CMSC 250 Homework 5 Spring 2006

Due Wed March 1 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

For each of the following, state either that the statement is true or that the statement is false. Then give a complete, convincing proof to justify your answer. You may use any rules on the "cheat sheet" and any formal definitions as they appear in the textbook and as presented in class. For the purposes of this homework, you may also use the fact that all integers are either odd or even and are never both.

1. For all integers x, if x is an even integer then $x^2 + x$ is an even integer.
2. The sum of any two rational numbers is rational.
3. The product of any two irrational numbers is irrational.
4. $\forall x, y, z \in Z, ((x|y) \wedge (x|z)) \rightarrow (x|y + z)$
5. The sum of any two consecutive integers is odd.
6. For all integers greater than 3 (x), $4|x(6 * 2^x)$
7. $\forall n \in Z, n \in Z^{even} \rightarrow n + n^2 + n^3 \in Z^{even}$
8. $\forall x, y \in Z^{odd}, x - y \in Z^{even}$
9. $\forall n \in Z, n^2 - n > 0 \rightarrow n \neq 1$
10. If n is the sum of any four consecutive integers, $\exists k \in Z, n = 4k + 2$.
11. For all integers n, n is an even number if and only if $5n^2 + 12$ is even.
12. $\forall m \in Z, \exists k \in Z, m^2 = 5k \vee m^2 = 5k + 1 \vee m^2 = 5k + 4$.