Name (PRINTED): _____

Student ID #: _____

Section # (or TA's name and time): _____

CMSC 250 Quiz #12 ANSWERS Wed, Apr. 19, 2006

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

Note: you only need to take the answer as far as a mathematical expression including addition, subtraction, multiplication, division, exponents and factorials.

1. [12 pnts.] Assume you have a set of beads. The set contains 100 beads where 57 of them are clear, 20 of them are red, 13 of them are green and 10 of them are purple. Assume two beads of the same color are indistinguishable from each other.

   a. If want you arrange all of the beads on the table in a single file line (assume none of them will roll off), how many ways could you do that?

   ANSWER:
   $\binom{100}{57}\binom{43}{20}\binom{23}{13}\binom{10}{10} = \frac{100!}{57!20!13!10!}$

   b. If you reach in and select 5 beads at random from the set (assuming they are all mixed up in a bag), what is the probability that you pull out all clear beads?

   ANSWER:
   $\frac{\binom{57}{5}}{\binom{100}{5}} = \frac{\frac{57!}{5!52!}}{\frac{100!}{95!5!}}$

   c. If you need to give all of the clear beads to the 6 children you have working on a project, but it does not matter if all have the same number or even if they all have any beads - how many ways can you distribute the clear beads?

   ANSWER:
   $\binom{57+(6-1)}{57} = \frac{(57+(6-1))!}{57!(6-1)!}$

   d. If you remove one bead at random and then put it back into the bag and then, again, remove a single bead at random, what is the probability that both have the same color?

   ANSWER:
   $\frac{57^2+20^2+13^2+10^2}{100^2}$

2. [18 pnts.] Assume you are playing with a single standard deck of playing cards. Answer the following questions. Reminder: A deck of cards has 13 values (A,2,3,4,5,6,7,8,9,J,Q and K) and 4 suits (hearts, diamonds, clubs and spades). Also, the hearts and diamonds are red while the clubs and spades are black.

   Note: you only need to take the answer as far as a mathematical expression including addition, subtraction, multiplication, division, exponents and factorials.

   a. If you draw one card at random, what is the probability that you get a card which is lower than the 4. (Assume the ace is considered low (as a 1).)

      ANSWER:
      $\frac{\binom{12}{1}}{\binom{52}{1}} = \frac{12}{52} = \frac{3}{13}$

   b. If you are dealt one five card hand, what is the probability that the lowest value for a card you have in your hand is a Ace (assume the ace is the lowest card value in the deck)?

      ANSWER:
      $1 - \frac{\binom{48}{5}}{\binom{52}{5}} = 1 - \frac{\frac{48!}{43!5!}}{\frac{52!}{47!5!}}$

   c. If you pick up that one five card hand you were already dealt, what is the probability that the cards appear (without your rearranging them) in ascending order. Here ascending order is defined as Ace through King and within a value: hearts are lowest, then diamonds, then spades, and lastly clubs. (Again, assume the ace is a 1 (only the lowest card).) (Note: you are answering this about the 5 card hand that the dealer has alreay dealt to you.)

      ANSWER:
      $\frac{1}{5!}$

   d. If you drop all of the cards of a full deck, what is the probability that they will all fall face down (so that you can't see the value of any of the cards)?

      ANSWER:
      $(\frac{1}{2})^{52}$

   e. If you deal all 52 cards to the four people sitting at the table using the proper method that the person to the left of the dealer gets the first card and then dealing continues clockwise, how many different ways could the cards be distributed to the 4 players (where each player gets 13 cards)?

      ANSWER:
      $\frac{52!}{(13!)^4}$