

CMSC 250 Homework 8 Spring 2003

Due Wed March 29 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

For each of the following statements say whether it is TRUE or FALSE. If true, use *mathematical induction* to prove it. If false, give a specific counter example.

1. $\forall n \in \mathbb{Z}^{\geq 1}, \sum_{i=1}^n (2i + 1) = (n + 1)^2 - 1.$

2. $\forall n \in \mathbb{Z}^{\geq 1}, 7 | 2^{3n} - 1.$

3. $\forall n \in \mathbb{Z}^+, \sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$

4. $\forall n \in \mathbb{Z}^{\geq 2}, 3 | 4^n + 1.$

5. $\forall n \in \mathbb{Z}^{\geq 3}, n^2 \geq 2n + 3$

6. $\forall n \in \mathbb{Z}^{\geq 0}, 2(n + 2) \leq (n + 2)^2$

7. $\forall n \in \mathbb{Z}^{\geq 2}, \sum_{k=2}^n \frac{1}{k-1} = \frac{n}{2}$

8. $\forall n \in \mathbb{Z}^{\geq 0}, 2 | (n + 2)!$