CMSC 250 Homework 10 Spring 2006

Due Wed April 12 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Prove each statement is true or find a specific counterexample. Assume all sets are subsets of a universal set U.

   (a) $\forall A, B \in \{sets\},\ A - B \neq B - A$

   (b) $\forall A, B, C \in \{sets\},\ (A \cup (B \cap C)) \subseteq (A \cup B \cup C)$

   (c) $\forall A, B, C \in \{sets\},\ (A - B) - C \subseteq (A - C)$

   (d) $\forall A, B, C \in \{sets\},\ (A \times B) \times C = A \times (B \times C)$

   (e) $\forall A, B, C \in \{sets\},\ A - (B \cup (A \cap C)) = (A - (B \cup C)) \cup (A \cap B \cap C)$

2. Find each of the following assuming $A = \{\{1\}, a\}$ and B=$\{$A,1$\}$

   (a) Find P(A)

   (b) Find P(B)

   (c) Answer the following questions:

      i. Is $A \in P(B)$?

      ii. Is $A \subseteq P(B)$?

      iii. Is $A \subseteq B$?

      iv. Is $A \in B$?

3. Let $A = \{a, b, c, d, e, f\}$, $A_1 = \{a, c\}$, $A_2 = \{b, d\}$, $A_3 = \{e, f\}$, $A_4 = \{a, c, d\}$.

   (a) Explain why $\{A_1, A_2, A_3, A_4\}$ does not form a partition of A.

   (b) Explain why $\{A_3, A_4\}$ does not forma a partition of A.

   (c) Explain why $\{A_1, A_2, A_3\}$ does form a partition of A.

   (d) Alter the set A_3 in two different ways so that the new collection of subsets $\{A_1, A_2, A_3\}$ fails to be a partition of A for a different reason for each of the alterations.

4. Your UMCP student ID is an 8 digit number.

   (a) How many different ID's are possible (assuming no other restrictions)?

   (b) How many different ID's are possible if there can be no 0's in the number?

   (c) How many different ID's are possible if the number can neither start nor end with a 0?

   (d) What is the probability that a ID chosen at random contains no 0's?