4
Mathematical Induction

- Used to verify a property of a sequence
- 2,4,6,8,… for i >= 1 $a_i = 2i$
  - infinite sequence with infinite distinct values
- for i >= 1 $b_i = (-1)^i$
  - infinite sequence with finite distinct values
- for 1<=i<=6 $c_i = i+5$
  - finite sequence (with finite distinct values)

Finding the Explicit Formula

- Figure the formula of this sequence

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$$

- Different sequences with same initial values

$$k >= 0$$

$$a_k = 2k + 1$$

$$b_k = (k-1)^3 + k + 2$$

Summation & Product Notation

- Sum of Items Specified

$$\sum_{k=1}^{6} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

- Product of Items Specified

$$\prod_{k=1}^{5} 2k = 2(1) * 2(2) * 2(3) * 2(4) * 2(5)$$

Variable ending point

- n as the index of the final term

$$\sum_{k=0}^{n} \frac{k+1}{n+k}$$

- for n = 2
- for n = 3

Nesting of Sum/Product Notation

- Variations (same or different??):

$$\sum_{j=1}^{J} \sum_{i=1}^{n_j} Y_{ij}^2 \qquad \sum_{j=1}^{J} \left(\sum_{i=1}^{n_j} Y_{ij}\right)^2 \qquad \left(\sum_{j=1}^{J} \sum_{i=1}^{n_j} Y_{ij}\right)^2$$

Telescoping Series

$$\sum_{k=1}^{n} \left(\frac{k}{k+1} - \frac{k+1}{k+2}\right)$$

$$\prod_{i=1}^{n} \left(\frac{i}{i+1}\right)$$

Properties

- Merging and Splitting

$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$	$\sum_{k=m}^{n} a_k = \sum_{k=m}^{i} a_k + \sum_{k=i+1}^{n} a_k$
$\prod_{k=m}^{n} a_k * \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (a_k * b_k)$	$\prod_{k=m}^{n} a_k = \prod_{k=m}^{i} a_k * \prod_{k=i+1}^{n} a_k$

- Distribution

$$c * \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} (c * a_k)$$

Factorial

- n! = n*(n-1)*(n-2)*…*2*1

- Definition
  0! = 1
  n! = n*(n-1)!