CMSC 250 Homework 6 Spring 2006

Due Wed Mar 8 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

For each of the following statements, prove that the statement is true, or give a counterexample to show that it is false.

1. There is an even prime number larger than two.
2. There are an infinite number of composite numbers that are not perfect squares.
3. $\forall n \in \mathbf{Z}^{odd}$, $n^2 \equiv_8 1$.
4. $\exists n \in \mathbf{Z}, n^2 \equiv_8 5$.
5. There are an infinite number of pairs of integers $\{n, m\}$ such that $(n^2 + m^2) \equiv_8 5$.
6. $\forall n \in \mathbf{Z}$, n^3 is odd if and only if n is odd.
7. $\forall p \in \mathbf{Z}^{prime}$, $\sqrt[3]{p}$ is irrational.
8. $\forall n \in \mathbf{Z}^{\geq 2}$, $\sqrt[n]{7}$ is irrational.
9. Let $p, q \in \mathbf{Z}^{prime}$ such that $p \neq q$. Then $\forall n \in \mathbf{Z}$ such that $0 < n < pq$, it is not the case that $p|n$ and $q|n$.
10. Let $a, b, c \in \mathbf{Z}$ such that $a^3 = b^4 = c$, and let p be a prime number. If $p|a$, then $p^3|b$.