# CMSC 250 Homework 11 Fall 2005

## Due Wed Nov 16 at the beginning of your discussion section.

**You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.**

Answer the following questions about your pizza restaurant. You must explain in english what formula(s) you are using and why - then you must show all work before the answer in order to receive full credit.

The answer only needs to be taken to a form that includes only addition, subtraction, multiplication, division, factorials, and exponents. You do not need to evaluate those to a number.

1. Assume you have received 20 pizza orders that need to be delivered to 20 distinct houses. Answer all of the following questions.

   (a) If there were exactly 8 pepperoni pizzas that you have made to fill these orders, how many ways could they be distributed to the 20 pizza orders that are being delivered. Assume there are other types of pizzas ordered also, but you are not concerned with them for this one question. Also, it could be the case that all one household ordered all 8 pepperoni pizzas or any number of them.

   This is $r$-combinations with repetition allowed:
   There are 8 pepperoni pizza orders so $r = 8$, and 20 houses so $n = 20$.

   $$\binom{r+n-1}{r} = \binom{8+20-1}{8} = \binom{27}{8} = \frac{27!}{8!(27-8)!} = \frac{27!}{8!19!}$$

   (b) If you have exactly 4 drivers to deliver your pizzas, and each driver will carry exactly 5 orders at a time, how many different ways could you distribute the orders to the 4 drivers.

   There are 20! arrangements of the 20 pizza orders, and each driver has 5! possible arrangements of his/her pizza order so the total number of ways to distribute the orders is:

   $$\frac{20!}{(5!)^4}$$

   Alternatively, the first driver can get any 5 of the 20 pizza orders so the number of combinations is $\binom{20}{5}$. The second driver can get any 5 of the remaining 15 pizza orders so the number of combinations is $\binom{15}{5}$. The third driver can get any 5 of the remaining 10 pizza orders so the number of combinations is $\binom{10}{5}$. The fourth driver can get any 5 of the remaining 5 pizza orders so the number of combinations is $\binom{5}{5}$.
   The total number of ways to distribute the orders is the product of these values:

   $$\binom{20}{5}\binom{15}{5}\binom{10}{5}\binom{5}{5} = \frac{20!}{(5!)^4}$$

(c) If you have exactly one driver to deliver your pizzas, and you must give that driver the exact order in which the pizza orders must be delivered, how many different delivery plans could you give him?

This is just the number of permutations of the 20 pizza orders:

$$20!$$

(d) If each pizza order is for exactly one pizza, that each of the orders is for a pizza with a different topping combination, and the driver delivers exactly one pizza to each of the 20 houses (the correct houses) - what is the probability that the correct pizza got delivered to each of the 20 houses?

Only one of the arrangements is correct, so the probability is:

$$\frac{1}{20!}$$

(e) If your store has 10 different toppings available (besides cheese which is just included on all of the pizzas but does not count as a topping), and people order pizzas with any combination of these toppings (or none at all) - how many different pizza topping combinations are available? (Assume you can not order double or more of the same topping - either a topping is on that pizza or it is not.)

$$2^{10}$$

(f) If your store has 10 different toppings available, and the sale is for a 3-topping pizza - how many different 3 topping pizzas are available from your store. (Assume, the same as the previous question about double or more toppings.)

It is the number of ways of taking 10 toppings 3 at a time:

$$\left( \begin{array}{c} 10 \\ 3 \end{array} \right) = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!}$$

(g) If a person orders a pizza at random, what is the probability that it is a 3-topping pizza?

It is the number of 3-topping pizzas divided by the total number of pizzas.

$$\frac{\frac{10!}{3!7!}}{2^{10}}$$

2. Answer the following questions about the spring camp you are running for children who are out of school for spring break. Assume you have 30 children and 5 councelors. All of the children are distinguishable from each other, and all of the councelors are also distinguishable from each other.

   (a) How many ways can you divide the children into groups where one group has exactly one councelor? For this one assume the groups must all be of the same size.

   There are 30! arrangements of the 30 campers, and each group has 6! possible arrangements of campers, so the total number of possible groupings:

$$\frac{30!}{(6!)^5}$$

   Alternatively, the first councelor can get any 6 of the 30 campers, the second councelor can get any 6 of the remaining 24 campers, etc. The total number of ways to group the campers is:

$$\binom{30}{6}\binom{24}{6}\binom{18}{6}\binom{12}{6}\binom{6}{6} = \frac{30!}{(6!)^5}$$

   (b) You need to line up the children and councelors to board the bus for a field trip. They need to make one single file like. How many ways can this line be formed if there are no other restrictions?

   This is the number of permutations of the 30 campers plus 5 councelors:

$$35!$$

   (c) How many ways can you form the line in the previous question if there must be a councelor at the beginning of the line and a councelor at the end of the line and the other councelors are not attending the field trip (and so are not in the line at all).

   This is a 2-permutation of the 5 councelors and a permutation of the 30 campers:

$$P(5,2)30! = 5 * 4 * 30!$$

   Alternatively, you can place the councelors directly. The front of the line has a choice of the 5 councelors, and the back of the line has a choice of the remaining 4 councelors.

$$5 * 4 * 30!$$

   (d) How many ways can you form the line in the previous question if there must be a councelor at the beginning of the line and a councelor at the end of the line and the other councelors are mixed in with the children in the rest of the line.

   The front of the line has a choice of the 5 councelors, the back of the line has a choice of the remaining 4 councelors, and the remaining 33 people (30 camplers and 3 councelors) can be in any order.

$$5 * 4 * 33!$$

(e) You have been given 30 identical tee-shirts to give to the children (they are all about the same size) - how many ways can you distribute the tee-shirts so that each child gets exactly one tee-shirt? (The tee-shirts are indistinguishable from each other, but the children are distinguishable from each other.)

Every distribution of shirts looks the same so the answer is

$$1$$

(f) You have to put the children into two teams to play kick ball. How many different ways can you divide the 30 children onto the two teams assuming the teams must be the same size.

Each team must have 15 children. It is the number of ways of taking 30 children 15 at a time. Once one team is chosen the other team is automatically defined.

$$\binom{30}{15} = \frac{30!}{15!(30-15)!} = \frac{30!}{15!15!}$$

(g) You have your team of 15 kick ball players, and you must decide the order of who will kick which (while you are offense). How many different kicking orders could you make up?

It is just the number of ways to arrange the 15 players:

$$15!$$

(h) Assume that kickball is played with only a 6 person team - the others will have to watch from the bench. How many different ways could you select the 6 person team from the 15 you were given in the previous divide?

It is the number of ways of taking 15 children 6 at a time:

$$\binom{15}{6} = \frac{15!}{6!(15-6)!} = \frac{15!}{6!9!}$$

(i) Assume again that kickball is played with only a 6 person team and that the others will have to watch from the bench. How many different ways could you create a kicking order for your team?

This is a 6-permutation of 15 children.

$$P(15,6) = \frac{15!}{(15-6)!} = \frac{15!}{9!}$$

(j) Assume you have 30 different parents who have each come to pick up exactly one child. You give each parent exactly one child without concerning yourself with if the parent actually gets their own child. What is the probabiltity that each child gets to their own parent?
There are 30! possible arrangements of the parents (or equivalently children). Only one is correct:

$$\frac{1}{30!}$$