Name (PRINTED): ______________________

Student ID #: ______________________

Section # (or TA's: name and time) ______________________

CMSC 250 Quiz #9 ANSWERS Wed., Mar 29, 2006

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly **20 minutes** to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [15 pnts.] Prove the following statement is true or give a specific counter example to prove that it is false.

$$\forall n \in Z^{\geq 4},\ \ 2^n \geq n^2$$

ANSWER:

Base Case: $(n = 4)$

RHS: $2^n = 2^4 = 16$

LHS: $n^2 = 4^2 = 16$

LHS $\geq$ RHS since $16 \geq 16$

Inductive Hypothesis: $(n = x)$

$2^x \geq x^2$

Inductive Step: $(n = x + 1)$

Show:

$2^{x+1} \geq (x + 1)^2$

Proof:

Part1: Find the b value

$2^x \geq x^2$ by the IH

$2^x + 2^x \geq x^2 + 2^x$ by adding 2^x to both sides

$2^{x+1} \geq x^2 + 2^x$ by algebra

Part 2:

show: $x^2 + 2^x \geq (x + 1)^2$

Proof:

Assume: $x^2 + 2^x < (x + 1)^2$

$x^2 + 2^x < x^2 + 2x + 1$

$2^x < 2x + 1$

Since 2^x is an exponentially increasing function that starts at 16 when $x = 4$

and $2x + 1$ is a linearly increasing function that starts at 9 when $x = 4$

Is it a Contradiction that $2^x < 2x + 1$

$x^2 + 2^x \geq (x + 1)^2$ by CCW with contradiction

Therefore, since part 1 showed that $2^{x+1} \geq x^2 + 2^x$

and part 2 showed that and part 2 showed that $x^2 + 2^x \geq (x + 1)^2$

we know by the transitive property that $2^{x+1} \geq (x + 1)^2$

↓ TURN OVER ↓

2. [15 pnts.] Prove the following statement is true or give a specific counter example to prove that it is false:

$$\forall n \in Z^+, \ \sum_{j=0}^{n}[3(2^j)] = 6(2^n) - 3$$

Base Case: $(n = 1)$

LHS: $\sum_{j=1}^{n}(3(2^j)) = \sum_{j=1}^{1}(3(2^j)) = (3(2^0)) + (3(2^1)) = 3 + 6 = 9$

RHS: $6(2^n) - 3 = 6(2^1) - 3 = 6(2) - 3 = 12 - 3 = 9$

RHS = LHS

Inductive Hypothesis: $(n = x)$

$\sum_{j=0}^{x}(3(2^j)) = 6(2^k) - 3$

Inductive Step: $(n = x + 1)$

Show:

$\sum_{j=0}^{x+1}(3(2^j)) = 6(2^{x+1}) - 3$

Proof:

$\sum_{j=0}^{x+1}(3(2^j)) = \sum_{j=0}^{x}(3(2^j)) + \sum_{j=x+1}^{x+1}(3(2^j))$ by splitting the summation

$= 6(2^x) - 3 + \sum_{j=x+1}^{x+1}(3(2^j))$ by the IH

$= 6(2^x) - 3 + 3(2^{x+1})$ by expanding the summation

$= 6(2^x) - 3 + 6(2^x)$ by algebra

$= (2)(6)(2^x) - 3$

$= 6(2^{x+1}) - 3$

QED