CMSC 250 Quiz #7 Wednesday, Mar. 8, 2006

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [20 pnts.] Disprove by counter example or Prove each of the following:

   a. The rational numbers are closed under division. (Note: This means "Dividing any rational by an rational must result in a rational number.")

   Let
   $a \in Q$, s.t. $a = \frac{c}{d}, c, d \in Z$
   $b \in Q$, s.t. $a = \frac{e}{f}, e, f \in Z$
   and that
   $def \neq 0$

   $\frac{a}{b} = \frac{\frac{c}{d}}{\frac{e}{f}} = \frac{cf}{de}$
   $cf, de \in Z$ by closure of integer over multiplication
   So $\frac{a}{b} \in Q$

   b. No perfect square has the form $2(2k+1)$.

   $$\forall n \in Z, n \in Z^{perfectsquare} \rightarrow \sim \exists k \in Z, n = 2(2k+1)$$

   $$\forall n \in Z, n \in Z^{perfectsquare} \rightarrow \forall k \in Z, n \neq 2(2k+1)$$

   $\forall n \in Z, \exists k, q \in Z$ s.t. $n = 4k + q, q = 0, 1, 2, 3$
   $n^2 \equiv_4 q^2$
   Case 1: $q = 0, n^2 \equiv_4 0$
   Case 2: $q = 1, n^2 \equiv_4 1$
   Case 3: $q = 2, n^2 \equiv_4 4 \equiv_4 0$
   Case 4: $q = 3, n^2 \equiv_4 9 \equiv_4 1$
   Thus, $\forall n \in Z, \sim (n^2 \equiv_4 2)$
   $\forall n \in Z, n \in Z^{perfectsquare} \rightarrow \sim (n \equiv_4 2)$
   $\forall n \in Z, n \in Z^{perfectsquare} \rightarrow \sim \exists k \in Z, n = 4k + 2$
   $\forall n \in Z, n \in Z^{perfectsquare} \rightarrow \sim \exists k \in Z, n = 2(2k+1)$

2. [4 pnts.] Write the standard factored form of 450: $450 = 2 * 3^2 * 5^2$

3. [6 pnts.] Use the unique factorization theorem and suppose that m is an integer such that

$$4 * 3 * 2 * m = 7 * 8 * 9 * 10$$

Circle Yes or No for each of the following: Yes means that this is something that must be true, No means it doesn't necessarily need to be true: $m = 7 * 3 * 10 = 2 * 3 * 5 * 7$

a)	$7 \mid m$	YES	
b)	$5 \mid m$	YES	
c)	$9 \mid m$		NO
d)	$3 \mid m$	YES	
e)	$15 \mid m$	YES	
f)	$2 \mid m$	YES	