

CMSC 250 Homework 2 ANSWERS Spring 2006

Due Wed Feb 8 at the beginning of your discussion section.

1. Give the negations of the following statements using DeMorgan's Law.

- (a) Either cats or dogs are allowed here.
- (b) Either poodles or dobermans don't make good pets.
- (c) Fish or birds are not allowed, or dogs are allowed.

2. For each of the following statements, state "inverse," "converse," "contrapositive," or "none." I may have changed verb tenses to make so that the statements sound right, but tell if the form matches.

- (a) Converse.
- (b) Inverse.
- (c) Inverse.
- (d) None.
- (e) Converse.
- (f) Contrapositive.
- (g) None.

3. Write the complete truth table for each of the following:

(a)

p	q	r	$p \rightarrow q$	$\sim r$	$\sim r \vee p$	$(p \rightarrow q) \rightarrow (\sim r \vee p)$
1	1	1	1	0	1	1
1	1	0	1	1	1	1
1	0	1	0	0	1	1
1	0	0	0	1	1	1
0	1	1	1	0	0	0
0	1	0	1	1	1	1
0	0	1	1	0	0	0
0	0	0	1	1	1	1

(b)

a	b	$a \wedge b$	$a \vee b$	$(a \wedge b) \leftrightarrow (a \vee b)$
1	1	1	1	1
1	0	0	1	0
0	1	0	1	0
0	0	0	0	1

4. For each of the following state the one single rule from Theorem 1.1.1 (page 14) or Table 1.3.1 (Page 40) that could be used to go directly from the statement(s) to the conclusion, or “none” if there is no one single rule. (note: the answer is “none” even if a series of rules can be applied)

- (a) None.
- (b) None.
- (c) Modus Ponens.
- (d) None.
- (e) Modus Tollens.
- (f) Disjunctive Syllogism.
- (g) None.
- (h) None.

5. Use a truth table to determine if the following argument is valid or not. State whether or not it is valid, and give your reasons why it is valid or not.

p	q	r	$p \vee q$	$\sim r$	$\sim r \rightarrow q$	$\sim p$	$\sim p \rightarrow \sim r$	p
1	1	1	1	0	1	0	1	1
1	1	0	1	1	1	0	1	1
1	0	1	1	0	1	0	1	1
1	0	0	1	1	0	0	1	1
0	1	1	1	0	1	1	0	0
0	1	0	1	1	1	1	1	0
0	0	1	0	0	1	1	0	0
0	0	0	0	1	0	1	1	0

NOT VALID. All three premises are true in rows 1, 2, 3 and 6, but the conclusion is not true in row 6.

6. Use the rules of inference you were given to prove the following:

(a)	P1	$p \vee ((\sim q) \wedge r)$		
	P2	$\sim (p \wedge s)$		
	P3	s		
	S1	$\sim p \vee \sim s$	DeMorgan's laws	P2
	S2	$\sim (\sim s)$	Double negative	P3
	S3	$\sim p$	Disjunctive Syllogism	S1 and S2
	S4	$\sim q \wedge r$	Disjunctive Syllogism	S3 and P1
	S5	$r \wedge \sim q$	Commutative law	S4
	S6	$\sim (r \rightarrow q)$	Definition of Implication	S5

(b)	P1	$p \vee q$		
	P2	$\sim p \vee r$		
	P3	$\sim r \vee s$		
	S1	$q \vee p$	Commutative law	P1
	S2	$\sim q \rightarrow p$	Def. of Implication and D.Neg.	S1
	S3	$p \rightarrow r$	Definition of Implication	P2
	S4	$r \rightarrow s$	Definition of Implication	P3
	S5	$p \rightarrow s$	Hypothetical Syllogism	S3 and S4
	S6	$\sim q \rightarrow s$	Hypothetical Syllogism	S2 and S5
	S7	$q \vee s$	Def. of Implication and D.Neg.	S6

(c)	P1	$p \rightarrow r$		
	P2	$q \rightarrow s$		
	P3	$x \wedge y$		
	S1		Assume $p \wedge q$	Conditional World
	S2		p	Conjunctive Simplification
	S3		r	Modus Ponens
	S4		q	Conjunctive Simplification
	S5		s	Modus Ponens
	S6		$r \wedge s$	Conjunctive Addition
	S7	$(p \wedge q) \rightarrow (r \wedge s)$		Closing C.W. without contradiction
	S8	x		Conjunctive Simplification
	S9	$x \vee m$		Disjunctive Addition
	S10	$((p \wedge q) \rightarrow (r \wedge s)) \wedge (x \vee m)$		Conjunctive Addition