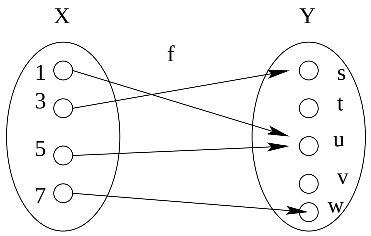
CMSC 250 Homework 13 Spring 2006

Due Wed May 3 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Let $X = \{1, 3, 5, 7\}$ and $Y = \{s, t, u, v, w\}$. Define $f : X \to Y$ by the following arrow diagram



   (a) write the domain and the co-domain of f.

   (b) what is the range of f?

   (c) what is the inverse image of s? Of u?

2. Define f: $R \to R$ by the rule

   $f(x) = 3x + 5$ for all $x \in R$.

   (a) Is f onto? Prove or give a counterexample.

   (b) Is f one-to-one? Prove or give a counterexample.

   (c) Find its inverse function or state why no such function can exist.

3. (a) How many one-to-one functions are there from a set with three elements to a set with four elements?

   (b) How many one-to-one functions are there from a set with six elements to a set with five elements?

   (c) How many onto functions are there from a set with three elements to a set with two elements?

   (d) How many onto functions are there from a set with four elements to a set with two elements?

4. (a) In a group of 687 people, must there be two who have same first name and last name initials? Why or why not?

   (b) If 5 distinct integers are chosen from between 2 and 10 inclusive, must at least 1 of them be a prime? Why or why not?

   (c) I have a bag filled with marbles. There are 15 red marbles, 9 blue marbles, 12 green marbles, 15 yellow marbles, 4 black marbles, and 3 white marbles. How many marbles would I have to pick out of the bag to guarantee that I have 5 marbles of the same color?

5. If each function F and G are defined by formulas find $G \circ F$ and $F \circ G$ and determine whether $G \circ F$ equals $F \circ G$

   (a) F(x)= x^2; G(x)= $x^2 + 1$

   (b) F(x)= x^3; G(x)= x^3.

6. Let $D = \{x \in \mathbf{Z} | 0 \leq x < 16\}$. Find examples of functions for each of the parts below. Find algebraic expressions for the functions. You may wish to use algebraic equations taken mod 16 to construct your functions. Do not list ordered pairs of elements, or the identity function for any of your answers.

   (a) Find $h : D \to D$ such that $h(h(h(x))) = h^{-1}(x)$.

   (b) Find $f : D \to D$ such that $\forall x \in D,\ f(x) = f(x+5)$, and f is not a constant function.

   (c) Find $g : D \to D$ such that $f(f(x)) = 0$, but $\exists x \in D,\ f(x) \neq 0$.