CMSC 250 Homework 9 Spring 2006

Due Wed Apr 5 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Use induction to prove or give a specific counter example to disprove each of the following statements:

   (a) Assume:

   $$a_1 = 3, \quad a_2 = 5, \quad a_3 = 1, \text{ and } \quad a_n = 2a_{n-1}1 + 3a_{n-2} + 4a_{n-3} \forall n \in Z \text{ where } n > 3$$

   Prove that

   $$\forall n \in Z^+, \quad a_n \in Z^{odd}$$

   (b) Suppose that e_0, e_1, e_2, ... is a sequence defined as follows:

   $$e_0 = 1, e_1 = 2, e_2 = 3,$$

   $$e_k = 2e_{k-1} + 4e_{k-2} + e_{k-3}$$

   for all integers $k \geq 3$.

   Prove that $e_n \leq 7^n$ for all integers $n \geq 0$.

2. Let $A = \{a, d, g, h\}$, $B = \{b, c, d, e\}$, $C = \{b, d, f, g\}$, $D = \{f, h\}$, $E\{e, g\}$. The Universal set $U = \{a, b, c, d, e, f, g, h\}$ Find the following:

   (a) $A \cap B =$

   (b) $A - C =$

   (c) $D \times E =$

   (d) $B^c =$

3. Show that $A \cap B \cup C$ is ambiguous by drawing Venn diagrams for $(A \cap B) \cup C$ and $A \cap (B \cup C)$ and showing that they are not the same.

4. Let $\Sigma = \{1, 2, 3\}$.

   (a) $\Sigma^0 =$

   (b) $\Sigma^1 =$

   (c) $\Sigma^2 =$